Pre-Calculus 12 Formula Sheet
ApexMath

1. Functions & Relations

Transformations of Functions: $af\big(b(x-h)\big)+k$

- Vertical stretch/compression: $af(x)$
- Horizontal stretch/compression: $f(bx)$
- Horizontal shift: $f(x-h)$
- Vertical shift: $f(x)+k$

Even and Odd Functions:

- **Even:** $f(-x)=f(x)$ — Symmetric about the y-axis.
- **Odd:** $f(-x)=-f(x)$ — Symmetric about the origin.

2. Trigonometry

Primary Ratios:

$$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}\qquad\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}\qquad\tan\theta=\frac{\text{opposite}}{\text{adjacent}}$$

Reciprocal Ratios:

$$\csc\theta=\frac{1}{\sin\theta}\qquad\sec\theta=\frac{1}{\cos\theta}\qquad\cot\theta=\frac{1}{\tan\theta}$$

Trigonometric Graph Transformations: $y=a\sin(bx-c)+d$

- a: amplitude
- b: affects period $\left(\text{Period}=\dfrac{2\pi}{|b|}\right)$
- c: phase shift (horizontal shift)
- d: vertical shift

Pythagorean Identity:

$$\sin^2\theta+\cos^2\theta=1$$

Sum and Difference Formulas:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angle Formulas:

$$\sin(2A) = 2 \sin A \cos A$$
$$\cos(2A) = \cos^2 A - \sin^2 A$$
$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

3. Polynomial Functions

Standard Form:

$$f(x) = a_n x^n + \cdots + a_1 x + a_0$$

Remainder Theorem:

- If $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.

4. Exponential & Logarithmic Functions

Exponential Form:

$$f(x) = a \cdot b^x, \qquad b > 0, \quad b \neq 1$$

Logarithm Definition:

$$\log_b(x) = y \quad \Longleftrightarrow \quad b^y = x$$

Laws of Logarithms:

$$\log_b(xy) = \log_b(x) + \log_b(y)$$
$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$
$$\log_b(x^n) = n \log_b(x)$$

Change of Base Formula:

$$\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$$

Exponential Growth / Decay:

$$A(t) = A_0\, e^{kt}$$

$$\text{Half-life:} \quad A = A_0 \left(\frac{1}{2}\right)^{t/h}$$

5. Sequences & Series

Arithmetic Sequence:

$$a_n = a_1 + (n-1)d \qquad S_n = \frac{n}{2}\big[2a_1 + (n-1)d\big]$$

Geometric Sequence:

$$a_n = a_1 \cdot r^{\,n-1} \qquad S_n = \frac{a_1(1-r^n)}{1-r}, \quad r \neq 1 \qquad S_\infty = \frac{a_1}{1-r}, \quad |r| < 1$$

Sigma Notation:

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + \cdots + f(b)$$

6. Conics

Parabola (Vertical Axis):

$$(x-h)^2 = 4p(y-k)$$

- Focus: $(h,\ k+p)$
- Directrix: $y = k - p$
- Axis of symmetry: $x = h$

Parabola (Horizontal Axis):

$$(y-k)^2 = 4p(x-h)$$

- Focus: $(h+p,\ k)$
- Directrix: $x = h - p$
- Axis of symmetry: $y = k$

Circle:

$$(x-h)^2 + (y-k)^2 = r^2 \qquad \text{Center: } (h,k), \quad \text{Radius: } r$$

Ellipse (Horizontal Major Axis): $a > b$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad c^2 = a^2 - b^2, \quad \text{Foci: } (h \pm c,\ k)$$

Ellipse (Vertical Major Axis): $a > b$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \qquad c^2 = a^2 - b^2, \quad \text{Foci: } (h,\ k \pm c)$$

Hyperbola (Horizontal Transverse Axis):

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad c^2 = a^2 + b^2, \quad \text{Foci: } (h \pm c,\ k)$$

Hyperbola (Vertical Transverse Axis):

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 \qquad c^2 = a^2 + b^2, \quad \text{Foci: } (h,\ k \pm c)$$

Eccentricity (all conics):

$$e = \frac{c}{a}$$

7. Rational Functions

Standard Form:

$$f(x) = \frac{p(x)}{q(x)}, \qquad q(x) \neq 0$$

Key Features:

- **Vertical Asymptote:** Set $q(x) = 0$ and solve. The function is undefined at these x-values.
- **Horizontal Asymptote:** Compare the degrees of $p(x)$ and $q(x)$:
  - $\deg(p) < \deg(q)$: $\quad y = 0$
  - $\deg(p) = \deg(q)$: $\quad y = \dfrac{\text{leading coeff of } p}{\text{leading coeff of } q}$
  - $\deg(p) > \deg(q)$: No horizontal asymptote
- **Oblique (Slant) Asymptote:** Occurs when $\deg(p) = \deg(q)+1$. Perform polynomial long division; the quotient (without remainder) is the oblique asymptote.
- **Hole (Removable Discontinuity):** Occurs when a factor cancels from both $p(x)$ and $q(x)$. Set the cancelled factor equal to zero to find the x-coordinate of the hole.
- **x-intercepts:** Set $p(x) = 0$ (after cancelling common factors).
- **y-intercept:** Evaluate $f(0)$, provided 0 is in the domain.

Basic Reciprocal Function:

$$f(x) = \frac{1}{x} \qquad \text{Domain: } x \neq 0 \qquad \text{Asymptotes: } x = 0,\ y = 0$$

Transformed Reciprocal:

$$f(x) = \frac{a}{b(x-h)} + k$$

- Vertical Asymptote: $x = h$
- Horizontal Asymptote: $y = k$

8. Inverses of Functions

Definition:

The inverse of a function $f(x)$, written $f^{-1}(x)$, reverses the input and output. If $f(a) = b$, then $f^{-1}(b) = a$.

How to Find an Inverse:

1. Replace $f(x)$ with y.

2. Swap x and y.

3. Solve for y.

4. Replace y with $f^{-1}(x)$.

Key Properties:

- The graph of $f^{-1}(x)$ is the reflection of $f(x)$ over the line $y = x$.
- $f\big(f^{-1}(x)\big) = x \quad \text{and} \quad f^{-1}\big(f(x)\big) = x$
- The domain of f^{-1} is the range of f, and vice versa.
- A function has an inverse that is also a function if and only if f passes the **Horizontal Line Test** (HLT).

Logarithm–Exponential Inverse Pair:

$$f(x) = b^x \quad \Longleftrightarrow \quad f^{-1}(x) = \log_b(x)$$

Radical–Power Inverse Pair:

$$f(x) = x^n \quad \Longleftrightarrow \quad f^{-1}(x) = x^{1/n} = \sqrt[n]{x} \qquad (x \geq 0 \text{ when } n \text{ is even})$$

9. Permutations & Combinations

Factorial:

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 \qquad 0! = 1$$

Permutations (order matters):

$$P(n,r) = \frac{n!}{(n-r)!}$$

The number of ways to arrange r items chosen from n distinct items.

Combinations (order does not matter):

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

The number of ways to choose r items from n distinct items without regard to order.

Fundamental Counting Principle:

If one event can occur in m ways and a second independent event can occur in n ways, then both events together can occur in $m \times n$ ways.

Permutations with Identical Elements:

If n objects are arranged and there are a identical objects of one type, b of another, etc.:

$$\text{Number of arrangements} = \frac{n!}{a!\,b!\,\cdots}$$

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}\, b^{k}$$

The general term (the $(k+1)$-th term) is:

$$T_{k+1} = \binom{n}{k}\, a^{n-k}\, b^{k}$$

Pascal's Triangle Identity:

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

Thank you for learning with ApexMath!

If you found this resource helpful, we'd love for you to share your feedback or leave a review.

Pre-Calculus 12 Formula Sheet • ApexMath
