Sequences & Series
Lesson 4: Applications of Sequences & Series

ApexMath

Notes

In real life, arithmetic and geometric sequences/series can model many problems:

- **Arithmetic:** Linear growth/decay (saving money by adding the same amount each month).
- **Geometric:** Exponential growth/decay (population growth, compound interest, bouncing ball heights).

Arithmetic Series Formula:

$$S_n = \frac{n}{2}(a_1 + a_n), \quad a_n = a_1 + (n-1)d$$

Geometric Series Formula:

$$S_n = a_1 \cdot \frac{1 - r^n}{1 - r}, \quad S_\infty = \frac{a_1}{1 - r} \quad (|r| < 1)$$

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Deep Walkthrough Examples

Example A. Saving money (Arithmetic)
You save \$50 the first month, then increase by \$10 each month. How much have you saved after 12 months?

Step 1. Identify: $a_1 = 50$, $d = 10$, $n = 12$.

Step 2. Find a_{12}:

$$a_{12} = 50 + (12 - 1)(10) = 160$$

Step 3. Use series formula:

$$S_{12} = \frac{12}{2}(50 + 160) = 1260$$

Final Answer: $\boxed{1260}$

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Example B. Compound interest (Geometric)
You invest \$1000 in a bank that gives 5% interest compounded yearly. How much after 10 years?

Step 1. Identify: $a_1 = 1000$, $r = 1.05$, $n = 10$.

Step 2. Formula for a_n:

$$a_{10} = 1000 \cdot (1.05)^9 \approx 1551.3$$

Step 3. Total value (sum of deposits with growth):

$$S_{10} = 1000 \cdot \frac{1 - (1.05)^{10}}{1 - 1.05} \approx 12578$$

Final Answer: $\boxed{12578}$

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Example C. Bouncing ball (Finite)
A ball is dropped from $h = 10$ m. Each bounce reaches $r = 0.6$ of the previous height. Find the total vertical distance traveled **after the 5th bounce**.

Step 1. Formula for distance after n bounces:

$$D = h + 2\left(\frac{a_1(1 - r^n)}{1 - r}\right)$$

Legend of Variables:

- D: total vertical distance after n bounces
- h: initial height (first drop)
- a_1: first bounce height ($a_1 = hr$)
- r: bounce ratio
- n: number of bounces

Step 2. Substitute values:

$$D = 10 + 2\left(\frac{10(0.6)(1 - (0.6)^5)}{1 - 0.6}\right)$$

Step 3. Simplify:

$$D = 10 + 2\left(\frac{6(1 - 0.07776)}{0.4}\right)$$

$$D = 10 + 2\left(\frac{6(0.92224)}{0.4}\right)$$

$$D = 10 + 2\left(\frac{5.53344}{0.4}\right)$$

$$D = 10 + 2(13.8336) = 10 + 27.6672 = 37.6672$$

Final Answer: $\boxed{37.67m}$

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Common Mistakes to Avoid

- Forgetting to double bounce heights (up + down) except for the first drop.
- Mixing arithmetic vs. geometric situations.
- Using finite formulas when $|r| < 1$ allows infinite sum.
- Confusing a_n (last term) with S_n (sum).

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Worksheet Problems

1. A person saves \$200 the first month and increases by \$25 each month. How much after 15 months?

2. A car depreciates by 12% each year from \$25,000. What is its value after 8 years?

3. A bouncing ball starts at 10 m and bounces to 60% of its height. Find the total distance after the 5th bounce.

4. A bouncing ball starts at 12 m and bounces to 50% of its height. Find the total distance traveled after the 3rd bounce.

5. An investor deposits \$500 at the start of each year for 10 years with 8% interest. What is the total amount?

6. A building has 30 floors. Each floor has 4 more windows than the floor below (floor 1 has 10 windows). How many total windows in the building?

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Detailed Answer Key

1. $a_1 = 200$, $d = 25$, $n = 15$.

$$a_{15} = 200 + (15 - 1)(25) = 550$$

$$S_{15} = \frac{15}{2}(200 + 550) = 5625$$

Final Answer: $\boxed{5625}$

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2. $a_1 = 25000$, $r = 0.88$, $n = 8$.

$$a_8 = 25000(0.88)^7 \approx 9837.5$$

Final Answer: $\boxed{9838}$

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3. $h = 10,\ r = 0.6,\ n = 5$.

$$D = h + 2\left(\frac{hr(1 - r^n)}{1 - r}\right)$$

$$D = 10 + 2\left(\frac{10(0.6)(1 - (0.6)^5)}{0.4}\right) = 37.67$$

Final Answer: $\boxed{37.67m}$

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4. $h = 12,\ r = 0.5,\ n = 3$.

$$D = 12 + 2\left(\frac{12(0.5)(1 - (0.5)^3)}{1 - 0.5}\right)$$

$$= 12 + 2\left(\frac{6(0.875)}{0.5}\right) = 12 + 21 = 33$$

Final Answer: $\boxed{33m}$

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5. $a_1 = 500$, $r = 1.08$, $n = 10$.

$$S_{10} = 500 \cdot \frac{1 - (1.08)^{10}}{1 - 1.08}$$

$$= 500 \cdot \frac{1 - 2.1589}{-0.08} = 7243$$

Final Answer: $\boxed{7243}$

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6. Arithmetic: $a_1 = 10$, $d = 4$, $n = 30$.

$$a_{30} = 10 + (29)(4) = 126$$

$$S_{30} = \frac{30}{2}(10 + 126) = 2040$$

Final Answer: $\boxed{2040}$

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