

Conics: Circles

Lesson 6: Understanding Circles

ApexMath

Notes

A circle is the set of all points equidistant from a fixed point called the center.

Standard form equation of a circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

- (h, k) = center of the circle - r = radius (must be positive)

Key points: - The distance from any point on the circle to the center is exactly r - Completing the square is often needed to rewrite a general form equation in standard form

Deep Walkthrough Examples

Example A: Standard form given

Equation: $(x - 3)^2 + (y + 2)^2 = 25$

Step 1: Compare with standard form

$$(x - h)^2 + (y - k)^2 = r^2$$

Step 2: Identify the center

$$h = 3, \quad k = -2 \implies \text{Center} = (3, -2)$$

Step 3: Identify the radius

$$r = \sqrt{25} = 5$$

Final Answer: $Center : (3, -2), Radius : 5$

Example B: Complete the square

Equation: $x^2 + y^2 - 6x + 8y + 9 = 0$

Step 1: Group terms

$$(x^2 - 6x) + (y^2 + 8y) + 9 = 0$$

Step 2: Complete the square

$$x^2 - 6x \rightarrow (x - 3)^2 - 9$$

$$y^2 + 8y \rightarrow (y + 4)^2 - 16$$

Step 3: Rewrite equation

$$(x - 3)^2 - 9 + (y + 4)^2 - 16 + 9 = 0 \implies (x - 3)^2 + (y + 4)^2 = 16$$

Step 4: Identify center and radius

$$\text{Center} = (3, -4), \quad r = \sqrt{16} = 4$$

Final Answer: $Center : (3, -4), Radius : 4$

Example C: Given center and radius

Center: (2,-3), Radius: 5

Step 1: Plug values into standard form

$$(x - 2)^2 + (y + 3)^2 = 5^2$$

Step 2: Simplify

$$(x - 2)^2 + (y + 3)^2 = 25$$

Final Answer: $(x - 2)^2 + (y + 3)^2 = 25$

Example D: Given center and a point on the circle

Center: (0,0), passes through (3,4)

Step 1: Compute the radius using distance formula

$$r = \sqrt{(3 - 0)^2 + (4 - 0)^2} = \sqrt{9 + 16} = 5$$

Step 2: Standard form

$$(x - 0)^2 + (y - 0)^2 = 5^2 \implies x^2 + y^2 = 25$$

Final Answer: $x^2 + y^2 = 25$

Common Mistakes to Avoid

- Forgetting to complete the square properly
 - Mixing up signs for the center (h, k)
 - Forgetting that r^2 must be positive
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Worksheet Problems

1. $(x + 2)^2 + (y - 5)^2 = 36$
 2. $x^2 + y^2 - 4x + 6y - 12 = 0$
 3. $(x - 1)^2 + (y + 3)^2 = 49$
 4. $x^2 + y^2 + 2x - 8y + 1 = 0$
 5. $(x + 4)^2 + (y - 1)^2 = 16$
 6. Center $(1, 2)$, radius 3 $\rightarrow$ find equation
 7. Center $(-2, 5)$, passes through $(1, 1) \rightarrow$ find equation
 8. Center $(0, -1)$, radius 4 $\rightarrow$ find equation
 9. Center $(5, 7)$, passes through $(2, 3) \rightarrow$ find equation
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Detailed Answer Key

1. $(x+2)^2+(y-5)^2 = 36$ Center: $(-2, 5)$, Radius: 6 **Final Answer:**

$Center : (-2, 5), Radius : 6$

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2. $x^2 + y^2 - 4x + 6y - 12 = 0$

Step 1: Group terms

$$(x^2 - 4x) + (y^2 + 6y) - 12 = 0$$

Step 2: Complete the square

$$x^2 - 4x \rightarrow (x - 2)^2 - 4, \quad y^2 + 6y \rightarrow (y + 3)^2 - 9$$

Step 3: Adjust constants and rewrite

$$(x-2)^2 - 4 + (y+3)^2 - 9 - 12 = 0 \implies (x-2)^2 + (y+3)^2 = 25$$

Center: (2,-3), Radius: 5 **Final Answer:** $Center : (2, -3), Radius : 5$

3. $(x-1)^2 + (y+3)^2 = 49$ Center: (1,-3), Radius: 7 **Final Answer:** $Center : (1, -3), Radius : 7$

4. $x^2 + y^2 + 2x - 8y + 1 = 0$

Step 1: Group terms

$$(x^2 + 2x) + (y^2 - 8y) + 1 = 0$$

Step 2: Complete the square

$$x^2 + 2x \rightarrow (x+1)^2 - 1, \quad y^2 - 8y \rightarrow (y-4)^2 - 16$$

Step 3: Rewrite equation

$$(x+1)^2 - 1 + (y-4)^2 - 16 + 1 = 0 \implies (x+1)^2 + (y-4)^2 = 16$$

Center: (-1,4), Radius: 4 **Final Answer:** $Center : (-1, 4), Radius : 4$

5. $(x+4)^2 + (y-1)^2 = 16$ Center: (-4,1), Radius: 4 **Final Answer:** $Center : (-4, 1), Radius : 4$

6. Center (1,2), radius 3 $\rightarrow (x-1)^2 + (y-2)^2 = 9$ **Final Answer:** $(x-1)^2 + (y-2)^2 = 9$

7. Center (-2,5), passes through (1,1) $\rightarrow$

Step 1: Compute radius

$$r = \sqrt{(1+2)^2 + (1-5)^2} = \sqrt{9+16} = 5$$

Step 2: Standard form

$$(x+2)^2 + (y-5)^2 = 25$$

Final Answer: $(x+2)^2 + (y-5)^2 = 25$

8. Center (0,-1), radius 4 $\rightarrow (x-0)^2 + (y+1)^2 = 16$ **Final Answer:** $x^2 + (y+1)^2 = 16$

9. Center (5,7), passes through (2,3) $\rightarrow$

Step 1: Compute radius

$$r = \sqrt{(2-5)^2 + (3-7)^2} = \sqrt{9+16} = 5$$

Step 2: Standard form

$$(x-5)^2 + (y-7)^2 = 25$$

Final Answer: $\boxed{(x-5)^2 + (y-7)^2 = 25}$

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