

Lesson 8: Parabolas

ApexMath

Notes

A **parabola** is the set of points equidistant from a fixed point (the *focus*) and a fixed line (the *directrix*).

Standard forms (use either form)

$$(x - h)^2 = 4p(y - k) \quad (\text{vertical parabola, opens up/down})$$

$$(y - k)^2 = 4p(x - h) \quad (\text{horizontal parabola, opens left/right})$$

Here p is the distance from the vertex to the focus (and to the directrix). If you prefer the a -form, note $a = \frac{1}{4p}$ so

$$y - k = a(x - h)^2 \iff (x - h)^2 = 4p(y - k), \quad p = \frac{1}{4a}.$$

Focus / Directrix formulas (centered at (h, k))

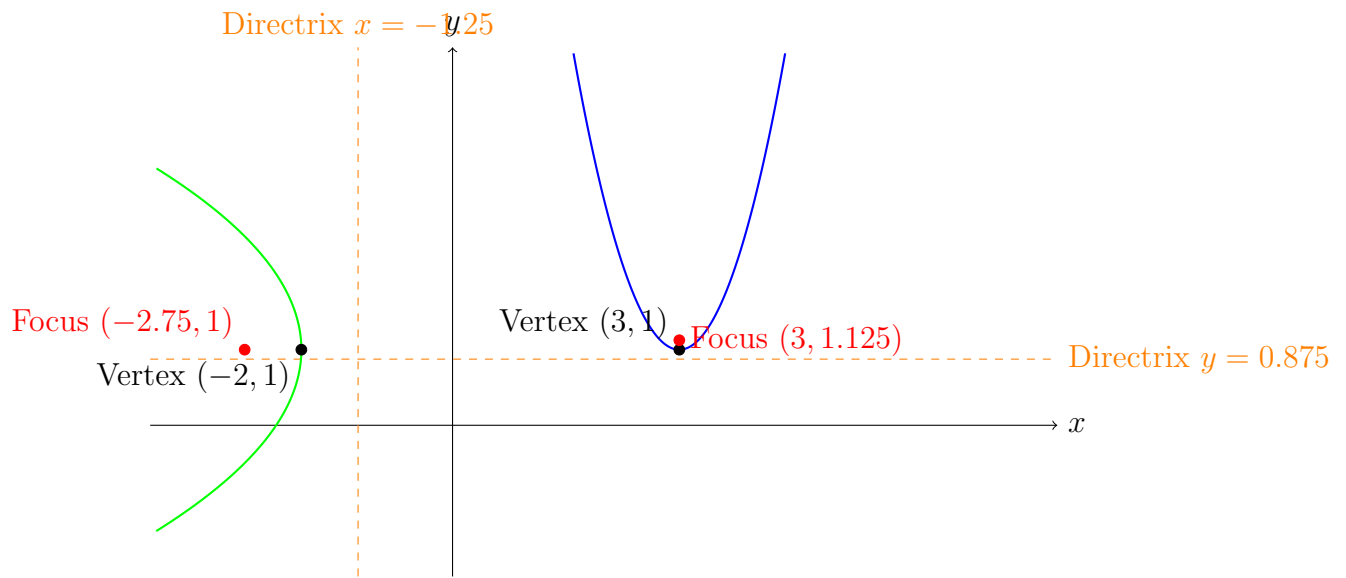
Vertical: focus $(h, k + p)$, directrix $y = k - p$.

Horizontal: focus $(h + p, k)$, directrix $x = h - p$.

Pro tips

- Check which variable is squared to decide orientation: $(x - h)^2$ vertical, $(y - k)^2$ horizontal.
- Use p (or $a = \frac{1}{4p}$) to place focus and directrix quickly.
- Sign of p (or sign of a) gives opening direction: positive $\Rightarrow$ up/right, negative $\Rightarrow$ down/left.

Visual example (in Notes only)



Deep Walkthrough Examples

Example A: Vertical parabola

Equation:

$$y - 1 = 2(x - 3)^2$$

Step 1. Identify vertex and coefficient:

$$\text{Vertex } (h, k) = (3, 1), \quad a = 2 \quad (\text{if you use } y - k = a(x - h)^2)$$

Step 2. Orientation:

Vertical (opens up) since the squared term is $(x - 3)^2$ and $a > 0$.

Step 3. Use $p = \frac{1}{4a}$ (or convert to $(x - h)^2 = 4p(y - k)$):

$$p = \frac{1}{4 \cdot 2} = \frac{1}{8} = 0.125$$

$$\text{Focus: } (h, k + p) = (3, 1.125)$$

$$\text{Directrix: } y = k - p = 1 - 0.125 = 0.875$$

Vertical parabola — Vertex (3, 1); Focus (3, 1.125); Directrix $y = 0.875$

Example B: Horizontal parabola

Equation:

$$x + 2 = -\frac{1}{3}(y - 1)^2$$

Step 1. Rewrite and identify vertex:

$$x - (-2) = -\frac{1}{3}(y - 1)^2 \Rightarrow \text{Vertex } (h, k) = (-2, 1), \quad a = -\frac{1}{3}.$$

Step 2. Orientation:

Horizontal (opens left) since $(y - 1)^2$ is squared and $a < 0$.

Step 3. Use $p = \frac{1}{4a}$:

$$p = \frac{1}{4(-1/3)} = -\frac{3}{4} = -0.75$$

$$\text{Focus: } (h + p, k) = (-2 - 0.75, 1) = (-2.75, 1)$$

$$\text{Directrix: } x = h - p = -2 - (-0.75) = -1.25$$

Horizontal parabola — Vertex $(-2, 1)$; Focus $(-2.75, 1)$; Directrix $x = -1.25$

Common Mistakes to Avoid

- Mixing up which variable is squared (decides orientation).
- Using the wrong sign for p (or a) — sign controls opening direction.
- Forgetting that $p = \frac{1}{4a}$ when you use $y - k = a(x - h)^2$.
- Swapping the focus and directrix positions accidentally.

Worksheet Problems

1. Identify the vertex, focus, and directrix:

$$(x - 1)^2 = 8(y + 2)$$

2. Identify the vertex, focus, and directrix:

$$(y + 3)^2 = -12(x - 4)$$

3. Identify the vertex, focus, and directrix:

$$y = (x - 2)^2 + 1$$

4. Convert to standard form and identify:

$$y^2 + 4y + 8x = 0$$

Detailed Answer Key

Problem 1

$$(x - 1)^2 = 8(y + 2)$$

Step 1. Vertex:

$$\text{Vertex } (1, -2)$$

Step 2. Identify p (since $4p=8$):

$$4p = 8 \Rightarrow p = 2$$

Step 3. Orientation:

Vertical (opens up)

Step 4. Focus and directrix:

$$\text{Focus: } (1, -2 + 2) = (1, 0)$$

$$\text{Directrix: } y = -2 - 2 = -4$$

Vertical parabola — Vertex $(1, -2)$; Focus $(1, 0)$; Directrix $y = -4$

Problem 2

$$(y + 3)^2 = -12(x - 4)$$

Step 1. Vertex:

$$\text{Vertex } (4, -3)$$

Step 2. Identify p (since $4p=-12$):

$$4p = -12 \Rightarrow p = -3$$

Step 3. Orientation:

Horizontal (opens left)

Step 4. Focus and directrix:

$$\text{Focus: } (4 - 3, -3) = (1, -3)$$

$$\text{Directrix: } x = 4 - (-3) = 7$$

Horizontal parabola — Vertex $(4, -3)$; Focus $(1, -3)$; Directrix $x = 7$

Problem 3

$$y = (x - 2)^2 + 1 \quad \Longrightarrow \quad (x - 2)^2 = y - 1$$

Step 1. Vertex:

Vertex $(2, 1)$

Step 2. Identify $4p=1 \Rightarrow p = \frac{1}{4}$, orientation vertical

$$\text{Focus: } (2, 1 + \frac{1}{4}) = (2, 1.25)$$

$$\text{Directrix: } y = 1 - \frac{1}{4} = 0.75$$

Vertical parabola — Vertex $(2, 1)$; Focus $(2, 1.25)$; Directrix $y = 0.75$

Problem 4

$$y^2 + 4y + 8x = 0$$

Step 1. Complete the square:

$$(y + 2)^2 = -8x + 4 \quad \Longrightarrow \quad (y + 2)^2 = -8\left(x - \frac{1}{2}\right)$$

Step 2. Vertex:

Vertex $(\frac{1}{2}, -2)$

Step 3. Identify $4p=-8 \Rightarrow p = -2$

Step 4. Focus and directrix:

$$\text{Focus: } (\tfrac{1}{2} - 2, -2) = (-\tfrac{3}{2}, -2)$$

$$\text{Directrix: } x = \tfrac{1}{2} - (-2) = \tfrac{5}{2}$$

Horizontal parabola — Vertex $(\frac{1}{2}, -2)$; Focus $(-\frac{3}{2}, -2)$; Directrix $x = \frac{5}{2}$

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