

ApexMath

Conics Review

Learning goals

By the end of this lesson you should be able to:

- Identify the type of conic from its general or standard equation.
- Convert general form equations to standard form by completing the square.
- Find key characteristics: center/vertex, axes, radii, foci, eccentricity, and asymptotes if applicable.
- Sketch each conic and write key formulas for each type.

Notes

Conic types and standard forms:

Circle (center (h, k) , radius r):

$$(x - h)^2 + (y - k)^2 = r^2$$

Ellipse horizontal:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1, \quad a > b > 0$$

Ellipse vertical:

$$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1, \quad a > b > 0$$

Hyperbola horizontal:

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Hyperbola vertical:

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

Parabola horizontal:

$$y - k = \frac{1}{4p}(x - h)^2$$

Parabola vertical:

$$x - h = \frac{1}{4p}(y - k)^2$$

Key formulas:

- Ellipse: $c^2 = a^2 - b^2$, eccentricity $e = c/a$
- Hyperbola: $c^2 = a^2 + b^2$, eccentricity $e = c/a > 1$
- Circle: $a = b = r$, $e = 0$
- Parabola: focus at distance p from vertex, directrix at distance p
- Hyperbola asymptotes: slope $\pm b/a$ (horizontal) or $\pm a/b$ (vertical)

Common mistakes

- Mixing formulas between ellipse and hyperbola. Fix: check the sign in the standard form.
- Choosing the wrong a^2 in ellipses/hyperbolas. Fix: a^2 is always under the “dominant” squared term.
- Forgetting to shift the center. Always include (h, k) in all formulas.
- Arithmetic errors when completing squares. Write constants explicitly and double-check.

Deep Walkthroughs

Example Circle

Equation:

$$(x + 1)^2 + (y - 2)^2 = 25$$

Identify center and radius:

$$(h, k) = (-1, 2), \quad r = \sqrt{25} = 5$$

Center: $(-1, 2)$, Radius: 5, Eccentricity: 0
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Example Ellipse from general form

Equation:

$$9x^2 + 16y^2 - 72x + 64y + 144 = 0$$

Group terms and complete squares:

$$9(x^2 - 8x) + 16(y^2 + 4y) = -144 \Rightarrow 9(x - 4)^2 + 16(y + 2)^2 = 64$$

Divide to get standard form:

$$\frac{(x - 4)^2}{64/9} + \frac{(y + 2)^2}{4} = 1$$

Compute parameters:

$$a^2 = 64/9 \Rightarrow a = 8/3, \quad b^2 = 4 \Rightarrow b = 2, \quad c^2 = a^2 - b^2 = 28/9 \Rightarrow c = \sqrt{28}/3$$

Center: $(4, -2)$, $a = 8/3$, $b = 2$, $c = \sqrt{28}/3$, Eccentricity ≈ 0.623
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Example Hyperbola from general form

Equation:

$$4y^2 - 9x^2 + 16y + 36x - 36 = 0$$

Group and complete squares:

$$4(y^2 + 4y) - 9(x^2 - 4x) - 36 = 0 \Rightarrow 4(y + 2)^2 - 9(x - 2)^2 - 16 = 0$$

Divide to get standard form:

$$\frac{(y + 2)^2}{4} - \frac{(x - 2)^2}{16/9} = 1$$

Compute parameters:

$$(h, k) = (2, -2), \quad a^2 = 4 \Rightarrow a = 2, \quad b^2 = 16/9 \Rightarrow b = 4/3, \quad c^2 = a^2 + b^2 = 52/9 \Rightarrow c = \sqrt{52}/3$$

Center: $(2, -2)$, $a = 2$, $b = 4/3$, $c = \sqrt{52}/3$, Eccentricity ≈ 1.202 , Asymptotes:
 $y + 2 = \pm 3/2(x - 2)$

Worksheet Problems

1. $x^2 + y^2 - 6x + 8y + 9 = 0$
2. $4x^2 - 9y^2 - 16x + 36y + 36 = 0$
3. $9x^2 + 25y^2 - 54x + 100y + 144 = 0$
4. $y^2 - 4y - 8x + 12 = 0$
5. $16x^2 - 25y^2 + 64x + 150y - 159 = 0$

Detailed Answer Key

Problem 1 Circle

Group and complete squares:

$$(x^2 - 6x) + (y^2 + 8y) = -9 \Rightarrow (x - 3)^2 + (y + 4)^2 = 16$$

Center: $(3, -4)$, Radius: 4, Eccentricity $e = 0$

Problem 2 Hyperbola

Group and complete squares:

$$4(x^2 - 4x) - 9(y^2 - 4y) + 36 = 0 \Rightarrow 4(x - 2)^2 - 9(y - 2)^2 = 4$$

Divide to get standard form:

$$\frac{(x - 2)^2}{1} - \frac{(y - 2)^2}{4/9} = 1$$

Parameters:

$$a^2 = 1 \Rightarrow a = 1, \quad b^2 = 4/9 \Rightarrow b = 2/3, \quad c^2 = a^2 + b^2 = 13/9 \Rightarrow c = \sqrt{13}/3$$

Asymptotes:

$$y - 2 = \pm \frac{b}{a}(x - 2) = \pm 2/3(x - 2)$$

Center: $(2, 2)$, $a = 1$, $b = 2/3$, $c = \sqrt{13}/3$, Eccentricity ≈ 1.201 , Asymptotes:
 $y - 2 = \pm 2/3(x - 2)$

Problem 3 Ellipse

Group and complete squares:

$$9(x^2 - 6x) + 25(y^2 + 4y) = -144 \Rightarrow 9(x - 3)^2 + 25(y + 2)^2 = 10$$

Divide to get standard form:

$$\frac{(x - 3)^2}{10/9} + \frac{(y + 2)^2}{10/25} = 1 \Rightarrow \frac{(x - 3)^2}{1.111} + \frac{(y + 2)^2}{0.4} = 1$$

Compute parameters:

$$a^2 \approx 1.111 \Rightarrow a \approx 1.054, \quad b^2 \approx 0.4 \Rightarrow b \approx 0.632, \quad c^2 = a^2 - b^2 \approx 0.711 \Rightarrow c \approx 0.843$$

Center: $(3, -2)$, $a \approx 1.054$, $b \approx 0.632$, $c \approx 0.843$, Eccentricity ≈ 0.8

Problem 4 Parabola

Complete the square:

$$y^2 - 4y - 8x + 12 = 0 \Rightarrow (y - 2)^2 = 8(x - 0.5)$$

Vertex, focus, directrix:

$$\text{Vertex}(0.5, 2), \text{Focus}(2.5, 2), \text{Directrix } x = -1.5, p = 2$$

Vertex: $(0.5, 2)$, Focus: $(2.5, 2)$, Directrix: $x = -1.5$, Conic Type: Parabola, $p = 2$

Problem 5 Hyperbola

Group and complete squares:

$$16(x^2 + 4x) - 25(y^2 - 6y) - 159 = 0 \Rightarrow 16(x + 2)^2 - 25(y - 3)^2 = -2$$

Multiply -1 and divide:

$$25(y - 3)^2 - 16(x + 2)^2 = 2 \Rightarrow \frac{(y - 3)^2}{2/25} - \frac{(x + 2)^2}{2/16} = 1$$

Parameters:

$$a^2 = 2/25 \Rightarrow a \approx 0.283, \quad b^2 = 2/16 \Rightarrow b \approx 0.354, \quad c^2 = a^2 + b^2 \approx 0.203 \Rightarrow c \approx 0.451$$

Asymptotes:

$$y - 3 = \pm a/b(x + 2) \approx \pm 0.8(x + 2)$$

Center: $(-2, 3)$, $a \approx 0.283$, $b \approx 0.354$, $c \approx 0.451$, Eccentricity ≈ 1.594 , Asymptotes
 $y - 3 \approx \pm 0.8(x + 2)$

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End of Lesson • ApexMath