

Sequences & Series

Lesson 1: Introduction to Sequences

ApexMath

Notes

A **sequence** is an ordered list of numbers that follow a certain pattern. The position is called the *term number* (n), and the value is called the *term* (a_n).

Types of sequences:

- **Arithmetic sequence:** Each term is obtained by adding a fixed number (d). General term:

$$a_n = a_1 + (n - 1)d$$

- **Geometric sequence:** Each term is obtained by multiplying by a fixed number (r). General term:

$$a_n = a_1 \cdot r^{n-1}$$

Key ideas:

1. Identify if the sequence changes by addition (d) or multiplication (r).
2. Write the general formula for a_n .
3. Use substitution to find terms or parameters.

Deep Walkthrough Examples

Example A. Arithmetic sequence

Sequence: 5, 8, 11, 14, ... Find the 15th term.

Step 1. Recognize type: Each term increases by 3. Arithmetic with $a_1 = 5, d = 3$.

Step 2. Apply formula:

$$a_n = a_1 + (n - 1)d$$

$$a_{15} = 5 + (15 - 1) \cdot 3$$

Step 3. Compute:

$$a_{15} = 5 + 14 \cdot 3 = 5 + 42 = 47$$

Final Answer: 47

Example B. Geometric sequence

Sequence: 2, 6, 18, 54, ... Find the 10th term.

Step 1. Recognize type: Each term multiplies by 3. Geometric with $a_1 = 2, r = 3$.

Step 2. Apply formula:

$$a_n = a_1 \cdot r^{n-1}$$

$$a_{10} = 2 \cdot 3^9$$

Step 3. Compute:

$$a_{10} = 2 \cdot 19683 = 39366$$

Final Answer: 39366

Example C. Mixed recognition

Sequence: 81, 27, 9, 3, ... Find the type and the 8th term.

Step 1. Recognize type: Each term divides by 3 (equivalently multiplies by $r = \frac{1}{3}$). So it is geometric with $a_1 = 81, r = \frac{1}{3}$.

Step 2. Apply formula:

$$a_n = 81 \cdot \left(\frac{1}{3}\right)^{n-1}$$

$$a_8 = 81 \cdot \left(\frac{1}{3}\right)^7$$

Step 3. Compute:

$$a_8 = 81 \cdot \frac{1}{2187} = \frac{81}{2187} = \frac{1}{27}$$

Final Answer: $\frac{1}{27}$

Common Mistakes to Avoid

- Confusing addition with multiplication (mixing arithmetic and geometric).
- Forgetting that n starts at 1 (not 0).
- Using r^n instead of r^{n-1} .
- Plugging wrong values of a_1 (always check the first term).

Worksheet Problems

1. Arithmetic: 4, 7, 10, ... Find the 20th term.
2. Geometric: 5, 10, 20, ... Find the 12th term.
3. Arithmetic: First term = 12, common difference -4 . Find a_{25} .
4. Geometric: First term = 81, ratio = $\frac{1}{3}$. Find a_6 .
5. Word problem: A teacher puts \$50 in a jar every month. Write the sequence for the first 5 months and the formula for the n^{th} month's contribution.
6. Arithmetic: A sequence starts at 15 and decreases by 7 each time. What is the 30th term?
7. Geometric: First term = 1, ratio = 2. Find the 15th term.
8. Word problem: A ball is dropped from 100 m and bounces back up to $\frac{3}{4}$ of its previous height each time. Write the first 4 terms of the sequence of bounce heights.
9. Arithmetic: Sequence: $-2, 1, 4, 7, \dots$. Find the explicit formula for a_n .
10. Geometric: Sequence: 16, 8, 4, 2, ... Find the explicit formula for a_n .

Detailed Answer Key

1. **Arithmetic:** 4, 7, 10, ... Find a_{20} .

Step 1. Difference check: $7 - 4 = 3$, $10 - 7 = 3$. Constant difference $\Rightarrow$ arithmetic.

Step 2. General formula:

$$a_n = a_1 + (n - 1)d = 4 + (n - 1) \cdot 3$$

Checkpoint: $a_2 = 4 + 1 \cdot 3 = 7$. Matches sequence

Step 3. Find a_{20} :

$$a_{20} = 4 + 19 \cdot 3 = 4 + 57 = 61$$

Final Answer: $a_n = 3n + 1$, $a_{20} = 61$

2. Geometric: $5, 10, 20, \dots$ Find a_{12} .

Step 1. Ratio check: $10/5 = 2$, $20/10 = 2$. Constant ratio $\Rightarrow$ geometric.

Step 2. General formula:

$$a_n = a_1 r^{n-1} = 5 \cdot 2^{n-1}$$

Checkpoint: $a_3 = 5 \cdot 2^2 = 20$. Matches sequence

Step 3. Find a_{12} :

$$a_{12} = 5 \cdot 2^{11} = 5 \cdot 2048 = 10240$$

Final Answer: $\boxed{a_n = 5 \cdot 2^{n-1}, \quad a_{12} = 10240}$

3. Arithmetic: $a_1 = 12, d = -4$. Find a_{25} .

Step 1. General formula:

$$a_n = 12 + (n - 1)(-4) = 16 - 4n$$

Step 2. Check formula with $n = 1$:

$$a_1 = 16 - 4(1) = 12$$

Correct

Step 3. Find a_{25} :

$$a_{25} = 16 - 4(25) = 16 - 100 = -84$$

Final Answer: $\boxed{a_n = 16 - 4n, \quad a_{25} = -84}$

4. Geometric: $a_1 = 81, r = \frac{1}{3}$. Find a_6 .

Step 1. General formula:

$$a_n = 81 \left(\frac{1}{3}\right)^{n-1}$$

Step 2. Check formula with $n = 2$:

$$a_2 = 81 \cdot \frac{1}{3} = 27$$

Matches

Step 3. Find a_6 :

$$a_6 = 81 \left(\frac{1}{3}\right)^5 = \frac{81}{243} = \frac{1}{3}$$

Final Answer: $\boxed{a_n = 81 \left(\frac{1}{3}\right)^{n-1}, \quad a_6 = \frac{1}{3}}$

5. Word problem: teacher deposits

Step 1. First 5 deposits:

50, 50, 50, 50, 50

Step 2. Recognize type: All terms are the same, so it is constant.

Step 3. Formula:

$$a_n = 50$$

Final Answer: 50, 50, 50, 50, 50 and $a_n = 50$

6. Arithmetic: starts at 15, decreases by 7.

Step 1. Parameters: $a_1 = 15$, $d = -7$.

Step 2. Formula:

$$a_n = 15 + (n - 1)(-7) = 22 - 7n$$

Checkpoint: $a_2 = 22 - 14 = 8$. Matches

Step 3. Find a_{30} :

$$a_{30} = 22 - 7(30) = 22 - 210 = -188$$

Final Answer: $a_n = 22 - 7n$, $a_{30} = -188$

7. Geometric: $a_1 = 1$, $r = 2$.

Step 1. Formula:

$$a_n = 2^{n-1}$$

Step 2. Check: $a_4 = 2^3 = 8$. Matches

Step 3. Find a_{15} :

$$a_{15} = 2^{14} = 16384$$

Final Answer: $a_n = 2^{n-1}$, $a_{15} = 16384$

8. Ball bounce heights.

Step 1. First height: 100.

Step 2. Multiply by $\frac{3}{4}$ each time:

100, 75, 56.25, 42.1875

Step 3. Formula:

$$a_n = 100 \left(\frac{3}{4}\right)^{n-1}$$

Final Answer: 100, 75, 56.25, 42.1875 and $a_n = 100 \left(\frac{3}{4}\right)^{n-1}$

9. Arithmetic: $-2, 1, 4, 7, \dots$

Step 1. Difference: $1 - (-2) = 3$.

Step 2. Formula:

$$a_n = -2 + (n - 1)(3) = 3n - 5$$

Step 3. Check: $a_2 = 3(2) - 5 = 1$. Matches

Step 4. Find a_{40} :

$$a_{40} = 3(40) - 5 = 120 - 5 = 115$$

Final Answer: $\boxed{a_n = 3n - 5, \quad a_{40} = 115}$

10. Geometric: $16, 8, 4, 2, \dots$

Step 1. Ratio: $8/16 = \frac{1}{2}$.

Step 2. Formula:

$$a_n = 16 \left(\frac{1}{2}\right)^{n-1}$$

Step 3. Check: $a_2 = 16 \cdot \frac{1}{2} = 8$. Matches

Step 4. Find a_{12} :

$$a_{12} = 16 \left(\frac{1}{2}\right)^{11} = \frac{16}{2048} = \frac{1}{128} = 0.0078125$$

Final Answer: $\boxed{a_n = 16 \left(\frac{1}{2}\right)^{n-1}, \quad a_{12} = \frac{1}{128}}$

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