

Sequences & Series

Lesson 2: Arithmetic Series & Sums

ApexMath

Notes

An **arithmetic sequence** is a list of numbers with a constant difference between consecutive terms.

$$a_n = a_1 + (n - 1)d$$

- a_1 : the first term
- d : the common difference
- n : the position (term number)
- a_n : the n^{th} term of the sequence

An **arithmetic series** is the sum of the first n terms of an arithmetic sequence.

$$S_n = \frac{n}{2}(2a_1 + (n - 1)d) \quad \text{or} \quad S_n = \frac{n}{2}(a_1 + a_n)$$

- S_n : the sum of the first n terms
- a_n : the n^{th} term (found using the term formula)

Key ideas:

1. Use $a_n = a_1 + (n - 1)d$ to find a specific term or to solve for n .
2. Use the series formulas to calculate sums once n , a_1 , and either d or a_n are known.
3. Always check whether the question asks for a term (a_n) or a sum (S_n).

Deep Walkthrough Examples

Example A. Simple sum

Find the sum of the first 20 terms of the sequence 3, 7, 11, 15, ...

Step 1. Identify parameters: $a_1 = 3$, $d = 4$, $n = 20$.

Step 2. Use term formula to find last term:

$$a_{20} = a_1 + (20 - 1)d = 3 + 19(4) = 79$$

Step 3. Apply series formula:

$$S_{20} = \frac{20}{2}(a_1 + a_{20}) = 10(3 + 79) = 10(82) = 820$$

Final Answer: 820

Example B. Using last term given

Find the sum of the arithmetic series $5 + 8 + 11 + \cdots + 50$.

Step 1. Identify parameters: $a_1 = 5$, $d = 3$, $a_n = 50$.

Step 2. Solve for n :

$$50 = 5 + (n - 1)(3) \quad \Rightarrow \quad 45 = 3(n - 1) \quad \Rightarrow \quad n = 16$$

Step 3. Apply sum formula:

$$S_{16} = \frac{16}{2}(a_1 + a_{16}) = 8(5 + 50) = 8(55) = 440$$

Final Answer: 440

Example C. Word problem

A worker's monthly salary starts at \$2000 and increases by \$150 each month. What is the total salary over the first 12 months?

Step 1. Identify parameters: $a_1 = 2000$, $d = 150$, $n = 12$.

Step 2. Find last term:

$$a_{12} = 2000 + (12 - 1)(150) = 2000 + 1650 = 3650$$

Step 3. Apply sum formula:

$$S_{12} = \frac{12}{2}(2000 + 3650) = 6(5650) = 33900$$

Final Answer: \$33,900

Common Mistakes to Avoid

- Forgetting to use the term formula before applying the sum formula.
 - Confusing a_n (a single term) with S_n (the sum).
 - Using $n - 1$ instead of n when calculating the sum.
 - Skipping units in word problems (e.g., money, seats, weeks).
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Worksheet Problems

1. Arithmetic: $2, 6, 10, \dots$. Find the 30^{th} term and the sum of the first 30 terms.
 2. Arithmetic: $7 + 10 + 13 + \dots + 100$. Find the number of terms and the sum.
 3. Sequence starts at 12 and increases by 8. Find a_{25} and the sum of the first 25 terms.
 4. A student saves \$5 the first week, \$10 the second week, and so on. Find the 20^{th} week's deposit and the total after 20 weeks.
 5. Sequence: $50, 47, 44, \dots$. Find the 40^{th} term and the sum of the first 40 terms.
 6. The n^{th} term of a sequence is $a_n = 4n + 1$. Find the first term, common difference, and the sum of the first 15 terms.
 7. Find the number of terms and the sum of $20 + 25 + 30 + \dots + 95$.
 8. A sequence starts at 100, and the sum of the first 12 terms is 780. Find the common difference.
 9. An auditorium has 25 seats in the first row, 28 in the second, 31 in the third, and so on. Find the seats in the 20th row and the total seats in the first 20 rows.
 10. A person earns \$100 in week 1, and each week earns \$20 more than the previous week. Find the earnings in week 15 and total earnings after 15 weeks.
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Detailed Answer Key

1. $a_1 = 2$, $d = 4$, $n = 30$.

$$a_{30} = 2 + (30 - 1)(4) = 2 + 116 = 118$$

$$S_{30} = \frac{30}{2}(2 + 118) = 15(120) = 1800$$

Final Answer: $a_{30} = 118$, $S_{30} = 1800$

2. $a_1 = 7$, $d = 3$, $a_n = 100$.

Find n :

$$100 = 7 + (n - 1)3 \Rightarrow n = 32$$

$$S_{32} = \frac{32}{2}(7 + 100) = 16(107) = 1712$$

Final Answer: $n = 32$, $S_{32} = 1712$

3. $a_1 = 12$, $d = 8$, $n = 25$.

$$a_{25} = 12 + (25 - 1)(8) = 12 + 192 = 204$$

$$S_{25} = \frac{25}{2}(12 + 204) = \frac{25}{2}(216) = 2700$$

Final Answer: $a_{25} = 204$, $S_{25} = 2700$

4. $a_1 = 5$, $d = 5$, $n = 20$.

$$a_{20} = 5 + (20 - 1)(5) = 5 + 95 = 100$$

$$S_{20} = \frac{20}{2}(5 + 100) = 10(105) = 1050$$

Final Answer: $a_{20} = 100$, $S_{20} = 1050$

5. $a_1 = 50$, $d = -3$, $n = 40$.

$$a_{40} = 50 + (40 - 1)(-3) = 50 - 117 = -67$$

$$S_{40} = \frac{40}{2}(50 + (-67)) = 20(-17) = -340$$

Final Answer: $a_{40} = -67$, $S_{40} = -340$

6. $a_n = 4n + 1$.

First term: $a_1 = 5$. Common difference: $d = 4$.

$$S_{15} = \frac{15}{2}(a_1 + a_{15})$$

$$a_{15} = 4(15) + 1 = 61$$

$$S_{15} = \frac{15}{2}(5 + 61) = \frac{15}{2}(66) = 495$$

Final Answer: $\boxed{a_1 = 5, d = 4, S_{15} = 495}$

7. $a_1 = 20, d = 5, a_n = 95$.

Find n :

$$95 = 20 + (n - 1)5 \Rightarrow n = 16$$

$$S_{16} = \frac{16}{2}(20 + 95) = 8(115) = 920$$

Final Answer: $\boxed{n = 16, S_{16} = 920}$

8. $a_1 = 100, S_{12} = 780, n = 12$.

$$S_{12} = \frac{12}{2}(200 + 11d) = 780 \Rightarrow 200 + 11d = 130$$

$$d = -\frac{70}{11}$$

Final Answer: $\boxed{d = -\frac{70}{11}}$

9. $a_1 = 25, d = 3, n = 20$.

$$a_{20} = 25 + (20 - 1)(3) = 25 + 57 = 82$$

$$S_{20} = \frac{20}{2}(25 + 82) = 10(107) = 1070$$

Final Answer: $\boxed{a_{20} = 82, S_{20} = 1070}$

10. $a_1 = 100, d = 20, n = 15$.

$$a_{15} = 100 + (15 - 1)(20) = 100 + 280 = 380$$

$$S_{15} = \frac{15}{2}(100 + 380) = \frac{15}{2}(480) = 3600$$

Final Answer: $\boxed{a_{15} = 380, S_{15} = 3600}$

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