

ApexMath

Sequences and Series Review

Learning goals

By the end of this lesson you should be able to:

- Identify arithmetic and geometric sequences and series.
- Compute n -th terms and partial sums.
- Apply formulas to word problems including bouncing ball scenarios.
- Recognize convergence/divergence of infinite geometric series.

Notes

Arithmetic Sequences and Series

- General term formula:

$$a_n = a_1 + (n - 1)d$$

- Sum of first n terms:

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequences and Series

- General term formula:

$$a_n = a_1 \cdot r^{n-1}$$

- Sum of first n terms:

$$S_n = a_1 \frac{1 - r^n}{1 - r}, \quad r \neq 1$$

- Infinite geometric series (convergent if $|r| < 1$):

$$S_\infty = \frac{a_1}{1 - r}$$

Bouncing Ball Problems

- The first drop counts once. Subsequent bounces are counted twice (up and down).
- Bounce heights form a geometric sequence:

$$h_n = h_1 \cdot r^{n-1}$$

- Total distance formula:

$$\text{Total distance} = h_0 + 2(h_1 + h_2 + \cdots + h_n)$$

Deep Walkthroughs

Example 1 Arithmetic Sequence

Question: The sequence 2, 5, 8, 11, ... continues. Find the 10th term and the sum of the first 10 terms.

Step 1: Identify the sequence type It is arithmetic with $a_1 = 2, d = 3$.

Step 2: Find the 10th term

$$a_{10} = 2 + (10 - 1) \cdot 3 = 29$$

10th term: 29

Step 3: Find the sum of the first 10 terms

$$S_{10} = \frac{10}{2}(2 + 29) = 155$$

Sum of first 10 terms: 155

Example 2 Geometric Sequence

Question: The sequence 3, 6, 12, 24, ... continues. Find the 8th term and the sum of the first 8 terms.

Step 1: Identify the sequence type It is geometric with $a_1 = 3, r = 2$.

Step 2: Find the 8th term

$$a_8 = 3 \cdot 2^7 = 384$$

8th term: 384

Step 3: Find the sum of first 8 terms

$$S_8 = 3 \cdot \frac{1 - 2^8}{1 - 2} = 765$$

Sum of first 8 terms: 765

Example 3 Infinite Geometric Series

Question: Find the sum to infinity of the series $5 + 2.5 + 1.25 + \dots$

Step 1: Identify values $a_1 = 5, r = 0.5$.

Step 2: Check convergence Since $|r| = 0.5 < 1$, series converges.

Step 3: Apply infinite sum formula

$$S_\infty = \frac{5}{1 - 0.5} = 10$$

Sum to infinity: 10

Example 4 Bouncing Ball

Question: A ball is dropped from 180 m, and each bounce reaches 75% of the previous height. Find the height after the 5th bounce, and the total distance travelled after the 7th bounce.

Step 1: Define first bounce and ratio First bounce height (after first hit) is $h_1 = 180 \cdot 0.75 = 135$. Set $r = 0.75$.

Step 2: Use the n -th term formula $h_n = h_1 \cdot r^{n-1}$ to find the 5th bounce height

$$h_5 = 135 \cdot 0.75^4 \approx 42.71 \text{ m}$$

Height after 5th bounce: 42.71 m

Step 3: Total distance after 7 bounces Compute the sum of the first 7 bounce heights:

$$\sum_{k=1}^7 h_k = h_1 \cdot \frac{1 - r^7}{1 - r} = 135 \cdot \frac{1 - 0.75^7}{1 - 0.75} \approx 467.92$$

Then total distance (first drop once, bounces twice):

$$\text{Distance} = 180 + 2 \cdot 467.9187 \dots \approx 1115.84 \text{ m}$$

Total distance after 7 bounces: 1115.84 m

Common Mistakes

- **Arithmetic vs. Geometric confusion:** Students often mix up the formulas. Remember: Arithmetic = add/subtract a constant (d) • Geometric = multiply/divide by a constant (r).
- **Forgetting parentheses in exponents:** For example, writing $a_8 = 3 \cdot 2^7$ incorrectly as $3 \cdot 2 \cdot 7$. Always evaluate powers first.
- **Infinite series errors:** Only use the infinite sum formula if $|r| < 1$. If $|r| \geq 1$, the series diverges (no sum exists).
- **Bouncing ball problems:** A common mistake is forgetting the first drop, or forgetting to double the bounces (since the ball travels up and down).
- **Mislabeling terms:** Students sometimes confuse the n -th term formula with the sum formula. Always double-check if the problem asks for a single term (a_n) or the total (S_n).
- **Sign mistakes with negative ratios:** In geometric sequences with negative r , the signs alternate. Missing this leads to wrong answers.

Worksheet

1. The sequence $7, 11, 15, \dots$ continues. Find the 12th term and the sum of the first 12 terms.
2. The sequence $2, 6, 18, \dots$ continues. Find the 6th term and the sum of the first 6 terms.
3. Find the sum to infinity of the geometric series $8, 4, 2, \dots$

4. A ball is dropped from 100 m and each bounce reaches 50% of the previous height. Find the height after the 4th bounce and the total distance travelled after the 6th bounce.
5. The sequence $3, -6, 12, \dots$ continues. Find the 7th term and the sum of the first 7 terms.
6. A farmer plants rows of trees. The first row has 5 trees, and each new row has 3 more trees than the row before. How many trees are in the 10th row and what is the total number of trees in 10 rows?
7. An investor invests \$1000 initially and doubles the investment each year. What will be the amount after 8 years, and what is the total amount accumulated over the 8 years?

Detailed Answer Key

Problem 1 Arithmetic

Step 1: Identify values $a_1 = 7, d = 4$.

Step 2: Find 12th term

$$a_{12} = 7 + (12 - 1) \cdot 4 = 51$$

12th term: 51

Step 3: Find sum of first 12 terms

$$S_{12} = \frac{12}{2}(7 + 51) = 348$$

Sum: 348

Problem 2 Geometric

Step 1: Identify values $a_1 = 2, r = 3$.

Step 2: Find 6th term

$$a_6 = 2 \cdot 3^5 = 486$$

6th term: 486

Step 3: Find sum of first 6 terms

$$S_6 = 2 \cdot \frac{1 - 3^6}{1 - 3} = 728$$

Sum: 728

Problem 3 Infinite Geometric

Step 1: Identify values $a_1 = 8, r = 0.5$.

Step 2: Check convergence $|r| = 0.5 < 1$ so converges.

Step 3: Apply formula

$$S_\infty = \frac{8}{1 - 0.5} = 16$$

Sum to infinity: 16

Problem 4 Bouncing Ball

Step 1: Identify values $h_0 = 100, h_1 = 100 \cdot 0.5 = 50, r = 0.5$.

Step 2: Find 4th bounce height

$$h_4 = 50 \cdot 0.5^3 = 6.25$$

Height after 4th bounce: 6.25 m

Step 3: Find total distance after 6 bounces

$$\sum_{k=1}^6 h_k = 50 \cdot \frac{1 - 0.5^6}{1 - 0.5} = 98.4375$$

$$\text{Distance} = 100 + 2(98.4375) = 296.875$$

Total distance after 6 bounces: 296.88 m

Problem 5 Geometric

Step 1: Identify values $a_1 = 3, r = -2$.

Step 2: Find 7th term

$$a_7 = 3 \cdot (-2)^6 = 192$$

7th term: 192

Step 3: Find sum of first 7 terms

$$S_7 = 3 \cdot \frac{1 - (-2)^7}{1 - (-2)} = 129$$

Sum: 129

Problem 6 Arithmetic Word Problem (Trees)

Step 1: Identify values $a_1 = 5, d = 3$.

Step 2: Find 10th term

$$a_{10} = 5 + (10 - 1) \cdot 3 = 32$$

Trees in 10th row: 32

Step 3: Find total in 10 rows

$$S_{10} = \frac{10}{2}(5 + 32) = 185$$

Total trees: 185

Problem 7 Geometric Word Problem (Investment)

Step 1: Identify values $a_1 = 1000, r = 2$.

Step 2: Find amount after 8 years

$$a_8 = 1000 \cdot 2^7 = 128,000$$

Amount after 8 years: \$128,000

Step 3: Find total amount

$$S_8 = 1000 \cdot \frac{1 - 2^8}{1 - 2} = 255,000$$

Total amount: \$255,000

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