Exponents & Logarithms Quiz
Answer Key
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1. Which of the following is an exponential function?

Explanation: An exponential function has the form $f(x) = a \cdot b^x$, where the *variable* is in the exponent and the *base* b is a positive constant ($b > 0$, $b \neq 1$). This is different from a power function like $f(x) = x^3$, where the variable is the base and the exponent is constant.

The function $f(x) = 3^x$ has a constant base (3) and a variable exponent (x). This is an exponential function.

$$\boxed{f(x) = 3^x}$$

2. Evaluate $f(x) = 2^x$ when $x = 5$.

Step 1: Substitute $x = 5$.

$$f(5) = 2^5$$

Step 2: Evaluate by repeated multiplication.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

$$\boxed{f(5) = 32}$$

3. What is the domain and range of $f(x) = 5^x$?

Domain: An exponential function b^x is defined for every real number x — there is no restriction on the exponent.

$$\text{Domain: all real numbers} \quad (-\infty,\ +\infty)$$

Range: The output of 5^x is always positive. No matter what x is, $5^x > 0$. The graph gets arbitrarily close to 0 (as $x \to -\infty$) but never reaches or goes below 0.

$$\text{Range: } y > 0 \quad (0,\ +\infty)$$

$$\boxed{\text{Domain: all reals, } \quad \text{Range: } y > 0}$$

4. Describe the transformation of $g(x) = 2^{(x-3)} + 2$ compared to $f(x) = 2^x$.

Explanation: Apply the standard transformation rules:

- Replacing x with $(x-3)$ inside the exponent shifts the graph **right by 3 units**. (Subtracting from x moves the graph to the right — this is counterintuitive but consistent with function transformation rules.)
- Adding $+2$ outside the exponential shifts the graph **up by 2 units**.

$$\boxed{\text{Shifted right 3 units and up 2 units}}$$

5. The population of a city grows according to $P(t) = 50{,}000(1.03)^t$. What is the population after 4 years?

Step 1: Substitute $t = 4$.

$$P(4) = 50{,}000 \times (1.03)^4$$

Step 2: Evaluate $(1.03)^4$.

$$(1.03)^2 = 1.0609 \qquad (1.03)^4 = (1.0609)^2 \approx 1.12551$$

Step 3: Multiply.

$$P(4) = 50{,}000 \times 1.12551 \approx 56{,}275$$

$$\boxed{P(4) \approx 56{,}275 \text{ people}}$$

6. What is the logarithmic form of $2^5 = 32$?

Explanation: Exponential and logarithmic forms are equivalent ways of expressing the same relationship. The conversion rule is:

$$b^y = x \iff \log_b(x) = y$$

Here $b = 2$, $y = 5$, and $x = 32$. Applying the rule:

$$2^5 = 32 \iff \log_2(32) = 5$$

$$\boxed{\log_2(32) = 5}$$

7. Evaluate $\log_3(27)$.

Step 1: The question asks: "to what power must 3 be raised to get 27?"

$$\log_3(27) = y \iff 3^y = 27$$

Step 2: Express 27 as a power of 3.

$$27 = 3^3$$

Step 3: Therefore $y = 3$.

$$\boxed{\log_3(27) = 3}$$

8. Evaluate $\log_{10}(1000)$.

Step 1: Ask: "to what power must 10 be raised to get 1000?"

$$\log_{10}(1000) = y \quad \iff \quad 10^y = 1000$$

Step 2: Express 1000 as a power of 10.

$$1000 = 10^3$$

Step 3: Therefore $y = 3$.

$$\boxed{\log_{10}(1000) = 3}$$

9. Evaluate $\ln(e^3)$.

Step 1: Recall that ln means $\log_e$, the natural logarithm with base $e \approx 2.71828$.
Step 2: Apply the inverse relationship between ln and e^x. Since logarithm and exponential are inverse functions:

$$\ln(e^y) = y \quad \text{for all real } y$$

Step 3: Therefore:

$$\ln(e^3) = 3$$

$$\boxed{\ln(e^3) = 3}$$

10. What is the domain and range of $f(x) = \log_2(x)$?

Domain: Logarithms are only defined for positive inputs. You cannot take the log of zero or a negative number.

$$\text{Domain: } x > 0 \quad (0,\ +\infty)$$

Range: The output of $\log_2(x)$ can be any real number. As $x \to 0^+$, $\log_2(x) \to -\infty$; as $x \to +\infty$, $\log_2(x) \to +\infty$.

$$\text{Range: all real numbers} \quad (-\infty,\ +\infty)$$

Note: the domain and range of $\log_2(x)$ are exactly the range and domain of 2^x, respectively — they are inverse functions.

$$\boxed{\text{Domain: } x > 0, \quad \text{Range: all real numbers}}$$

11. Simplify $\log_5(3) + \log_5(7)$ using logarithm properties.

Step 1: Apply the Product Rule for logarithms:

$$\log_b(M) + \log_b(N) = \log_b(M \cdot N)$$

Step 2: The bases are the same (both base 5), so the rule applies directly.

$$\log_5(3) + \log_5(7) = \log_5(3 \times 7) = \log_5(21)$$

$$\boxed{\log_5(21)}$$

12. Simplify $\log_3(18) - \log_3(2)$ using logarithm properties.

Step 1: Apply the Quotient Rule for logarithms:

$$\log_b(M) - \log_b(N) = \log_b\left(\frac{M}{N}\right)$$

Step 2: Apply with $M = 18$ and $N = 2$.

$$\log_3(18) - \log_3(2) = \log_3\left(\frac{18}{2}\right) = \log_3(9)$$

Step 3: Evaluate $\log_3(9)$ since $9 = 3^2$.

$$\log_3(9) = 2$$

$$\boxed{\log_3(9) = 2}$$

13. Simplify $3\log_2(4)$ using logarithm properties.

Step 1: Apply the Power Rule for logarithms:

$$n \cdot \log_b(M) = \log_b(M^n)$$

Step 2: Move the coefficient 3 up as the exponent of 4.

$$3\log_2(4) = \log_2(4^3) = \log_2(64)$$

Step 3: Evaluate. Since $64 = 2^6$:

$$\log_2(64) = 6$$

$$\boxed{\log_2(64) = 6}$$

14. Convert $\log_5(12)$ to natural logarithm form using the change of base formula.

Step 1: The Change of Base Formula allows any logarithm to be rewritten using any other base. Using natural log:

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$

Step 2: Apply with $b = 5$ and $x = 12$.

$$\log_5(12) = \frac{\ln(12)}{\ln(5)}$$

This is also equivalent to $\frac{\log(12)}{\log(5)}$ using common (base-10) logarithms. Both forms are correct.

$$\boxed{\log_5(12) = \frac{\ln(12)}{\ln(5)}}$$

15. Expand $\log_2(8x^3)$ using logarithm properties.

Step 1: Apply the Product Rule to separate the product inside the log.

$$\log_2(8x^3) = \log_2(8) + \log_2(x^3)$$

Step 2: Apply the Power Rule to $\log_2(x^3)$.

$$\log_2(x^3) = 3\log_2(x)$$

Step 3: Evaluate $\log_2(8)$. Since $8 = 2^3$, we have $\log_2(8) = 3$.
Step 4: Write the final expanded form.

$$\log_2(8x^3) = \log_2(8) + 3\log_2(x) = 3 + 3\log_2(x)$$

$$\boxed{\log_2(8) + 3\log_2(x)}$$

16. Condense $2\log_3(x) + \log_3(5)$ into a single logarithm.

Step 1: Apply the Power Rule to move the coefficient 2 up as an exponent.

$$2\log_3(x) = \log_3(x^2)$$

Step 2: Apply the Product Rule to combine the two logs.

$$\log_3(x^2) + \log_3(5) = \log_3(x^2 \cdot 5)$$

$$\boxed{\log_3(5x^2)}$$

17. Solve $2^x = 16$.

Step 1: Express 16 as a power of 2.

$$16 = 2^4$$

Step 2: Rewrite the equation with the same base on both sides.

$$2^x = 2^4$$

Step 3: Since the bases are equal, set the exponents equal.

$$x = 4$$

$$\boxed{x = 4}$$

18. Solve $3^{2x} = 27$.

Step 1: Express 27 as a power of 3.

$$27 = 3^3$$

Step 2: Rewrite the equation with the same base.

$$3^{2x} = 3^3$$

Step 3: Set the exponents equal.

$$2x = 3$$

Step 4: Divide both sides by 2.

$$x = \frac{3}{2} = 1.5$$

$$\boxed{x = 1.5}$$

19. Solve $5^x = 120$. Round to 3 decimal places.

Step 1: The bases cannot be matched easily, so take the natural logarithm of both sides.

$$\ln(5^x) = \ln(120)$$

Step 2: Apply the Power Rule for logarithms to bring the exponent down.

$$x \ln(5) = \ln(120)$$

Step 3: Isolate x by dividing both sides by $\ln(5)$.

$$x = \frac{\ln(120)}{\ln(5)}$$

Step 4: Evaluate numerically.

$$x = \frac{\ln(120)}{\ln(5)} \approx \frac{4.7875}{1.6094} \approx 2.975$$

$$\boxed{x \approx 2.975}$$

20. Solve $\log_2(x) = 5$.

Step 1: Convert from logarithmic form to exponential form using the equivalence $\log_b(x) = y \Leftrightarrow b^y = x$.

$$\log_2(x) = 5 \quad \Longrightarrow \quad x = 2^5$$

Step 2: Evaluate.

$$x = 2^5 = 32$$

$$\boxed{x = 32}$$

21. Solve $\log_3(x + 2) = 2$.

Step 1: Convert from logarithmic form to exponential form.

$$\log_3(x + 2) = 2 \quad \Longrightarrow \quad 3^2 = x + 2$$

Step 2: Evaluate the left side.

$$9 = x + 2$$

Step 3: Subtract 2 from both sides.

$$x = 7$$

Step 4: Check that the argument is positive: $x + 2 = 9 > 0$. ✓

$$\boxed{x = 7}$$

22. Solve $\log_2(x) + \log_2(4) = 5$.

Step 1: Apply the Product Rule to combine the left side into a single logarithm.

$$\log_2(x \cdot 4) = 5 \qquad \Rightarrow \qquad \log_2(4x) = 5$$

Step 2: Convert to exponential form.

$$4x = 2^5 = 32$$

Step 3: Divide both sides by 4.

$$x = \frac{32}{4} = 8$$

Step 4: Check: $\log_2(8) + \log_2(4) = 3 + 2 = 5$. ✓

$$\boxed{x = 8}$$

23. A bacteria culture doubles every 3 hours. Starting with 100 bacteria, how long until there are 3200?

Step 1: Write the doubling model. After n doublings, there are 100×2^n bacteria.

$$100 \times 2^n = 3200$$

Step 2: Divide both sides by 100.

$$2^n = 32$$

Step 3: Express 32 as a power of 2.

$$32 = 2^5 \qquad \Rightarrow \qquad n = 5$$

Step 4: Each doubling takes 3 hours. Convert the number of doublings to time.

$$t = 5 \times 3 = 15 \text{ hours}$$

$$\boxed{15 \text{ hours}}$$

24. If $\text{pH} = -\log_{10}[\text{H}^+]$ is 7, what is the hydrogen ion concentration $[\text{H}^+]$?

Step 1: Substitute $\text{pH} = 7$ into the formula.

$$7 = -\log_{10}[\text{H}^+]$$

Step 2: Multiply both sides by -1.

$$-7 = \log_{10}[\text{H}^+]$$

Step 3: Convert from logarithmic form to exponential form.

$$[\text{H}^+] = 10^{-7}$$

$$\boxed{[\text{H}^+] = 10^{-7} \text{ mol/L}}$$

25. Which statement is TRUE?

Statement: If $\log_5(x) = 2$, then $x = 25$.

Verification: Convert from logarithmic form to exponential form.

$$\log_5(x) = 2 \quad \Longrightarrow \quad x = 5^2 = 25$$

This is correct. The logarithm asks "to what power must 5 be raised to get x?" The answer is 2, and $5^2 = 25$.

$$\boxed{\text{If } \log_5(x) = 2, \text{ then } x = 25}$$

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