Functions & Relations Quiz
Answer Key
ApexMath

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1. Which of the following is a function?

Explanation: A relation is a function if and only if every x-value appears exactly once — no input maps to more than one output. The set $\{(1,2),\ (2,3),\ (3,4)\}$ has three distinct x-values: 1, 2, and 3, each appearing exactly once. Every input has a unique output, so this is a function.

$$\boxed{\{(1,\ 2),\ (2,\ 3),\ (3,\ 4)\}}$$

2. What is the domain of $f(x) = \sqrt{x-5}$?

Step 1: The square root of a negative number is not a real number, so the expression inside the radical must be greater than or equal to zero.

$$x - 5 \geq 0$$

Step 2: Solve the inequality by adding 5 to both sides.

$$x \geq 5$$

$$\boxed{x \geq 5}$$

3. What is the domain of $f(x) = \dfrac{1}{x^2 - 16}$?

Step 1: A fraction is undefined when its denominator equals zero. Set the denominator equal to zero and solve.

$$x^2 - 16 = 0$$

Step 2: Factor as a difference of squares.

$$(x-4)(x+4) = 0$$

Step 3: Solve for x.

$$x = 4 \qquad \text{or} \qquad x = -4$$

Step 4: Exclude both values from the domain.

$$\boxed{x \in \mathbb{R}, \quad x \neq \pm 4}$$

4. If $f(x) = 3x^2 - 2x + 5$, find $f(3)$.

Step 1: Replace every x with 3. Use brackets to stay organized.

$$f(3) = 3(3)^2 - 2(3) + 5$$

Step 2: Evaluate each term.

$$3(9) - 6 + 5 = 27 - 6 + 5$$

Step 3: Combine.

$$= 26$$

$$\boxed{f(3) = 26}$$

5. If $g(x) = 2x + 1$, find $g(a - 2)$.

Step 1: Replace every x with the expression $(a - 2)$.

$$g(a - 2) = 2(a - 2) + 1$$

Step 2: Distribute the 2.

$$= 2a - 4 + 1$$

Step 3: Combine the constants.

$$= 2a - 3$$

$$\boxed{g(a - 2) = 2a - 3}$$

6. If $f(x) = x^2 + 3$ and $g(x) = 2x - 1$, find $(f + g)(x)$.

Step 1: The combined function $(f + g)(x)$ means simply add the two functions together.

$$(f + g)(x) = f(x) + g(x) = (x^2 + 3) + (2x - 1)$$

Step 2: Remove the brackets and collect like terms.

$$= x^2 + 2x + 3 - 1 = x^2 + 2x + 2$$

$$\boxed{(f + g)(x) = x^2 + 2x + 2}$$

7. If $f(x) = 5x + 2$ and $g(x) = x + 3$, find $(f - g)(x)$.

Step 1: Subtract $g(x)$ from $f(x)$. Be careful to distribute the negative sign across all terms of $g(x)$.

$$(f-g)(x) = f(x) - g(x) = (5x+2) - (x+3)$$

Step 2: Distribute the negative and collect like terms.

$$= 5x + 2 - x - 3 = 4x - 1$$

$$\boxed{(f-g)(x) = 4x - 1}$$

8. If $f(x) = x + 2$ and $g(x) = x - 3$, find $(f \cdot g)(x)$.

Step 1: Multiply the two functions together.

$$(f \cdot g)(x) = (x+2)(x-3)$$

Step 2: Expand using FOIL (First, Outer, Inner, Last).

$$= x^2 - 3x + 2x - 6$$

Step 3: Combine like terms.

$$= x^2 - x - 6$$

$$\boxed{(f \cdot g)(x) = x^2 - x - 6}$$

9. If $f(x) = x^2 - 9$ and $g(x) = x - 3$, find $\left(\frac{f}{g}\right)(x)$ for $x \neq 3$.

Step 1: Write the quotient.

$$\left(\frac{f}{g}\right)(x) = \frac{x^2 - 9}{x - 3}$$

Step 2: Factor the numerator as a difference of squares.

$$= \frac{(x-3)(x+3)}{x-3}$$

Step 3: Cancel the common factor $(x-3)$, valid since $x \neq 3$.

$$= x + 3$$

$$\boxed{\left(\frac{f}{g}\right)(x) = x + 3, \quad x \neq 3}$$

10. If $f(x) = 2x + 1$ and $g(x) = x^2$, find $(f \circ g)(x)$.

Step 1: The composition $(f \circ g)(x)$ means $f(g(x))$ — substitute $g(x)$ into f in place of x.

$$(f \circ g)(x) = f(g(x)) = f\left(x^2\right)$$

Step 2: Replace every x in $f(x) = 2x + 1$ with x^2.

$$= 2(x^2) + 1 = 2x^2 + 1$$

$$\boxed{(f \circ g)(x) = 2x^2 + 1}$$

11. If $f(x) = 3x - 2$ and $g(x) = x + 4$, find $(f \circ g)(2)$.

Step 1: Work from the inside out. First evaluate $g(2)$.

$$g(2) = 2 + 4 = 6$$

Step 2: Now substitute the result into f.

$$f(g(2)) = f(6) = 3(6) - 2 = 18 - 2 = 16$$

$$\boxed{(f \circ g)(2) = 16}$$

12. If $f(x) = 2x + 3$, what is $f^{-1}(x)$?

Step 1: Replace $f(x)$ with y.

$$y = 2x + 3$$

Step 2: Swap x and y to reflect across the line $y = x$.

$$x = 2y + 3$$

Step 3: Solve for y. Subtract 3 from both sides.

$$x - 3 = 2y$$

Step 4: Divide both sides by 2.

$$y = \frac{x - 3}{2}$$

$$\boxed{f^{-1}(x) = \frac{x - 3}{2}}$$

13. If $f(x) = \dfrac{x}{3} - 2$ and $f^{-1}(x) = 3(x + 2)$, find $f(f^{-1}(4))$.

Step 1: Recognize that f and f^{-1} are inverse functions. By the definition of inverse functions, applying a function and then its inverse (or vice versa) always returns the original input.

$$f\big(f^{-1}(x)\big) = x \quad \text{for all valid } x$$

Step 2: Apply this property directly with $x = 4$.

$$f\big(f^{-1}(4)\big) = 4$$

We can verify: $f^{-1}(4) = 3(4 + 2) = 18$, then $f(18) = \dfrac{18}{3} - 2 = 6 - 2 = 4$. ✓

$$\boxed{f\big(f^{-1}(4)\big) = 4}$$

14. What is the domain and range of $f(x) = \sqrt{x+2} - 3$?

Domain:
Step 1: The expression inside the radical must be ≥ 0.

$$x + 2 \geq 0 \qquad \Rightarrow \qquad x \geq -2$$

Range:
Step 2: The square root function $\sqrt{x+2}$ always produces values ≥ 0. The minimum output of $\sqrt{x+2}$ is 0, which occurs at $x = -2$.
Step 3: Subtract 3 from that minimum.

$$f(-2) = \sqrt{0} - 3 = -3$$

Since $\sqrt{x+2} \geq 0$, we have $f(x) = \sqrt{x+2} - 3 \geq -3$.

$$\boxed{\text{Domain: } x \geq -2 \qquad \text{Range: } y \geq -3}$$

15. Simplify $\sqrt[3]{27x^3y^6}$.

Step 1: Separate the cube root across each factor.

$$\sqrt[3]{27x^3y^6} = \sqrt[3]{27} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{y^6}$$

Step 2: Evaluate each cube root individually.

$$\sqrt[3]{27} = 3 \qquad \sqrt[3]{x^3} = x \qquad \sqrt[3]{y^6} = y^{6/3} = y^2$$

Step 3: Multiply the results.

$$= 3 \cdot x \cdot y^2 = 3xy^2$$

$$\boxed{3xy^2}$$

16. Factor $4x^3 + 8x^2 - 12x$ completely.

Step 1: Identify and factor out the greatest common factor (GCF). Each term shares a factor of $4x$.

$$4x^3 + 8x^2 - 12x = 4x(x^2 + 2x - 3)$$

Step 2: Factor the trinomial $x^2 + 2x - 3$. Find two numbers that multiply to -3 and add to $+2$. Those numbers are $+3$ and -1.

$$x^2 + 2x - 3 = (x+3)(x-1)$$

Step 3: Write the fully factored form.

$$4x^3 + 8x^2 - 12x = 4x(x+3)(x-1)$$

$$\boxed{4x(x+3)(x-1)}$$

17. Factor $x^2 - 7x + 12$.

Step 1: Find two numbers that multiply to $+12$ (the constant term) and add to -7 (the coefficient of x).

The pair that works is -3 and -4, since $(-3)(-4) = 12$ and $(-3) + (-4) = -7$.

Step 2: Write the factored form.

$$x^2 - 7x + 12 = (x-3)(x-4)$$

$$\boxed{(x-3)(x-4)}$$

18. Factor $x^3 - 2x^2 - 9x + 18$ by grouping.

Step 1: Split the four terms into two pairs.

$$\left(x^3 - 2x^2\right) + (-9x + 18)$$

Step 2: Factor out the GCF from each pair.

$$x^2(x-2) - 9(x-2)$$

Step 3: Factor out the common binomial $(x-2)$.

$$= (x-2)(x^2-9)$$

Step 4: Factor $x^2 - 9$ as a difference of squares.

$$= (x-2)(x-3)(x+3)$$

$$\boxed{(x-2)(x-3)(x+3)}$$

19. Divide $x^3 + 5x^2 + 2x - 8$ by $(x+1)$. State the quotient and remainder.

Step 1: Use polynomial long division. Divide the leading term of the dividend by the leading term of the divisor.

$$x^3 \div x = x^2$$

Step 2: Multiply x^2 by $(x+1)$ and subtract.

$$x^3 + 5x^2 + 2x - 8 - x^2(x+1) = x^3 + 5x^2 + 2x - 8 - \left(x^3 + x^2\right) = 4x^2 + 2x - 8$$

Step 3: Repeat. Divide $4x^2$ by x.

$$4x^2 \div x = 4x$$

Multiply $4x$ by $(x+1)$ and subtract.

$$4x^2 + 2x - 8 - 4x(x+1) = 4x^2 + 2x - 8 - (4x^2 + 4x) = -2x - 8$$

Step 4: Repeat. Divide $-2x$ by x.

$$-2x \div x = -2$$

Multiply -2 by $(x+1)$ and subtract.

$$-2x - 8 - (-2)(x+1) = -2x - 8 - (-2x - 2) = -6$$

Step 5: The process ends because the degree of the remainder (-6) is less than the degree of the divisor. Collect the results.

$$\text{Quotient: } x^2 + 4x - 2 \qquad \text{Remainder: } -6$$

$$\boxed{\text{Quotient: } x^2 + 4x - 2, \quad \text{Remainder: } -6}$$

20. Find the remainder when $f(x) = 2x^3 - 3x^2 + x - 5$ is divided by $(x - 2)$.

Step 1: Use the Remainder Theorem. When a polynomial $f(x)$ is divided by $(x - c)$, the remainder is simply $f(c)$.

Step 2: Identify $c = 2$ and evaluate $f(2)$.

$$f(2) = 2(2)^3 - 3(2)^2 + (2) - 5$$

Step 3: Evaluate each term.

$$= 2(8) - 3(4) + 2 - 5 = 16 - 12 + 2 - 5$$

Step 4: Combine.

$$= 1$$

$$\boxed{\text{Remainder} = 1}$$

21. For $f(x) = (x - 1)^2(x + 2)(x - 3)^3$, identify the zero with multiplicity 3.

Step 1: The zeros of a polynomial in factored form come from setting each factor equal to zero.

- $(x - 1)^2 = 0 \Rightarrow x = 1$, multiplicity 2
- $(x + 2) = 0 \Rightarrow x = -2$, multiplicity 1
- $(x - 3)^3 = 0 \Rightarrow x = 3$, multiplicity 3

Step 2: The exponent on the factor tells you the multiplicity. The factor $(x - 3)$ has exponent 3.

$$\boxed{x = 3 \text{ (multiplicity 3)}}$$

22. If a zero at $x = 2$ has multiplicity 2 and a zero at $x = -1$ has multiplicity 1, what does the graph do at each zero?

Explanation: The multiplicity of a zero determines the graph's behaviour at that x-intercept.

- **Even multiplicity** (e.g., 2): the graph *touches* the x-axis and bounces back without crossing. It behaves like the vertex of a parabola at that point.
- **Odd multiplicity** (e.g., 1, 3): the graph *crosses* through the x-axis, changing sign.

Applying these rules: the zero at $x = 2$ (multiplicity 2) causes a touch-and-bounce, and the zero at $x = -1$ (multiplicity 1) causes the graph to cross straight through.

$$\boxed{\text{Touches at } x = 2; \text{ crosses at } x = -1}$$

23. For $f(x) = -2x^4 + 5x^2 - 3$, describe the end behaviour.

Step 1: End behaviour is determined entirely by the leading term. Identify it.

$$\text{Leading term: } -2x^4$$

Step 2: Apply the end behaviour rules:

- The degree is 4 (even), so both ends of the graph point in the same direction.
- The leading coefficient is -2 (negative), so both ends point *downward*.

Step 3: Conclude: as $x \to +\infty$, $f(x) \to -\infty$, and as $x \to -\infty$, $f(x) \to -\infty$.

$$\boxed{\text{Down on both ends (both sides go to } -\infty)}$$

24. Revenue is $R(x) = 50x - 0.5x^2$. What is the revenue when 40 units are sold?

Step 1: Substitute $x = 40$ into the revenue function.

$$R(40) = 50(40) - 0.5(40)^2$$

Step 2: Evaluate each term.

$$= 2000 - 0.5(1600) = 2000 - 800$$

Step 3: Subtract.

$$R(40) = 1200$$

$$\boxed{R(40) = \$1{,}200}$$

25. A garden has a perimeter of 40 m. The area is $A(x) = x(20 - x)$. What is the maximum area?

Step 1: Expand the area function so its shape is clear.

$$A(x) = x(20 - x) = -x^2 + 20x$$

This is a downward-opening parabola (negative leading coefficient), so it has a maximum value at its vertex.

Step 2: Find the x-coordinate of the vertex using $x = -\dfrac{b}{2a}$ where $a = -1$ and $b = 20$.

$$x = -\frac{20}{2(-1)} = -\frac{20}{-2} = 10$$

Step 3: Substitute $x = 10$ back into the area function to find the maximum area.

$$A(10) = 10(20 - 10) = 10 \times 10 = 100$$

$$\boxed{A(10) = 100 \text{ m}^2}$$

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