Sequences & Conics Quiz
Answer Key
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1. What is the general term (nth term) of the sequence $3, 7, 11, 15, \ldots$?

Step 1: Confirm this is arithmetic. Check that consecutive terms differ by the same amount.

$$7 - 3 = 4 \qquad 11 - 7 = 4 \qquad 15 - 11 = 4$$

The common difference is $d = 4$.

Step 2: Use the arithmetic general term formula $a_n = a_1 + (n-1)d$.

$$a_n = 3 + (n-1)(4) = 3 + 4n - 4 = 4n - 1$$

Step 3: Verify with $n = 1$: $a_1 = 4(1) - 1 = 3$. ✓

$$\boxed{a_n = 4n - 1}$$

2. If $a_n = 5n^2 - 2$, what is the 4th term of the sequence?

Step 1: Substitute $n = 4$ into the formula.

$$a_4 = 5(4)^2 - 2$$

Step 2: Evaluate the exponent, then multiply, then subtract.

$$a_4 = 5(16) - 2 = 80 - 2 = 78$$

$$\boxed{a_4 = 78}$$

3. Which of the following is an arithmetic sequence? What is the common difference?

Explanation: An arithmetic sequence has a constant difference between consecutive terms. Check the sequence $10, 7, 4, 1, \ldots$:

$$7 - 10 = -3 \qquad 4 - 7 = -3 \qquad 1 - 4 = -3$$

The difference is constant at -3, confirming this is arithmetic.

$$\boxed{10,\ 7,\ 4,\ 1,\ \ldots \quad (d = -3)}$$

4. Find the sum of the first 20 terms of the arithmetic series $2 + 5 + 8 + 11 + \cdots$

Step 1: Identify the values: $a_1 = 2$, $d = 3$, $n = 20$.
Step 2: Use the arithmetic series sum formula.

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

Step 3: Substitute.

$$S_{20} = \frac{20}{2}\left[2(2) + (20-1)(3)\right] = 10\left[4 + 57\right] = 10 \times 61 = 610$$

$$\boxed{S_{20} = 610}$$

5. In an arithmetic series, $a_1 = 5$ and $d = 3$. Find the sum of the first 15 terms.

Step 1: Identify the values: $a_1 = 5$, $d = 3$, $n = 15$.
Step 2: Apply the sum formula.

$$S_{15} = \frac{15}{2}\left[2(5) + (15-1)(3)\right] = \frac{15}{2}\left[10 + 42\right] = \frac{15}{2} \times 52 = 15 \times 26 = 390$$

$$\boxed{S_{15} = 390}$$

6. Identify the geometric sequence and its common ratio.

Explanation: A geometric sequence has a constant ratio between consecutive terms. Check $3, 6, 12, 24, \ldots$:

$$\frac{6}{3} = 2 \qquad \frac{12}{6} = 2 \qquad \frac{24}{12} = 2$$

The ratio is constant at $r = 2$, confirming this is geometric.

$$\boxed{3,\ 6,\ 12,\ 24,\ \ldots \quad (r = 2)}$$

7. If the first term of a geometric sequence is 2 and the common ratio is 3, what is the 5th term?

Step 1: Use the geometric general term formula.

$$a_n = a_1 \cdot r^{n-1}$$

Step 2: Substitute $a_1 = 2$, $r = 3$, $n = 5$.

$$a_5 = 2 \cdot 3^{5-1} = 2 \cdot 3^4 = 2 \cdot 81 = 162$$

$$\boxed{a_5 = 162}$$

8. Find the sum of the geometric series $2 + 6 + 18 + 54 + 162$.

Step 1: Identify the values: $a_1 = 2$, $r = 3$, $n = 5$.
Step 2: Use the geometric series sum formula.

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$

Step 3: Substitute.

$$S_5 = \frac{2(3^5 - 1)}{3 - 1} = \frac{2(243 - 1)}{2} = \frac{2 \times 242}{2} = 242$$

$$\boxed{S_5 = 242}$$

9. Find the sum of the first 8 terms of the geometric series with $a_1 = 1$ and $r = 2$.

Step 1: Identify the values: $a_1 = 1$, $r = 2$, $n = 8$.
Step 2: Apply the geometric series sum formula.

$$S_8 = \frac{1(2^8 - 1)}{2 - 1} = \frac{256 - 1}{1} = 255$$

$$\boxed{S_8 = 255}$$

10. For which value of r does the infinite geometric series converge?

Explanation: An infinite geometric series converges if and only if $|r| < 1$. When $|r| \geq 1$, the terms do not shrink toward zero, so the sum grows without bound.

Among the options, $r = 0.4$ satisfies $|0.4| = 0.4 < 1$, so the series converges.

$$\boxed{r = 0.4}$$

11. Find the sum of the infinite geometric series $8 + 4 + 2 + 1 + \cdots$

Step 1: Identify the values. $a_1 = 8$ and $r = \frac{4}{8} = \frac{1}{2}$.
Step 2: Confirm convergence: $\left|\frac{1}{2}\right| = 0.5 < 1$. ✓
Step 3: Apply the infinite geometric series formula.

$$S_\infty = \frac{a_1}{1-r} = \frac{8}{1-\frac{1}{2}} = \frac{8}{\frac{1}{2}} = 8 \times 2 = 16$$

$$\boxed{S_\infty = 16}$$

12. A sum of \$2,000 is invested at 5% annual compound interest. What is the amount after 3 years?

Step 1: The compound interest formula is $A = P(1+r)^n$.
Step 2: Identify the values: $P = 2000$, $r = 0.05$, $n = 3$.
Step 3: Substitute and evaluate.

$$A = 2000(1.05)^3$$

Step 4: Compute $(1.05)^3 = 1.157625$.

$$A = 2000 \times 1.157625 = 2315.25$$

$$\boxed{\$2{,}315.25}$$

13. A person deposits \$500 at the end of each year for 5 years at 4% interest. What is the total value of the annuity?

Step 1: Use the future value of an ordinary annuity formula.

$$FV = PMT \cdot \frac{(1+i)^n - 1}{i}$$

Step 2: Identify the values: $PMT = 500$, $i = 0.04$, $n = 5$.
Step 3: Substitute.

$$FV = 500 \cdot \frac{(1.04)^5 - 1}{0.04}$$

Step 4: Evaluate $(1.04)^5 \approx 1.21665$.

$$FV = 500 \cdot \frac{1.21665 - 1}{0.04} = 500 \cdot \frac{0.21665}{0.04} = 500 \times 5.4163 \approx 2708.16$$

$$\boxed{\$2{,}708.16}$$

14. A ball is dropped from 10 m. After each bounce it reaches 60% of the previous height. What is the total distance traveled?

Step 1: Break the total distance into two parts: the initial drop, and the infinite series of bounces (each bounce goes up then back down).

Step 2: The ball falls 10 m initially.

Step 3: After the first bounce, it travels up 6 m and back down 6 m. The upward distances form an infinite geometric series with $a_1 = 6$ and $r = 0.6$.

$$S_{\text{up}} = \frac{6}{1 - 0.6} = \frac{6}{0.4} = 15 \text{ m}$$

By symmetry, the total downward bouncing distance also equals 15 m.

Step 4: Add all contributions.

$$\text{Total} = 10 + 2 \times 15 = 10 + 30 = 40 \text{ m}$$

$$\boxed{40 \text{ metres}}$$

15. Which of the following best describes a conic section?

Explanation: Conic sections get their name from how they are formed geometrically. When a flat plane slices through a double cone at different angles, the resulting cross-sections are curves: circles, ellipses, parabolas, and hyperbolas. Each curve is determined by the angle and position of the cutting plane relative to the cone.

$$\boxed{\text{The intersection of a plane with a cone}}$$

16. What is the radius of the circle $(x - 3)^2 + (y + 2)^2 = 49$?

Step 1: The standard form of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

Step 2: Compare with the given equation. The right-hand side is $49 = r^2$.

Step 3: Take the square root.

$$r = \sqrt{49} = 7$$

$$\boxed{r = 7}$$

17. Find the center and radius of $x^2 + y^2 - 4x + 6y - 12 = 0$.

Step 1: Rearrange and complete the square for both x and y. Group the terms.

$$(x^2 - 4x) + (y^2 + 6y) = 12$$

Step 2: Complete the square for x. Take half of -4, square it: $(-2)^2 = 4$.

$$(x^2 - 4x + 4) + (y^2 + 6y) = 12 + 4$$

Step 3: Complete the square for y. Take half of 6, square it: $(3)^2 = 9$.

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$$

Step 4: Write in standard form.

$$(x - 2)^2 + (y + 3)^2 = 25$$

Step 5: Read off the center $(h, k) = (2, -3)$ and radius $r = \sqrt{25} = 5$.

$$\boxed{\text{Center } (2,\ -3), \quad r = 5}$$

18. Identify the vertex and direction of the parabola $y = 2(x - 3)^2 + 1$.

Step 1: The equation is in vertex form $y = a(x - h)^2 + k$, where (h, k) is the vertex.
Step 2: Identify $h = 3$ and $k = 1$, so the vertex is $(3, 1)$.
Step 3: The coefficient $a = 2 > 0$, so the parabola opens **upward**.

$$\boxed{\text{Vertex } (3,\ 1),\ \text{opens upward}}$$

19. A parabola has focus at $(0, 2)$ and directrix $y = -2$. Find the equation.

Step 1: The focus is above the origin and the directrix is below, so the parabola opens upward with vertex at the origin $(0, 0)$.
Step 2: The value p is the distance from the vertex to the focus.

$$p = 2$$

Step 3: For an upward-opening parabola with vertex at the origin, the standard form is $x^2 = 4py$.

$$x^2 = 4(2)y = 8y$$

$$\boxed{x^2 = 8y}$$

20. For the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, find the lengths of the major and minor axes.

Step 1: The standard form is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a^2 > b^2$. The major axis has length $2a$ (along the x-axis) and the minor axis has length $2b$.
Step 2: Identify $a^2 = 25$ and $b^2 = 9$.

$$a = \sqrt{25} = 5 \qquad b = \sqrt{9} = 3$$

Step 3: Calculate the axis lengths.

$$\text{Major axis} = 2a = 10 \qquad \text{Minor axis} = 2b = 6$$

$$\boxed{\text{Major axis} = 10, \quad \text{Minor axis} = 6}$$

21. For the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$, find the coordinates of the foci.
Step 1: Identify $a^2 = 36$ and $b^2 = 20$ (since $36 > 20$, the major axis is along the x-axis).
Step 2: Use the ellipse focal length formula $c^2 = a^2 - b^2$.

$$c^2 = 36 - 20 = 16 \qquad \Rightarrow \qquad c = \sqrt{16} = 4$$

Step 3: The foci lie on the major axis (x-axis) at $(\pm c, 0)$.

$$\boxed{\text{Foci at } (\pm 4,\ 0)}$$

22. What is the standard form of a hyperbola with center at the origin and transverse axis along the x-axis?

Explanation: A hyperbola with its transverse (main) axis along the x-axis has two branches opening left and right. Its standard form is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where a is the distance from center to each vertex, and b relates to the asymptote slopes. Note the minus sign distinguishes it from an ellipse.

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

23. For the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, identify the vertices and the asymptotes.
Step 1: Identify $a^2 = 16$ and $b^2 = 9$.

$$a = \sqrt{16} = 4 \qquad b = \sqrt{9} = 3$$

Step 2: Since the x^2 term is positive, the transverse axis is along the x-axis. The vertices are at $(\pm a, 0)$.

$$\text{Vertices: } (\pm 4,\ 0)$$

Step 3: The asymptotes of this form of hyperbola are $y = \pm\frac{b}{a}x$.

$$y = \pm\frac{3}{4}x$$

$$\boxed{\text{Vertices: } (\pm 4,\ 0); \quad \text{Asymptotes: } y = \pm\frac{3}{4}x}$$

24. Determine which conic section the equation $4x^2 + 9y^2 = 36$ represents.

Step 1: Divide both sides by 36 to write the equation in standard form.

$$\frac{4x^2}{36} + \frac{9y^2}{36} = 1 \qquad \Rightarrow \qquad \frac{x^2}{9} + \frac{y^2}{4} = 1$$

Step 2: This matches the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a^2 = 9$ and $b^2 = 4$. Both denominators are different and both terms are positive, so this is an **ellipse**.

$$\boxed{\text{Ellipse}}$$

25. The equation $x^2 - y^2 = 4$ represents which conic section, and what are the asymptotes?

Step 1: Divide both sides by 4 to write in standard form.

$$\frac{x^2}{4} - \frac{y^2}{4} = 1$$

Step 2: This matches $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with $a^2 = b^2 = 4$. The subtraction of the two squared terms confirms this is a **hyperbola**.

Step 3: The asymptote formula is $y = \pm\frac{b}{a}x$. Here $a = b = 2$.

$$y = \pm\frac{2}{2}x = \pm x$$

$$\boxed{\text{Hyperbola; asymptotes: } y = \pm x}$$

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