Trigonometry Challenge Quiz
Answer Key
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1. Convert 210° to radians. Express your answer in simplest form.

Step 1: Use the degree-to-radian conversion factor.

$$210^\circ \times \frac{\pi}{180^\circ}$$

Step 2: Simplify the fraction. Divide numerator and denominator by their GCF of 30.

$$\frac{210\pi}{180} = \frac{7\pi}{6}$$

$$\boxed{\frac{7\pi}{6} \text{ radians}}$$

2. Convert $\frac{3\pi}{4}$ radians to degrees.

Step 1: Use the radian-to-degree conversion factor.

$$\frac{3\pi}{4} \times \frac{180^\circ}{\pi}$$

Step 2: Cancel π and simplify.

$$\frac{3 \times 180^\circ}{4} = \frac{540^\circ}{4} = 135^\circ$$

$$\boxed{135^\circ}$$

3. What is the coordinate of the point on the unit circle at angle $\frac{5\pi}{4}$?

Step 1: Find the reference angle. Since $\pi < \frac{5\pi}{4} < \frac{3\pi}{2}$, the angle is in Quadrant III.

$$\theta_R = \frac{5\pi}{4} - \pi = \frac{\pi}{4} \quad (45^\circ)$$

Step 2: The unit circle coordinates at $\frac{\pi}{4}$ are $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

Step 3: In Quadrant III, both x and y are negative. Apply the signs.

$$\left(-\frac{\sqrt{2}}{2},\ -\frac{\sqrt{2}}{2}\right)$$

$$\boxed{\left(-\frac{\sqrt{2}}{2},\ -\frac{\sqrt{2}}{2}\right)}$$

4. If a point on the unit circle has coordinates $\left(\frac{1}{2},\ \frac{\sqrt{3}}{2}\right)$, what is the angle in radians?

Step 1: On the unit circle, a point $(\cos\theta,\ \sin\theta)$ corresponds to angle θ. Identify the values.

$$\cos\theta = \frac{1}{2} \qquad \sin\theta = \frac{\sqrt{3}}{2}$$

Step 2: Both values are positive, placing the angle in Quadrant I. From the 30-60-90 special triangle, these values correspond to $\theta = \frac{\pi}{3}$ (60°).

$$\boxed{\theta = \frac{\pi}{3}}$$

5. In a 45-45-90 triangle with hypotenuse 8 units, what is the length of each leg?

Step 1: In a 45-45-90 triangle, the sides are in the ratio $1 : 1 : \sqrt{2}$. So the hypotenuse equals a leg times $\sqrt{2}$.

$$\text{hypotenuse} = \text{leg} \times \sqrt{2}$$

Step 2: Solve for the leg by dividing both sides by $\sqrt{2}$.

$$\text{leg} = \frac{8}{\sqrt{2}}$$

Step 3: Rationalize the denominator.

$$\text{leg} = \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

$$\boxed{4\sqrt{2}\text{ units}}$$

6. In a 30-60-90 triangle with hypotenuse 12, what is the length of the side opposite the 60° angle?

Step 1: In a 30-60-90 triangle, the sides are in the ratio $1 : \sqrt{3} : 2$. The hypotenuse corresponds to 2, the short leg to 1, and the long leg (opposite 60°) to $\sqrt{3}$.

Step 2: Set up the ratio. The scale factor is $\frac{12}{2} = 6$.

Step 3: Multiply the long leg ratio by the scale factor.

$$\text{side opposite } 60^\circ = 6 \times \sqrt{3} = 6\sqrt{3}$$

$$\boxed{6\sqrt{3} \text{ units}}$$

7. An angle of 135° is in which quadrant, and what is its reference angle?

Step 1: Identify the quadrant. Since $90^\circ < 135^\circ < 180^\circ$, the angle is in **Quadrant II**.
Step 2: For a Quadrant II angle, the reference angle is found by subtracting from 180°.

$$\theta_R = 180^\circ - 135^\circ = 45^\circ$$

$$\boxed{\text{Quadrant II; reference angle} = 45^\circ}$$

8. A point is at distance 5 from the origin at an angle of 240° in standard position. What are its coordinates?

Step 1: For a point at distance r from the origin at angle θ, the coordinates are $(r\cos\theta,\ r\sin\theta)$.
Step 2: Find the reference angle. Since $180^\circ < 240^\circ < 270^\circ$, the angle is in Quadrant III.

$$\theta_R = 240^\circ - 180^\circ = 60^\circ$$

Step 3: Use the exact values $\cos 60^\circ = \dfrac{1}{2}$ and $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$.
Step 4: In Quadrant III, both x and y are negative. Apply $r = 5$.

$$x = -5 \times \frac{1}{2} = -\frac{5}{2} \qquad y = -5 \times \frac{\sqrt{3}}{2} = -\frac{5\sqrt{3}}{2}$$

$$\boxed{\left(-\frac{5}{2},\ -\frac{5\sqrt{3}}{2}\right)}$$

9. What is the amplitude of the function $f(x) = 3\sin(2x)$?

Explanation: The general form of a sine function is $f(x) = A\sin(Bx + C) + D$. The amplitude is the absolute value of the coefficient A, representing the maximum distance the function reaches from its midline.

$$\text{Amplitude} = |A| = |3| = 3$$

$$\boxed{\text{Amplitude} = 3}$$

10. What is the period of the function $g(x) = \cos\left(\frac{\pi x}{2}\right)$?

Step 1: Identify the coefficient B inside the argument. Here $B = \frac{\pi}{2}$.
Step 2: Use the period formula.

$$\text{Period} = \frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{2}} = 2\pi \times \frac{2}{\pi} = 4$$

$$\boxed{\text{Period} = 4}$$

11. Identify the vertical asymptotes of $f(x) = \tan(x)$ on the interval $[0,\ 2\pi)$.

Step 1: Recall that $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$. Vertical asymptotes occur wherever $\cos(x) = 0$, because division by zero is undefined.

Step 2: Find where $\cos(x) = 0$ in $[0, 2\pi)$.

$$x = \frac{\pi}{2} \qquad \text{and} \qquad x = \frac{3\pi}{2}$$

$$\boxed{x = \frac{\pi}{2} \quad \text{and} \quad x = \frac{3\pi}{2}}$$

12. What is the range of the inverse sine function $\arcsin(x)$?

Explanation: The sine function is not one-to-one over its full domain, so to define an inverse, we restrict it to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ where it is both one-to-one and covers all values from -1 to 1. The output (range) of $\arcsin(x)$ is therefore this restricted interval.

$$\boxed{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$$

13. Evaluate $\arccos\left(\dfrac{\sqrt{2}}{2}\right)$.

Step 1: arccos asks: "what angle in $[0, \pi]$ has a cosine equal to $\frac{\sqrt{2}}{2}$?"

Step 2: From the 45-45-90 special triangle, $\cos(45^\circ) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

Step 3: Since $\frac{\pi}{4}$ is in the valid range $[0, \pi]$, this is the answer.

$$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$$\boxed{\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}}$$

14. Which of the following is a correct fundamental trigonometric identity?

Explanation: The Pythagorean identity is derived from the equation of the unit circle $x^2 + y^2 = 1$. Since $x = \cos\theta$ and $y = \sin\theta$ on the unit circle, substituting gives:

$$\cos^2(x) + \sin^2(x) = 1 \qquad \text{equivalently written as} \qquad \sin^2(x) + \cos^2(x) = 1$$

This identity holds for all values of x and is one of the most fundamental identities in trigonometry.

$$\boxed{\sin^2(x) + \cos^2(x) = 1}$$

15. Using the sum formula, find $\sin(75°) = \sin(45° + 30°)$.

Step 1: Write the sine sum formula.

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Step 2: Set $\alpha = 45°$ and $\beta = 30°$, then substitute exact values.

$$\sin(75°) = \sin 45° \cos 30° + \cos 45° \sin 30°$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

Step 3: Multiply the fractions.

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

Step 4: Combine over a common denominator.

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\boxed{\sin(75°) = \frac{\sqrt{6} + \sqrt{2}}{4}}$$

16. Find $\cos(105°)$ using the difference formula: $\cos(150° - 45°)$.

Step 1: Write the cosine difference formula.

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Step 2: Set $\alpha = 150°$ and $\beta = 45°$, then substitute exact values.

$$\cos(105°) = \cos 150° \cos 45° + \sin 150° \sin 45°$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

Step 3: Multiply.

$$= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

Step 4: Combine.

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\boxed{\cos(105°) = \frac{\sqrt{2} - \sqrt{6}}{4}}$$

17. If $\sin(\theta) = \frac{3}{5}$ and θ is in Quadrant I, find $\sin(2\theta)$.

Step 1: Use the Pythagorean identity to find $\cos(\theta)$. In Quadrant I, cosine is positive.

$$\cos^2(\theta) = 1 - \sin^2(\theta) = 1 - \frac{9}{25} = \frac{16}{25} \qquad \Rightarrow \qquad \cos(\theta) = \frac{4}{5}$$

Step 2: Apply the double angle identity for sine.

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

Step 3: Substitute and simplify.

$$\sin(2\theta) = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

$$\boxed{\sin(2\theta) = \frac{24}{25}}$$

18. If $\cos(\theta) = \frac{1}{3}$, find $\cos(2\theta)$ using the double angle formula.

Step 1: Write the double angle identity for cosine. There are three equivalent forms; the most direct when $\cos(\theta)$ is known is:

$$\cos(2\theta) = 2\cos^2(\theta) - 1$$

Step 2: Substitute $\cos(\theta) = \frac{1}{3}$.

$$\cos(2\theta) = 2\left(\frac{1}{3}\right)^2 - 1 = 2 \times \frac{1}{9} - 1 = \frac{2}{9} - 1$$

Step 3: Convert 1 to ninths and subtract.

$$\cos(2\theta) = \frac{2}{9} - \frac{9}{9} = -\frac{7}{9}$$

$$\boxed{\cos(2\theta) = -\frac{7}{9}}$$

19. Solve $\sin(x) = \frac{1}{2}$ for $x \in [0,\ 2\pi)$.

Step 1: Find the reference angle where $\sin\theta = \frac{1}{2}$.

$$\theta_R = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Step 2: Determine which quadrants have positive sine. Sine is positive in Quadrant I and Quadrant II.

Step 3: Find both solutions.

- Quadrant I: $x = \frac{\pi}{6}$
- Quadrant II: $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$$\boxed{x = \frac{\pi}{6} \quad \text{and} \quad x = \frac{5\pi}{6}}$$

20. Solve $2\cos^2(x) - \cos(x) - 1 = 0$ for $x \in [0,\ 2\pi)$.

Step 1: Recognize this as a quadratic equation in $\cos(x)$. Let $u = \cos(x)$ and factor.

$$2u^2 - u - 1 = (2u+1)(u-1) = 0$$

Step 2: Solve each factor.

$$u - 1 = 0 \quad \Rightarrow \quad \cos(x) = 1 \qquad \text{and} \qquad 2u + 1 = 0 \quad \Rightarrow \quad \cos(x) = -\frac{1}{2}$$

Step 3: Find all $x \in [0, 2\pi)$ for each case.
For $\cos(x) = 1$:

$$x = 0$$

For $\cos(x) = -\frac{1}{2}$ (cosine is negative in QII and QIII, reference angle $= \frac{\pi}{3}$):

$$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \qquad \text{and} \qquad x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\boxed{x = 0, \quad x = \frac{2\pi}{3}, \quad x = \frac{4\pi}{3}}$$

21. Solve $\sin(x)\cos(x) = \frac{1}{4}$ for $x \in [0,\ 2\pi)$.

Step 1: Use the double angle identity. Recall that $\sin(2x) = 2\sin(x)\cos(x)$, so:

$$\sin(x)\cos(x) = \frac{\sin(2x)}{2}$$

Step 2: Substitute into the equation.

$$\frac{\sin(2x)}{2} = \frac{1}{4} \qquad \Rightarrow \qquad \sin(2x) = \frac{1}{2}$$

Step 3: Let $u = 2x$. Since $x \in [0,\ 2\pi)$, we have $u \in [0,\ 4\pi)$. Find all u where $\sin(u) = \frac{1}{2}$.

$$u = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{\pi}{6} + 2\pi = \frac{13\pi}{6}, \quad \frac{5\pi}{6} + 2\pi = \frac{17\pi}{6}$$

Step 4: Divide each by 2 to get x.

$$x = \frac{\pi}{12}, \quad \frac{5\pi}{12}, \quad \frac{13\pi}{12}, \quad \frac{17\pi}{12}$$

$$\boxed{x = \frac{\pi}{12}, \quad \frac{5\pi}{12}, \quad \frac{13\pi}{12}, \quad \frac{17\pi}{12}}$$

22. A mass oscillates with displacement $d(t) = 5\cos(2\pi t)$. What is the maximum displacement?

Explanation: The function has the general form $d(t) = A\cos(Bt)$. The amplitude $|A|$ gives the maximum distance from the equilibrium position, since cos oscillates between -1 and 1.

$$d_{\text{max}} = |A| \times 1 = |5| \times 1 = 5$$

This maximum occurs whenever $\cos(2\pi t) = 1$, i.e., at $t = 0, 1, 2, \ldots$

$$\boxed{\text{Maximum displacement} = 5 \text{ units}}$$

23. The height of a Ferris wheel seat is modelled by $h(t) = 25 + 20\sin\left(\frac{\pi t}{15}\right)$. What is the minimum height?

Step 1: The minimum value of $\sin(\cdot)$ is -1. Substitute this to find the minimum height.

$$h_{\text{min}} = 25 + 20(-1) = 25 - 20 = 5$$

Step 2: Interpret the answer. The Ferris wheel seat is at its lowest point, 5 metres above the ground.

$$\boxed{h_{\text{min}} = 5 \text{ m}}$$

24. By graphing, approximately how many solutions does $\sin(x) = 0.3$ have on $[0,\ 2\pi)$?

Explanation: Sketch $y = \sin(x)$ over $[0,\ 2\pi)$ and draw the horizontal line $y = 0.3$.

Since $0 < 0.3 < 1$, the horizontal line lies above the x-axis. The sine curve rises above 0 in Quadrant I and Quadrant II. The horizontal line at $y = 0.3$ will intersect the sine curve exactly twice in $[0, 2\pi)$: once on the way up (near $x \approx 0.305$) and once on the way down (near $x \approx \pi - 0.305 \approx 2.837$).

$$\boxed{\text{2 solutions}}$$

25. Find the approximate solutions to $\tan(x) = 2$ on $[0,\ 2\pi)$ to 3 decimal places.

Step 1: Find the reference angle using the inverse tangent.

$$x_R = \arctan(2) \approx 1.107 \text{ radians}$$

Step 2: Determine the quadrants where tangent is positive. Tangent is positive in Quadrant I and Quadrant III.

Step 3: Find both solutions.

- Quadrant I: $x_1 = 1.107$
- Quadrant III: $x_2 = \pi + 1.107 \approx 3.14159 + 1.107 \approx 4.249$

$$\boxed{x \approx 1.107 \quad \text{and} \quad x \approx 4.249}$$

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