Absolute Value & Reciprocal Functions
Quiz Answer Key
ApexMath

Answer Key

1. Evaluate: $|-12| + |5| - |-3|$.

Step 1: Evaluate each absolute value. Absolute value gives the non-negative distance from zero.

$$|-12| = 12, \qquad |5| = 5, \qquad |-3| = 3$$

Step 2: Substitute and compute.

$$12 + 5 - 3 = 14$$

$$\boxed{14}$$

2. What is the vertex of $f(x) = |x + 4| - 7$?

Step 1: Identify the vertex form. An absolute value function has the form $f(x) = a|x - h| + k$, where the vertex is (h, k).

Step 2: Rewrite to match the form.

$$f(x) = |x - (-4)| + (-7)$$

So $h = -4$ and $k = -7$.

$$\boxed{\text{Vertex} = (-4,\ -7)}$$

3. Solve for x: $|x - 5| = 11$.

Step 1: Split into two cases. An equation $|A| = c$ (with $c > 0$) gives $A = c$ or $A = -c$.

$$x - 5 = 11 \qquad \text{or} \qquad x - 5 = -11$$

Step 2: Solve each case.

$$x = 16 \qquad \text{or} \qquad x = -6$$

$$\boxed{x = 16 \quad \text{and} \quad x = -6}$$

4. Solve for x: $|2x + 3| = 7$.

Step 1: Split into two cases.

$$2x + 3 = 7 \qquad \text{or} \qquad 2x + 3 = -7$$

Step 2: Solve each case.

$$2x = 4 \implies x = 2 \qquad \text{or} \qquad 2x = -10 \implies x = -5$$

$$\boxed{x = 2 \quad \text{and} \quad x = -5}$$

5. Solve for x: $|x + 2| = -3$.

The absolute value of any real expression is always **non-negative** — it represents a distance and can never be less than zero. Therefore it is impossible for $|x + 2|$ to equal -3.

$$\boxed{\text{No solution}}$$

6. Solve the inequality: $|x| < 5$.

Step 1: Apply the rule for $|A| < c$. When the absolute value is less than a positive number c, the solution is a single interval:

$$-c < A < c$$

Step 2: Substitute.

$$-5 < x < 5$$

$$\boxed{-5 < x < 5}$$

7. Solve the inequality: $|x - 2| \geq 3$.

Step 1: Apply the rule for $|A| \geq c$. When the absolute value is greater than or equal to a positive number c, the solution splits into two separate intervals:

$$A \geq c \qquad \text{or} \qquad A \leq -c$$

Step 2: Substitute and solve each branch.

$$x - 2 \geq 3 \implies x \geq 5$$

$$x - 2 \leq -3 \implies x \leq -1$$

$$\boxed{x \geq 5 \quad \text{or} \quad x \leq -1}$$

8. Given $f(x) = x - 3$, what is the vertical asymptote of $y = \dfrac{1}{f(x)}$?

Step 1: A vertical asymptote occurs where the denominator equals zero.

$$f(x) = 0 \implies x - 3 = 0 \implies x = 3$$

At $x = 3$, the denominator is zero so the reciprocal is undefined, producing a vertical asymptote.

$$\boxed{x = 3}$$

9. What is the horizontal asymptote of $y = \dfrac{1}{x+2}$?

Step 1: Consider the behaviour as $x \to \pm\infty$. As x grows very large (positive or negative), the denominator $(x+2)$ grows very large in magnitude, so the fraction approaches zero.

$$\lim_{x \to \pm\infty} \frac{1}{x+2} = 0$$

The graph gets closer and closer to $y = 0$ but never touches it.

$$\boxed{y = 0}$$

10. Invariant points of a reciprocal function occur where $f(x)$ equals:

An invariant point is a point shared by both $y = f(x)$ and $y = \dfrac{1}{f(x)}$. For both to have the same y-value:

$$f(x) = \frac{1}{f(x)} \implies [f(x)]^2 = 1 \implies f(x) = 1 \text{ or } f(x) = -1$$

$$\boxed{f(x) = 1 \quad \text{or} \quad f(x) = -1}$$

11. Find the invariant points for $f(x) = 2x - 5$ and $y = \dfrac{1}{f(x)}$.

Step 1: Set $f(x) = 1$ and solve.

$$2x - 5 = 1 \implies 2x = 6 \implies x = 3$$

Step 2: Set $f(x) = -1$ and solve.

$$2x - 5 = -1 \implies 2x = 4 \implies x = 2$$

Step 3: State the invariant points.

$$x = 3: \quad f(3) = 1 \implies (3,\ 1)$$

$$x = 2: \quad f(2) = -1 \implies (2,\ -1)$$

$$\boxed{x = 3 \text{ and } x = 2 \quad \big((3,\ 1) \text{ and } (2,\ -1)\big)}$$

12. If $f(x)$ is a quadratic with two x-intercepts, how many vertical asymptotes does $y = \dfrac{1}{f(x)}$ have?

Vertical asymptotes occur wherever $f(x) = 0$. If the quadratic has two x-intercepts, then $f(x) = 0$ at two distinct values of x, producing **two** vertical asymptotes.

$$\boxed{\text{Two vertical asymptotes}}$$

13. $y = |x|$ is shifted 3 units left and 2 units up. New equation?

Step 1: Apply horizontal shift. Shifting left by 3 replaces x with $(x + 3)$:

$$y = |x + 3|$$

Step 2: Apply vertical shift. Shifting up by 2 adds 2 outside:

$$y = |x + 3| + 2$$

$$\boxed{y = |x + 3| + 2}$$

14. What is the range of $f(x) = -|x - 1| + 4$?

Step 1: Identify the vertex. The vertex is at $(1, 4)$.

Step 2: Determine direction. The leading coefficient is $-1 < 0$, so the "V" opens **downward**. The vertex is the highest point.

Step 3: State the range. The function never exceeds 4.

$$\boxed{y \leq 4}$$

15. What is the vertical asymptote for $y = \dfrac{1}{2x - 8} + 5$?

Step 1: Set the denominator equal to zero.

$$2x - 8 = 0 \implies 2x = 8 \implies x = 4$$

$$\boxed{x = 4}$$

16. What is the horizontal asymptote for $y = \dfrac{1}{2x-8} + 5$?

Step 1: Consider the end behaviour. As $x \to \pm\infty$:

$$\frac{1}{2x-8} \to 0$$

Step 2: Apply the vertical shift. The $+5$ shifts the base horizontal asymptote ($y = 0$) up by 5.

$$y = 0 + 5 = 5$$

$$\boxed{y = 5}$$

17. Solve: $|2x - 1| < 7$.

Step 1: Apply the rule for $|A| < c$.

$$-7 < 2x - 1 < 7$$

Step 2: Add 1 to all three parts.

$$-6 < 2x < 8$$

Step 3: Divide all three parts by 2.

$$-3 < x < 4$$

$$\boxed{-3 < x < 4}$$

18. For $f(x) = |x - 2|$, what is the piecewise expression when $x < 2$?

Step 1: Apply the definition of absolute value. When the expression inside the absolute value is negative (i.e. $x - 2 < 0$, which occurs when $x < 2$), the absolute value flips its sign:

$$|x - 2| = -(x - 2)$$

$$\boxed{f(x) = -(x-2) \quad \text{for } x < 2}$$

19. If $f(x) = x^2 + 1$, how many vertical asymptotes does $y = \dfrac{1}{f(x)}$ have?

Step 1: Find where $f(x) = 0$.

$$x^2 + 1 = 0 \implies x^2 = -1$$

There is no real number whose square is -1. Since $f(x)$ is always at least 1 for all real x, it never equals zero.

Step 2: Conclude. With no real zeros in the denominator, there are no vertical asymptotes.

$$\boxed{\text{None}}$$

20. If $f(2) = 0.25$, what is $y = \dfrac{1}{f(x)}$ at $x = 2$?

Step 1: Substitute the known value.

$$y = \frac{1}{f(2)} = \frac{1}{0.25}$$

Step 2: Evaluate. $0.25 = \dfrac{1}{4}$, so $\dfrac{1}{0.25} = 4$.

$$\boxed{y = 4}$$

21. Solve: $|x^2 - 5| = 4$.

Step 1: Split into two cases.

$$x^2 - 5 = 4 \qquad \text{or} \qquad x^2 - 5 = -4$$

Step 2: Solve Case 1.

$$x^2 = 9 \implies x = \pm 3$$

Step 3: Solve Case 2.

$$x^2 = 1 \implies x = \pm 1$$

$$\boxed{x = \pm 3 \quad \text{and} \quad x = \pm 1}$$

22. A machined part has a target size of 10 mm with a tolerance of $\pm\, 0.05$ mm. Write an absolute value inequality for the acceptable size x.

Step 1: Interpret tolerance. The part is acceptable if its size is within 0.05 mm of the target. The distance between x and 10 must be at most 0.05.

Step 2: Write the inequality.

$$|x - 10| \leq 0.05$$

This means $9.95 \leq x \leq 10.05$.

$$\boxed{|x - 10| \leq 0.05}$$

23. In the graph of $y = \dfrac{1}{f(x)}$, if $f(x)$ increases, the reciprocal $\dfrac{1}{f(x)}$:

As $f(x)$ gets larger, 1 is being divided by a larger and larger number, so the result gets smaller. For example: $\frac{1}{2} > \frac{1}{5} > \frac{1}{100}$. This inverse relationship means a reciprocal function **decreases** wherever the original function increases.

$$\boxed{\text{Decreases}}$$

24. Find the invariant points for $f(x) = x^2$ and $y = \dfrac{1}{f(x)}$.

Step 1: Set $f(x) = 1$.

$$x^2 = 1 \implies x = \pm 1$$

Step 2: Set $f(x) = -1$.

$$x^2 = -1$$

There is no real solution since $x^2 \geq 0$ for all real x.

Step 3: State the invariant points.

$$x = 1 : \quad f(1) = 1 \implies (1,\ 1)$$

$$x = -1 : \quad f(-1) = 1 \implies (-1,\ 1)$$

$$\boxed{x = \pm 1 \quad \big((1,\,1) \text{ and } (-1,\,1)\big)}$$

25. True or False: $|a - b|$ is the same as $|b - a|$.

Step 1: Reason from the definition. Note that $b - a = -(a - b)$. Applying absolute value to both sides:

$$|b - a| = |-(a - b)| = |a - b|$$

Since the absolute value of a number and its negative are always equal, the two expressions are identical.

$$\boxed{\text{True}}$$

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