Arithmetic & Geometric Sequences and Series
Quiz Answer Key
ApexMath

Answer Key

1. Find the common difference d for the sequence: $5, 2, -1, -4, \ldots$

Step 1: Subtract any term from the term that follows it.

$$d = 2 - 5 = -3$$

Verify with the next pair:

$$-1 - 2 = -3 \quad \checkmark \qquad -4 - (-1) = -3 \quad \checkmark$$

$$\boxed{d = -3}$$

2. Find the 20th term of an arithmetic sequence where $a_1 = 3$ and $d = 4$.

Step 1: Use the general term formula.

$$a_n = a_1 + (n-1)d$$

Step 2: Substitute $n = 20$, $a_1 = 3$, $d = 4$.

$$a_{20} = 3 + (20-1)(4) = 3 + 19 \cdot 4 = 3 + 76 = 79$$

$$\boxed{a_{20} = 79}$$

3. In an arithmetic sequence, $a_1 = 10$ and $a_7 = 28$. Find d.

Step 1: Use the general term formula with $n = 7$.

$$a_7 = a_1 + (7-1)d$$

$$28 = 10 + 6d$$

Step 2: Solve for d.

$$6d = 18 \implies d = 3$$

$$\boxed{d = 3}$$

4. Calculate the sum of the first 15 terms of: $2 + 5 + 8 + \dots$

Step 1: Identify $a_1 = 2$ and $d = 3$.

Step 2: Use the arithmetic series sum formula.

$$S_n = \frac{n}{2}\big(2a_1 + (n-1)d\big)$$

Step 3: Substitute $n = 15$.

$$S_{15} = \frac{15}{2}\big(2(2) + 14(3)\big) = \frac{15}{2}(4 + 42) = \frac{15}{2}(46) = 15 \cdot 23 = 345$$

$$\boxed{S_{15} = 345}$$

5. Find the sum of the arithmetic series: $10 + 15 + 20 + \cdots + 100$.

Step 1: Identify $a_1 = 10$, $d = 5$, $a_n = 100$.

Step 2: Find n.

$$a_n = a_1 + (n-1)d \implies 100 = 10 + (n-1)(5)$$
$$90 = 5(n-1) \implies n - 1 = 18 \implies n = 19$$

Step 3: Use the sum formula $S_n = \dfrac{n}{2}(a_1 + a_n)$.

$$S_{19} = \frac{19}{2}(10 + 100) = \frac{19}{2}(110) = 19 \cdot 55 = 1045$$

$$\boxed{S = 1045}$$

6. Find the common ratio r for: $3, -6, 12, -24, \dots$

Step 1: Divide any term by the term before it.

$$r = \frac{-6}{3} = -2$$

Verify:

$$\frac{12}{-6} = -2 \quad \checkmark \qquad \frac{-24}{12} = -2 \quad \checkmark$$

$$\boxed{r = -2}$$

7. Find the 6th term of a geometric sequence where $a_1 = 5$ and $r = 2$.

Step 1: Use the general term formula.

$$a_n = a_1 \cdot r^{n-1}$$

Step 2: Substitute $n = 6$.

$$a_6 = 5 \cdot 2^{6-1} = 5 \cdot 2^5 = 5 \cdot 32 = 160$$

$$\boxed{a_6 = 160}$$

8. Find the 8th term of: $1000, 500, 250, \ldots$

Step 1: Find the common ratio.

$$r = \frac{500}{1000} = \frac{1}{2}$$

Step 2: Apply the general term formula with $n = 8$.

$$a_8 = 1000 \cdot \left(\frac{1}{2}\right)^7 = \frac{1000}{128} = \frac{125}{16}$$

Step 3: Convert to decimal.

$$\frac{125}{16} = 7.8125$$

$$\boxed{a_8 = \frac{125}{16} = 7.8125}$$

9. Calculate the sum of the first 8 terms of $2 + 6 + 18 + \ldots$

Step 1: Identify $a_1 = 2$ and $r = 3$.

Step 2: Use the geometric series sum formula.

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$

Step 3: Substitute $n = 8$.

$$S_8 = \frac{2(3^8 - 1)}{3 - 1} = \frac{2(6561 - 1)}{2} = 6560$$

$$\boxed{S_8 = 6560}$$

10. Does the infinite geometric series $10 + 5 + 2.5 + \dots$ converge?

Step 1: Find the common ratio.

$$r = \frac{5}{10} = 0.5$$

Step 2: Apply the convergence rule. An infinite geometric series converges if and only if $|r| < 1$.

$$|0.5| = 0.5 < 1 \quad \checkmark$$

$$\boxed{\text{Converges because } |r| = 0.5 < 1}$$

11. Calculate the sum of the infinite geometric series: $12 + 4 + \frac{4}{3} + \dots$

Step 1: Find the common ratio.

$$r = \frac{4}{12} = \frac{1}{3}$$

Step 2: Confirm $|r| < 1$.

$$\left|\frac{1}{3}\right| = \frac{1}{3} < 1 \quad \checkmark$$

Step 3: Apply the infinite sum formula.

$$S_\infty = \frac{a_1}{1-r} = \frac{12}{1-\frac{1}{3}} = \frac{12}{\frac{2}{3}} = 12 \cdot \frac{3}{2} = 18$$

$$\boxed{S_\infty = 18}$$

12. Evaluate: $\sum_{k=1}^{4}(k^2 - 1)$.

Step 1: Substitute each value of k and compute.

$$k = 1: \quad 1^2 - 1 = 0$$

$$k = 2: \quad 2^2 - 1 = 3$$

$$k = 3: \quad 3^2 - 1 = 8$$

$$k = 4: \quad 4^2 - 1 = 15$$

Step 2: Add the results.

$$0 + 3 + 8 + 15 = 26$$

$$\boxed{26}$$

13. Find the missing arithmetic mean between 4 and 16.

Step 1: The arithmetic mean between two numbers is their average.

$$\text{Arithmetic mean} = \frac{4+16}{2} = \frac{20}{2} = 10$$

Check: $4, 10, 16$ has common difference $d = 6$. ✓

$$\boxed{10}$$

14. Find the positive geometric mean between 4 and 16.

Step 1: The geometric mean between two numbers is the square root of their product.

$$\text{Geometric mean} = \sqrt{4 \cdot 16} = \sqrt{64} = 8$$

Check: $4, 8, 16$ has common ratio $r = 2$. ✓

$$\boxed{8}$$

15. How many terms are in the arithmetic sequence $7, 11, 15, \ldots, 127$?

Step 1: Identify $a_1 = 7$, $d = 4$, $a_n = 127$.

Step 2: Use the general term formula and solve for n.

$$127 = 7 + (n-1)(4)$$

$$120 = 4(n-1) \implies n - 1 = 30 \implies n = 31$$

$$\boxed{31 \text{ terms}}$$

16. A geometric series has $a_1 = 3$, $r = 2$, and $S_n = 93$. Find n.

Step 1: Use the geometric series sum formula.

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$

$$93 = \frac{3(2^n - 1)}{2 - 1} = 3(2^n - 1)$$

Step 2: Solve for n.

$$2^n - 1 = 31 \implies 2^n = 32 = 2^5 \implies n = 5$$

$$\boxed{n = 5}$$

17. The sum of an infinite geometric series is 20 and $r = 0.6$. Find a_1.

Step 1: Use the infinite sum formula $S_\infty = \dfrac{a_1}{1-r}$ and solve for a_1.

$$20 = \frac{a_1}{1 - 0.6} = \frac{a_1}{0.4}$$

Step 2: Multiply both sides by 0.4.

$$a_1 = 20 \times 0.4 = 8$$

$$\boxed{a_1 = 8}$$

18. Write $3 + 6 + 9 + 12 + 15$ in sigma notation.

Step 1: Identify the pattern. Each term equals $3k$ for $k = 1, 2, 3, 4, 5$.

$$3(1) = 3, \quad 3(2) = 6, \quad 3(3) = 9, \quad 3(4) = 12, \quad 3(5) = 15$$

Step 2: Write in sigma notation.

$$\sum_{k=1}^{5} 3k$$

$$\boxed{\sum_{k=1}^{5} 3k}$$

19. Which of the following is a geometric sequence? $1, 2, 4, 8, \ldots$

Step 1: Check for a constant ratio between consecutive terms.

$$\frac{2}{1} = 2, \qquad \frac{4}{2} = 2, \qquad \frac{8}{4} = 2$$

The ratio is constant at $r = 2$, confirming this is a geometric sequence.

$$\boxed{1, 2, 4, 8, \ldots \text{ with common ratio } r = 2}$$

20. Find the sum of: $1 + \frac{1}{4} + \frac{1}{16} + \dots$

Step 1: Identify $a_1 = 1$ and find r.

$$r = \frac{1/4}{1} = \frac{1}{4}$$

Step 2: Confirm convergence. $\left|\frac{1}{4}\right| < 1$ ✓

Step 3: Apply the infinite sum formula.

$$S_\infty = \frac{1}{1 - \frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\boxed{S_\infty = \frac{4}{3}}$$

21. A theater has 20 rows. The first row has 15 seats and each row has 2 more seats than the previous. How many seats are in the last row?

Step 1: Identify $a_1 = 15$, $d = 2$, $n = 20$.

Step 2: Find a_{20}.

$$a_{20} = 15 + (20 - 1)(2) = 15 + 38 = 53$$

$$\boxed{\text{53 seats in the last row}}$$

22. A ball drops from 10 m and bounces to 80% of its previous height each time. How high is the 4th bounce?

Step 1: Identify the geometric sequence. Each bounce height is a term with $a_1 = 10$, $r = 0.8$.

Step 2: The *1st bounce* reaches $a_1 \cdot r^1$, the *2nd bounce* reaches $a_1 \cdot r^2$, and so on. The 4th bounce is therefore:

$$h_4 = 10 \cdot (0.8)^4 = 10 \cdot 0.4096 = 4.096$$

$$\boxed{4.096 \text{ m}}$$

23. Find the number of terms in $\sum_{k=3}^{12} (k + 1)$.

Step 1: Count the integer values of k from 3 to 12 inclusive.

$$\text{Number of terms} = 12 - 3 + 1 = 10$$

$$\boxed{\text{10 terms}}$$

24. If the reciprocals of the terms form an arithmetic sequence, the sequence is called:

A **harmonic sequence** is defined as a sequence whose reciprocals form an arithmetic sequence. For example, $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ is harmonic because $1, 2, 3, 4, \ldots$ is arithmetic.

$$\boxed{\text{A harmonic sequence}}$$

25. What is the difference between a sequence and a series?

A **sequence** is an ordered list of numbers (terms). A **series** is the sum of the terms of a sequence. For example, $2, 5, 8, 11$ is a sequence, while $2+5+8+11 = 26$ is the corresponding series.

$$\boxed{\text{A sequence is a list of terms; a series is their sum.}}$$

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