Quadratic Equations & Functions
Quiz Answer Key
ApexMath

Answer Key

1. Factor the expression: $x^2 - 9x + 20$.

Step 1: Find two numbers that multiply to 20 and add to -9.

List factor pairs of 20: $(1, 20)$, $(2, 10)$, $(4, 5)$. The pair -4 and -5 gives $(-4)(-5) = 20$ and $(-4) + (-5) = -9$. ✓

Step 2: Write the factored form.

$$x^2 - 9x + 20 = (x - 4)(x - 5)$$

$$\boxed{(x - 4)(x - 5)}$$

2. Factor the expression: $2x^2 + 7x + 3$.

Step 1: Use the ac-method. Here $a = 2$, $b = 7$, $c = 3$, so $ac = 6$.

Find two numbers that multiply to 6 and add to 7: those are 1 and 6.

Step 2: Rewrite the middle term and factor by grouping.

$$2x^2 + 7x + 3 = 2x^2 + x + 6x + 3$$

$$= x(2x + 1) + 3(2x + 1)$$

$$= (2x + 1)(x + 3)$$

$$\boxed{(2x + 1)(x + 3)}$$

3. Solve for x: $x^2 + 4x - 12 = 0$.

Step 1: Factor the left side. Find two numbers that multiply to -12 and add to 4: those are 6 and -2.

$$(x + 6)(x - 2) = 0$$

Step 2: Apply the Zero Product Property.

$$x + 6 = 0 \implies x = -6$$

$$x - 2 = 0 \implies x = 2$$

$$\boxed{x = 2, \quad x = -6}$$

4. Solve for x: $3x^2 - 12x = 0$.

Step 1: Factor out the greatest common factor.

$$3x(x-4) = 0$$

Step 2: Apply the Zero Product Property.

$$3x = 0 \implies x = 0$$

$$x - 4 = 0 \implies x = 4$$

$$\boxed{x = 0, \quad x = 4}$$

5. Identify the vertex of the parabola: $y = -2(x+3)^2 - 7$.

Step 1: Identify the vertex form. The equation is already in vertex form $y = a(x-h)^2+k$, where the vertex is (h, k).

Step 2: Read off the vertex. Here $(x+3) = (x-(-3))$, so $h = -3$ and $k = -7$.

$$\boxed{\text{Vertex} = (-3,\ -7)}$$

6. What is the axis of symmetry for $y = 4(x-5)^2 + 2$?

Step 1: Use the vertex form. The axis of symmetry is the vertical line passing through the vertex. In $y = a(x-h)^2 + k$, the axis of symmetry is $x = h$.

Step 2: Read off h. Here $h = 5$.

$$\boxed{x = 5}$$

7. Convert $y = x^2 - 8x + 10$ to vertex form by completing the square.

Step 1: Group the x-terms.

$$y = (x^2 - 8x) + 10$$

Step 2: Complete the square inside the parentheses. Take half of -8, which is -4, then square it: $(-4)^2 = 16$. Add and subtract 16 inside to keep the expression equal.

$$y = (x^2 - 8x + 16 - 16) + 10$$

$$y = (x^2 - 8x + 16) - 16 + 10$$

Step 3: Write the perfect square trinomial as a squared binomial.

$$y = (x-4)^2 - 6$$

$$\boxed{y = (x-4)^2 - 6}$$

8. Convert $y = 2x^2 + 12x + 5$ to vertex form.

Step 1: Factor out the leading coefficient from the x-terms only.

$$y = 2(x^2 + 6x) + 5$$

Step 2: Complete the square inside the parentheses. Take half of 6, which is 3, then square it: $3^2 = 9$. Add and subtract 9 inside the parentheses.

$$y = 2(x^2 + 6x + 9 - 9) + 5$$

$$y = 2(x^2 + 6x + 9) - 2(9) + 5$$

Step 3: Write the squared binomial and simplify.

$$y = 2(x + 3)^2 - 18 + 5$$

$$y = 2(x + 3)^2 - 13$$

$$\boxed{y = 2(x + 3)^2 - 13}$$

9. Solve using the quadratic formula: $x^2 - 5x + 3 = 0$.

Step 1: Identify a, b, c.

$$a = 1, \quad b = -5, \quad c = 3$$

Step 2: Calculate the discriminant.

$$D = b^2 - 4ac = (-5)^2 - 4(1)(3) = 25 - 12 = 13$$

Step 3: Apply the quadratic formula.

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-5) \pm \sqrt{13}}{2(1)} = \frac{5 \pm \sqrt{13}}{2}$$

$$\boxed{x = \frac{5 \pm \sqrt{13}}{2}}$$

10. Find the discriminant of $2x^2 - 4x + 5 = 0$.

Step 1: Identify a, b, c.

$$a = 2, \quad b = -4, \quad c = 5$$

Step 2: Apply the discriminant formula $D = b^2 - 4ac$.

$$D = (-4)^2 - 4(2)(5) = 16 - 40 = -24$$

$$\boxed{D = -24}$$

11. Based on $D = -24$, what is the nature of the roots?

The discriminant tells us how many real roots exist.

- $D > 0$: two distinct real roots
- $D = 0$: one repeated real root
- $D < 0$: no real roots (the parabola does not cross the x-axis)

Since $D = -24 < 0$, the equation has no real roots.

$$\boxed{\text{No real roots}}$$

12. If the discriminant is 0, the parabola:

When $D = 0$, the quadratic formula gives $x = \dfrac{-b}{2a}$ — a single repeated root. This means the parabola just grazes the x-axis at exactly one point: its vertex. The parabola is **tangent** to the x-axis.

$$\boxed{\text{Is tangent to the } x\text{-axis (touches at exactly one point)}}$$

13. Write a quadratic in standard form with roots $x = 2$ and $x = -5$.

Step 1: Write factors from the roots. If $x = 2$ is a root, then $(x - 2)$ is a factor. If $x = -5$ is a root, then $(x + 5)$ is a factor.

$$y = (x - 2)(x + 5)$$

Step 2: Expand.

$$y = x^2 + 5x - 2x - 10 = x^2 + 3x - 10$$

$$\boxed{y = x^2 + 3x - 10}$$

14. What is the y-intercept of $f(x) = 3x^2 - 4x + 7$?

Step 1: Set $x = 0$. The y-intercept is always the point where $x = 0$.

$$f(0) = 3(0)^2 - 4(0) + 7 = 7$$

$$\boxed{y\text{-intercept} = 7}$$

15. Does $y = -3x^2 + 12x - 5$ have a maximum or minimum, and what is it?

Step 1: Determine maximum or minimum. Since $a = -3 < 0$, the parabola opens downward, so it has a **maximum** value.

Step 2: Find the x-coordinate of the vertex.

$$x = -\frac{b}{2a} = -\frac{12}{2(-3)} = -\frac{12}{-6} = 2$$

Step 3: Substitute $x = 2$ to find the maximum y-value.

$$y = -3(2)^2 + 12(2) - 5 = -3(4) + 24 - 5 = -12 + 24 - 5 = 7$$

$$\boxed{\text{Maximum value of 7}}$$

16. What is the range of $y = (x - 2)^2 + 5$?

Step 1: Identify the vertex. The equation is in vertex form with $h = 2$ and $k = 5$. The vertex is $(2, 5)$.

Step 2: Determine direction. Since $a = 1 > 0$, the parabola opens upward. The vertex is the lowest point on the graph.

Step 3: State the range. The function never goes below $y = 5$.

$$\boxed{y \geq 5}$$

17. A ball is thrown with height $h = -5t^2 + 20t$. How long until max height?

Step 1: Recognize that the maximum occurs at the vertex. Here $a = -5$ and $b = 20$.

Step 2: Use the vertex formula.

$$t = -\frac{b}{2a} = -\frac{20}{2(-5)} = -\frac{20}{-10} = 2$$

$$\boxed{t = 2 \text{ seconds}}$$

18. Using $h = -5t^2 + 20t$, what is the maximum height?

Step 1: Substitute $t = 2$ (from Question 17) into the height equation.

$$h = -5(2)^2 + 20(2) = -5(4) + 40 = -20 + 40 = 20$$

$$\boxed{\text{20 m}}$$

19. Solve for x: $(x-3)^2 = 16$.

Step 1: Take the square root of both sides. Remember to include $\pm$.

$$\sqrt{(x-3)^2} = \pm\sqrt{16}$$

$$x - 3 = \pm 4$$

Step 2: Solve both cases.

$$x - 3 = 4 \implies x = 7$$

$$x - 3 = -4 \implies x = -1$$

$$\boxed{x = 7, \quad x = -1}$$

20. If a quadratic equation has roots $x = 1 \pm \sqrt{2}$, these roots are:

The roots contain $\sqrt{2}$, which is not a perfect square. That means the roots cannot be written as a simple fraction or integer — they are **irrational**. Both roots are real numbers (no imaginary part), so they are **real and irrational**.

$$\boxed{\text{Real and irrational}}$$

21. Factor the difference of squares: $4x^2 - 49$.

Step 1: Recognize the pattern. The difference of squares formula is:

$$A^2 - B^2 = (A-B)(A+B)$$

Step 2: Identify A and B.

$$4x^2 = (2x)^2, \qquad 49 = 7^2$$

So $A = 2x$ and $B = 7$.

Step 3: Apply the formula.

$$4x^2 - 49 = (2x-7)(2x+7)$$

$$\boxed{(2x-7)(2x+7)}$$

22. In $y = a(x-h)^2 + k$, if a increases from 1 to 5, the parabola:

The value of a controls the vertical stretch of the parabola. A larger $|a|$ means the parabola rises (or falls) more steeply for each step away from the vertex. Increasing a from 1 to 5 makes the parabola **narrower**.

$$\boxed{\text{Becomes narrower}}$$

23. If k is decreased, the parabola shifts:

In $y = a(x-h)^2 + k$, the value k is the y-coordinate of the vertex. It shifts the entire parabola vertically. Decreasing k moves the vertex lower, so the parabola shifts **down**.

$$\boxed{\text{Down}}$$

24. Find two numbers whose sum is 10 and whose product is a maximum.

Step 1: Set up variables. Let the two numbers be x and $10 - x$.

Step 2: Write the product as a function of x.

$$P(x) = x(10 - x) = 10x - x^2$$

Step 3: Find the vertex. This is a downward-opening parabola ($a = -1$), so its vertex gives the maximum.

$$x = -\frac{b}{2a} = -\frac{10}{2(-1)} = 5$$

Step 4: Find both numbers.

$$x = 5, \qquad 10 - x = 5$$

$$P = 5 \times 5 = 25$$

$$\boxed{\text{Both numbers are 5, with maximum product 25}}$$

25. Find the equation of a parabola with vertex $(0, 0)$ passing through $(2, 8)$.

Step 1: Use vertex form. Since the vertex is $(0, 0)$, we have $h = 0$ and $k = 0$, so the equation simplifies to:

$$y = ax^2$$

Step 2: Substitute the point $(2, 8)$ to find a.

$$8 = a(2)^2 = 4a$$

$$a = \frac{8}{4} = 2$$

Step 3: Write the equation.

$$y = 2x^2$$

$$\boxed{y = 2x^2}$$

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