Radical Expressions & Equations
Quiz Answer Key
ApexMath

Answer Key

1. Which of the following best describes the principal square root of a non-negative number a?

The **principal square root** of a non-negative number a is defined as the **non-negative** number that, when squared, equals a. In other words, if $b = \sqrt{a}$, then $b \geq 0$ and $b^2 = a$. The word "principal" specifically rules out the negative root.

$$\boxed{\text{The non-negative number } b \geq 0 \text{ such that } b^2 = a}$$

2. Simplify $\sqrt{72}$.

Step 1: Find the largest perfect-square factor of 72.

$$72 = 36 \times 2$$

Step 2: Split the radical and simplify.

$$\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$$

$$\boxed{\sqrt{72} = 6\sqrt{2}}$$

3. Simplify $\sqrt{98}$.

Step 1: Find the largest perfect-square factor of 98.

$$98 = 49 \times 2$$

Step 2: Split and simplify.

$$\sqrt{98} = \sqrt{49 \times 2} = \sqrt{49} \cdot \sqrt{2} = 7\sqrt{2}$$

$$\boxed{\sqrt{98} = 7\sqrt{2}}$$

4. Simplify $\sqrt{18}+\sqrt{50}$.

Step 1: Simplify each radical separately.

$$\sqrt{18}=\sqrt{9\times 2}=3\sqrt{2}$$

$$\sqrt{50}=\sqrt{25\times 2}=5\sqrt{2}$$

Step 2: Add like radicals. Both terms have $\sqrt{2}$, so they can be combined just like like terms.

$$3\sqrt{2}+5\sqrt{2}=(3+5)\sqrt{2}=8\sqrt{2}$$

$$\boxed{\sqrt{18}+\sqrt{50}=8\sqrt{2}}$$

5. Simplify $5\sqrt{12}-2\sqrt{27}$.

Step 1: Simplify each radical.

$$\sqrt{12}=\sqrt{4\times 3}=2\sqrt{3}$$

$$\sqrt{27}=\sqrt{9\times 3}=3\sqrt{3}$$

Step 2: Substitute back and simplify.

$$5\sqrt{12}-2\sqrt{27}=5(2\sqrt{3})-2(3\sqrt{3})=10\sqrt{3}-6\sqrt{3}=4\sqrt{3}$$

$$\boxed{5\sqrt{12}-2\sqrt{27}=4\sqrt{3}}$$

6. Simplify $\sqrt{20}+\sqrt{45}-\sqrt{80}$.

Step 1: Simplify each radical.

$$\sqrt{20}=\sqrt{4\times 5}=2\sqrt{5}$$

$$\sqrt{45}=\sqrt{9\times 5}=3\sqrt{5}$$

$$\sqrt{80}=\sqrt{16\times 5}=4\sqrt{5}$$

Step 2: Combine all like radicals.

$$2\sqrt{5}+3\sqrt{5}-4\sqrt{5}=(2+3-4)\sqrt{5}=1\sqrt{5}=\sqrt{5}$$

$$\boxed{\sqrt{20}+\sqrt{45}-\sqrt{80}=\sqrt{5}}$$

7. Evaluate $\sqrt{2} \cdot \sqrt{32}$.

Step 1: Multiply the radicands. The rule is $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$.

$$\sqrt{2} \cdot \sqrt{32} = \sqrt{2 \times 32} = \sqrt{64}$$

Step 2: Evaluate.

$$\sqrt{64} = 8$$

$$\boxed{\sqrt{2} \cdot \sqrt{32} = 8}$$

8. Simplify $(2\sqrt{3})(3\sqrt{6})$.

Step 1: Multiply the whole-number parts together and the radical parts together.

$$(2\sqrt{3})(3\sqrt{6}) = (2 \times 3)\left(\sqrt{3} \times \sqrt{6}\right) = 6\sqrt{18}$$

Step 2: Simplify $\sqrt{18}$.

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

Step 3: Substitute back.

$$6\sqrt{18} = 6 \cdot 3\sqrt{2} = 18\sqrt{2}$$

$$\boxed{(2\sqrt{3})(3\sqrt{6}) = 18\sqrt{2}}$$

9. Evaluate $(\sqrt{5} + 2)(\sqrt{5} - 2)$.

Step 1: Recognize the difference-of-squares pattern. This has the form $(A+B)(A-B) = A^2 - B^2$.

Here $A = \sqrt{5}$ and $B = 2$.

Step 2: Apply the formula.

$$(\sqrt{5} + 2)(\sqrt{5} - 2) = (\sqrt{5})^2 - (2)^2 = 5 - 4 = 1$$

$$\boxed{(\sqrt{5} + 2)(\sqrt{5} - 2) = 1}$$

10. Simplify $\dfrac{\sqrt{24}}{\sqrt{6}}$.

Step 1: Use the quotient rule for radicals. $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$.

$$\frac{\sqrt{24}}{\sqrt{6}} = \sqrt{\frac{24}{6}} = \sqrt{4}$$

Step 2: Evaluate.

$$\sqrt{4} = 2$$

$$\boxed{\frac{\sqrt{24}}{\sqrt{6}} = 2}$$

11. Simplify $\dfrac{\sqrt{75}}{\sqrt{3}}$.

Step 1: Apply the quotient rule.

$$\frac{\sqrt{75}}{\sqrt{3}} = \sqrt{\frac{75}{3}} = \sqrt{25}$$

Step 2: Evaluate.

$$\sqrt{25} = 5$$

$$\boxed{\frac{\sqrt{75}}{\sqrt{3}} = 5}$$

12. Rationalize the denominator: $\dfrac{5}{\sqrt{5}}$.

Step 1: Multiply numerator and denominator by $\sqrt{5}$. This clears the radical from the denominator without changing the value of the expression.

$$\frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{(\sqrt{5})^2} = \frac{5\sqrt{5}}{5}$$

Step 2: Cancel the common factor of 5.

$$\frac{5\sqrt{5}}{5} = \sqrt{5}$$

$$\boxed{\frac{5}{\sqrt{5}} = \sqrt{5}}$$

13. Rationalize the denominator: $\dfrac{3}{\sqrt{2}}$.

Step 1: Multiply numerator and denominator by $\sqrt{2}$.

$$\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{(\sqrt{2})^2} = \frac{3\sqrt{2}}{2}$$

The denominator is now rational. There is no common factor to cancel.

$$\boxed{\frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}}$$

14. Rationalize the denominator: $\dfrac{4}{\sqrt{6}-2}$.

Step 1: Identify the conjugate. When the denominator is a binomial containing a radical, multiply by its conjugate. The conjugate of $(\sqrt{6}-2)$ is $(\sqrt{6}+2)$.

Step 2: Multiply numerator and denominator by the conjugate.

$$\frac{4}{\sqrt{6}-2} \cdot \frac{\sqrt{6}+2}{\sqrt{6}+2} = \frac{4(\sqrt{6}+2)}{(\sqrt{6})^2 - (2)^2} = \frac{4(\sqrt{6}+2)}{6-4} = \frac{4(\sqrt{6}+2)}{2}$$

Step 3: Simplify.

$$\frac{4(\sqrt{6}+2)}{2} = 2(\sqrt{6}+2) = 2\sqrt{6}+4$$

$$\boxed{\frac{4}{\sqrt{6}-2} = 2\sqrt{6}+4}$$

15. Rationalize the denominator: $\dfrac{1}{\sqrt{3}+\sqrt{2}}$.

Step 1: Identify the conjugate. The conjugate of $(\sqrt{3}+\sqrt{2})$ is $(\sqrt{3}-\sqrt{2})$.

Step 2: Multiply by the conjugate.

$$\frac{1}{\sqrt{3}+\sqrt{2}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \frac{\sqrt{3}-\sqrt{2}}{1}$$

Step 3: Simplify.

$$= \sqrt{3}-\sqrt{2}$$

$$\boxed{\frac{1}{\sqrt{3}+\sqrt{2}} = \sqrt{3}-\sqrt{2}}$$

16. Solve for x: $\sqrt{x} = 7$.

Step 1: Square both sides to eliminate the radical.

$$(\sqrt{x})^2 = 7^2$$

$$x = 49$$

Step 2: Check. Substitute back into the original equation.

$$\sqrt{49} = 7 \quad \checkmark$$

$$\boxed{x = 49}$$

17. Solve for x: $\sqrt{x+5} = 3$.

Step 1: Square both sides.

$$(\sqrt{x+5})^2 = 3^2$$

$$x + 5 = 9$$

Step 2: Solve for x.

$$x = 9 - 5 = 4$$

Step 3: Check.

$$\sqrt{4+5} = \sqrt{9} = 3 \quad \checkmark$$

$$\boxed{x = 4}$$

18. Solve for x: $\sqrt{3x+1} = 4$.

Step 1: Square both sides.

$$(\sqrt{3x+1})^2 = 4^2$$

$$3x + 1 = 16$$

Step 2: Solve for x.

$$3x = 15$$

$$x = 5$$

Step 3: Check.

$$\sqrt{3(5)+1} = \sqrt{16} = 4 \quad \checkmark$$

$$\boxed{x = 5}$$

19. Solve for x: $\sqrt{2x-1} = x - 2$.

Step 1: Square both sides.

$$(\sqrt{2x-1})^2 = (x-2)^2$$

$$2x - 1 = x^2 - 4x + 4$$

Step 2: Rearrange into standard quadratic form. Move all terms to one side.

$$0 = x^2 - 4x - 2x + 4 + 1$$

$$0 = x^2 - 6x + 5$$

Step 3: Factor.

$$0 = (x-1)(x-5)$$

$$x = 1 \qquad \text{or} \qquad x = 5$$

Step 4: Check both solutions. Squaring can introduce extraneous roots, so checking is required.

Check $x = 1$:

$$\sqrt{2(1)-1} = \sqrt{1} = 1 \qquad \text{and} \qquad x - 2 = 1 - 2 = -1$$

$1 \neq -1$, so $x = 1$ is **extraneous** (rejected).

Check $x = 5$:

$$\sqrt{2(5)-1} = \sqrt{9} = 3 \qquad \text{and} \qquad x - 2 = 5 - 2 = 3$$

$3 = 3$ ✓

$$\boxed{x = 5 \text{ only} \quad (x = 1 \text{ is extraneous})}$$

20. Which of the following is the solution to $\sqrt{x} = -3$?

The principal square root of any real number is always **non-negative** by definition. There is no real number x for which $\sqrt{x}$ equals a negative value. Therefore, this equation has no solution.

$$\boxed{\text{No real solution}}$$

21. Find the domain restriction for $\sqrt{2x-10}$.

Step 1: Set the radicand greater than or equal to zero. The expression under a square root must be non-negative.

$$2x - 10 \geq 0$$

Step 2: Solve for x.

$$2x \geq 10$$

$$x \geq 5$$

$$\boxed{x \geq 5}$$

22. Determine the domain of $f(x) = \dfrac{1}{\sqrt{x-4}}$.

Step 1: Identify the restrictions. There are two issues to consider.

First, the expression under the square root must be non-negative:

$$x - 4 \geq 0 \implies x \geq 4$$

Second, the square root is in the *denominator*, so it cannot equal zero (division by zero is undefined):

$$\sqrt{x-4} \neq 0 \implies x - 4 \neq 0 \implies x \neq 4$$

Step 2: Combine both restrictions. $x \geq 4$ and $x \neq 4$ together give $x > 4$.

$$\boxed{x > 4}$$

23. Simplify $\sqrt[3]{54}$.

Step 1: Find the largest perfect-cube factor of 54.

$$54 = 27 \times 2$$

Step 2: Split and simplify.

$$\sqrt[3]{54} = \sqrt[3]{27 \times 2} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$$

$$\boxed{\sqrt[3]{54} = 3\sqrt[3]{2}}$$

24. Simplify $\sqrt[4]{48}$.

Step 1: Find the largest perfect-fourth-power factor of 48.

$$48 = 16 \times 3$$

Step 2: Split and simplify.

$$\sqrt[4]{48} = \sqrt[4]{16 \times 3} = \sqrt[4]{16} \cdot \sqrt[4]{3} = 2\sqrt[4]{3}$$

$$\boxed{\sqrt[4]{48} = 2\sqrt[4]{3}}$$

25. Simplify $\sqrt{x^2+6x+9}$.

Step 1: Factor the expression under the radical. Recognize this as a perfect-square trinomial.

$$x^2+6x+9=(x+3)^2$$

Step 2: Apply the square root.

$$\sqrt{(x+3)^2}$$

Step 3: Simplify using the rule $\sqrt{u^2}=|u|$. Since x can be any real number, $(x+3)$ could be negative. The square root must be non-negative, so we use the absolute value.

$$\sqrt{(x+3)^2}=|x+3|$$

$$\boxed{\sqrt{x^2+6x+9}=|x+3|}$$

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