Rational Expressions & Equations
Quiz Answer Key
ApexMath

Answer Key

1. Which of the following describes a rational expression?

A **rational expression** is a fraction whose numerator and denominator are both polynomials. Just as a rational number is the quotient of two integers, a rational expression is the quotient of two polynomials.

$$\boxed{\text{The quotient of two polynomials}}$$

2. Simplify $\dfrac{x^2-9}{x-3}$.

Step 1: Factor the numerator. Recognize $x^2 - 9$ as a difference of squares.

$$x^2 - 9 = (x+3)(x-3)$$

Step 2: Rewrite and cancel. The factor $(x-3)$ appears in both the numerator and denominator.

$$\frac{(x+3)(x-3)}{x-3} = x+3$$

Note: $x \neq 3$ because the original denominator is zero there.

$$\boxed{x+3, \quad x \neq 3}$$

3. Simplify $\dfrac{x^2-5x+6}{x-2}$.

Step 1: Factor the numerator. Find two numbers that multiply to 6 and add to -5: those are -2 and -3.

$$x^2 - 5x + 6 = (x-2)(x-3)$$

Step 2: Cancel the common factor.

$$\frac{(x-2)(x-3)}{x-2} = x-3$$

Note: $x \neq 2$.

$$\boxed{x-3, \quad x \neq 2}$$

4. Identify the non-permissible values for $\dfrac{x+1}{x^2-4}$.

Step 1: Set the denominator equal to zero and solve.

$$x^2 - 4 = 0$$

Step 2: Factor.

$$(x+2)(x-2) = 0$$

$$x = -2 \qquad \text{or} \qquad x = 2$$

These are the values that make the expression undefined.

$$\boxed{x \neq \pm 2}$$

5. Simplify the product $\dfrac{x}{2} \cdot \dfrac{4}{x^2}$.

Step 1: Multiply numerators together and denominators together.

$$\frac{x}{2} \cdot \frac{4}{x^2} = \frac{x \cdot 4}{2 \cdot x^2} = \frac{4x}{2x^2}$$

Step 2: Cancel common factors. Divide numerator and denominator by $2x$.

$$\frac{4x}{2x^2} = \frac{2}{x}$$

Note: $x \neq 0$.

$$\boxed{\frac{2}{x}, \quad x \neq 0}$$

6. Multiply and simplify $\dfrac{x+1}{x-2} \cdot \dfrac{x-2}{x+3}$.

Step 1: Multiply across.

$$\frac{(x+1)(x-2)}{(x-2)(x+3)}$$

Step 2: Cancel the common factor $(x-2)$.

$$\frac{(x+1)(x-2)}{(x-2)(x+3)} = \frac{x+1}{x+3}$$

Non-permissible values: $x \neq 2$ (original denominator) and $x \neq -3$.

$$\boxed{\frac{x+1}{x+3}, \quad x \neq 2,\ -3}$$

7. Simplify the quotient $\frac{x}{5} \div \frac{x^2}{10}$.

Step 1: Rewrite division as multiplication by the reciprocal.

$$\frac{x}{5} \div \frac{x^2}{10} = \frac{x}{5} \cdot \frac{10}{x^2}$$

Step 2: Multiply and simplify.

$$\frac{x \cdot 10}{5 \cdot x^2} = \frac{10x}{5x^2} = \frac{2}{x}$$

Note: $x \neq 0$.

$$\boxed{\frac{2}{x}, \quad x \neq 0}$$

8. Divide and simplify $\frac{x^2-1}{x} \div \frac{x+1}{x^2}$.

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{x^2-1}{x} \cdot \frac{x^2}{x+1}$$

Step 2: Factor the numerator $x^2 - 1$.

$$x^2 - 1 = (x+1)(x-1)$$

Step 3: Substitute and cancel.

$$\frac{(x+1)(x-1)}{x} \cdot \frac{x^2}{x+1} = \frac{(x+1)(x-1) \cdot x^2}{x(x+1)}$$

$$= \frac{(x+1)(x-1) \cdot x^2}{x\,(x+1)} = x(x-1)$$

Non-permissible values: $x \neq 0$ and $x \neq -1$.

$$\boxed{x(x-1), \quad x \neq 0,\ -1}$$

9. Add $\frac{3}{x} + \frac{2}{x}$.

Step 1: The denominators are the same, so add the numerators directly.

$$\frac{3}{x} + \frac{2}{x} = \frac{3+2}{x} = \frac{5}{x}$$

Note: $x \neq 0$.

$$\boxed{\frac{5}{x}, \quad x \neq 0}$$

10. Add and simplify $\frac{3}{x+2} + \frac{2}{x-1}$.

Step 1: Find the lowest common denominator (LCD). Since the denominators share no common factors, the LCD is $(x+2)(x-1)$.

Step 2: Rewrite each fraction with the LCD.

$$\frac{3}{x+2} \cdot \frac{x-1}{x-1} + \frac{2}{x-1} \cdot \frac{x+2}{x+2} = \frac{3(x-1)}{(x+2)(x-1)} + \frac{2(x+2)}{(x-1)(x+2)}$$

Step 3: Add the numerators.

$$\frac{3(x-1)+2(x+2)}{(x+2)(x-1)} = \frac{3x-3+2x+4}{(x+2)(x-1)} = \frac{5x+1}{(x+2)(x-1)}$$

$$\boxed{\frac{5x+1}{(x+2)(x-1)}}$$

11. Subtract $\frac{5}{x} - \frac{3}{x}$.

Step 1: The denominators are the same, so subtract the numerators directly.

$$\frac{5}{x} - \frac{3}{x} = \frac{5-3}{x} = \frac{2}{x}$$

Note: $x \neq 0$.

$$\boxed{\frac{2}{x}, \quad x \neq 0}$$

12. Simplify $\frac{1}{x-1} - \frac{1}{x+1}$.

Step 1: Find the LCD. The LCD is $(x-1)(x+1)$.

Step 2: Rewrite each fraction.

$$\frac{1(x+1)}{(x-1)(x+1)} - \frac{1(x-1)}{(x+1)(x-1)}$$

Step 3: Subtract the numerators.

$$\frac{(x+1)-(x-1)}{(x-1)(x+1)} = \frac{x+1-x+1}{(x-1)(x+1)} = \frac{2}{(x-1)(x+1)}$$

Step 4: Recognize the denominator as a difference of squares.

$$(x-1)(x+1) = x^2 - 1$$

$$\boxed{\frac{2}{x^2-1}}$$

13. Simplify the complex fraction $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}$.

Step 1: Simplify the numerator. The LCD for $\frac{1}{x}$ and $\frac{1}{y}$ is xy.

$$\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\frac{y+x}{xy}$$

Step 2: Simplify the denominator.

$$\frac{1}{x}-\frac{1}{y}=\frac{y}{xy}-\frac{x}{xy}=\frac{y-x}{xy}$$

Step 3: Divide the numerator by the denominator.

$$\frac{\frac{y+x}{xy}}{\frac{y-x}{xy}}=\frac{y+x}{xy}\cdot\frac{xy}{y-x}=\frac{(y+x)\,xy}{xy\,(y-x)}$$

$$=\frac{y+x}{y-x}$$

$$\boxed{\frac{y+x}{y-x}}$$

14. Solve $\dfrac{x}{3}=\dfrac{4}{6}$.

Step 1: Simplify the right side.

$$\frac{4}{6}=\frac{2}{3}$$

Step 2: Since both sides now have the same denominator, set numerators equal.

$$\frac{x}{3}=\frac{2}{3}\implies x=2$$

Step 3: Check. $\dfrac{2}{3}=\dfrac{4}{6}$ ✓

$$\boxed{x=2}$$

15. Solve $\dfrac{2x+1}{x-3} = 5$.

Step 1: Multiply both sides by $(x-3)$ to clear the denominator.

$$2x + 1 = 5(x-3)$$

Step 2: Expand the right side.

$$2x + 1 = 5x - 15$$

Step 3: Isolate x.

$$1 + 15 = 5x - 2x$$
$$16 = 3x$$
$$x = \frac{16}{3}$$

Step 4: Check. $x = \frac{16}{3} \neq 3$, so it is permissible.

$$\frac{2\left(\frac{16}{3}\right)+1}{\frac{16}{3}-3} = \frac{\frac{32}{3}+\frac{3}{3}}{\frac{16}{3}-\frac{9}{3}} = \frac{\frac{35}{3}}{\frac{7}{3}} = \frac{35}{7} = 5 \quad \checkmark$$

$$\boxed{x = \frac{16}{3}}$$

16. Solve $\dfrac{1}{x} + \dfrac{1}{2} = \dfrac{3}{4}$.

Step 1: Multiply every term by the LCD. The LCD of x, 2, and 4 is $4x$.

$$4x \cdot \frac{1}{x} + 4x \cdot \frac{1}{2} = 4x \cdot \frac{3}{4}$$
$$4 + 2x = 3x$$

Step 2: Solve for x.

$$4 = 3x - 2x = x$$
$$x = 4$$

Step 3: Check.

$$\frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4} \quad \checkmark$$
$$\boxed{x = 4}$$

17. Solve $\dfrac{x}{x-1} = 1 + \dfrac{1}{x-1}$.

Step 1: Multiply every term by $(x-1)$ to clear the denominators.

$$(x-1)\cdot\frac{x}{x-1} = (x-1)\cdot 1 + (x-1)\cdot\frac{1}{x-1}$$

$$x = (x-1)+1$$

$$x = x$$

Step 2: Interpret the result. The equation $x = x$ is true for every value of x. This means the equation is an identity — except that $x = 1$ makes the original denominator $(x-1)$ equal to zero, so $x = 1$ is non-permissible and must be excluded. With that restriction applied, there is no specific solution.

$$\boxed{\text{No solution}\quad (x = 1 \text{ is non-permissible})}$$

18. Solve $\dfrac{x+2}{x-2} = 0$.

Step 1: A fraction equals zero when its numerator equals zero and its denominator does not equal zero.

Set the numerator equal to zero:

$$x + 2 = 0 \implies x = -2$$

Step 2: Verify the denominator is not zero at $x = -2$.

$$x - 2 = -2 - 2 = -4 \neq 0 \quad \checkmark$$

$$\boxed{x = -2}$$

19. What are the restrictions for $\dfrac{x^2-1}{x^2+1}$?

Step 1: Set the denominator equal to zero.

$$x^2 + 1 = 0 \implies x^2 = -1$$

Step 2: Determine whether real solutions exist. Since $x^2 \geq 0$ for all real x, there is no real number whose square equals -1. The denominator is always positive and can never be zero.

$$\boxed{\text{No real restrictions}}$$

20. Simplify $\frac{x/y}{x^2/y^3}$.

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{x}{y} \div \frac{x^2}{y^3} = \frac{x}{y} \cdot \frac{y^3}{x^2}$$

Step 2: Multiply and cancel.

$$\frac{x \cdot y^3}{y \cdot x^2} = \frac{y^3}{y \cdot x} = \frac{y^2}{x}$$

$$\boxed{\frac{y^2}{x}}$$

21. Simplify $\frac{2x+4}{x^2-4}$.

Step 1: Factor the numerator.

$$2x + 4 = 2(x + 2)$$

Step 2: Factor the denominator. Recognize a difference of squares.

$$x^2 - 4 = (x + 2)(x - 2)$$

Step 3: Cancel the common factor $(x + 2)$.

$$\frac{2(x+2)}{(x+2)(x-2)} = \frac{2}{x-2}$$

Non-permissible values: $x \neq \pm 2$.

$$\boxed{\frac{2}{x-2}, \quad x \neq \pm 2}$$

22. Simplify $\dfrac{x^2+3x+2}{x^2+x}$.

Step 1: Factor the numerator. Find two numbers that multiply to 2 and add to 3: those are 1 and 2.

$$x^2+3x+2=(x+1)(x+2)$$

Step 2: Factor the denominator.

$$x^2+x=x(x+1)$$

Step 3: Cancel the common factor $(x+1)$.

$$\frac{(x+1)(x+2)}{x(x+1)}=\frac{x+2}{x}$$

Non-permissible values: $x \neq 0$ and $x \neq -1$.

$$\boxed{\frac{x+2}{x}, \quad x \neq 0,\ -1}$$

23. Pipe A fills a tank in 2 hours and Pipe B fills it in 3 hours. How long does it take together?

Step 1: Find each pipe's rate. Rate is the fraction of the tank filled per hour.

$$\text{Rate of A} = \frac{1}{2}\ \text{tank/hr}, \qquad \text{Rate of B} = \frac{1}{3}\ \text{tank/hr}$$

Step 2: Add the rates to get the combined rate.

$$\text{Combined rate} = \frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}\ \text{tank/hr}$$

Step 3: Time = 1 tank ÷ combined rate.

$$t=\frac{1}{\frac{5}{6}}=\frac{6}{5}=1.2\ \text{hours}$$

$$\boxed{t=\frac{6}{5}=1.2\ \text{hours}}$$

24. Solve $\dfrac{3}{x} - \dfrac{1}{2} = \dfrac{1}{x}$.

Step 1: Multiply every term by the LCD. The LCD of x and 2 is $2x$.

$$2x \cdot \frac{3}{x} - 2x \cdot \frac{1}{2} = 2x \cdot \frac{1}{x}$$

$$6 - x = 2$$

Step 2: Solve for x.

$$6 - 2 = x$$

$$x = 4$$

Step 3: Check.

$$\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4} = \frac{1}{x} \quad \checkmark$$

$$\boxed{x = 4}$$

25. Find the domain of $f(x) = \dfrac{x-5}{x^2-25}$.

Step 1: Set the denominator equal to zero and solve.

$$x^2 - 25 = 0$$

Step 2: Factor.

$$(x+5)(x-5) = 0$$

$$x = -5 \qquad \text{or} \qquad x = 5$$

These are the values where the function is undefined.

Note: Even though $(x-5)$ is a factor of both the numerator and denominator (and would cancel to give $\frac{1}{x+5}$), the restriction $x \neq 5$ still applies to the *original* function.

$$\boxed{\{x \mid x \neq \pm 5\}}$$

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