# Systems of Equations & Inequalities
# Quiz Answer Key
# ApexMath

## Answer Key

**1.** Solve the system: $x + y = 10$ and $x - y = 4$.

**Step 1: Add the two equations to eliminate $y$.**

$$(x + y) + (x - y) = 10 + 4$$

$$2x = 14 \implies x = 7$$

**Step 2: Substitute $x = 7$ into the first equation to find $y$.**

$$7 + y = 10 \implies y = 3$$

$$\boxed{x = 7, \quad y = 3}$$

**2.** Solve for $y$ using substitution: $2x - y = 1$ and $x = y + 2$.

**Step 1: Substitute $x = y + 2$ into the first equation.**

$$2(y + 2) - y = 1$$

**Step 2: Expand and simplify.**

$$2y + 4 - y = 1$$

$$y + 4 = 1$$

$$y = -3$$

$$\boxed{y = -3}$$

**3.** Solve the system: $3x + 2y = 12$ and $4x - 2y = 2$.

**Step 1: Add the two equations.** The $y$-terms cancel because $+2y$ and $-2y$ are opposites.

$$(3x + 2y) + (4x - 2y) = 12 + 2$$

$$7x = 14 \implies x = 2$$

**Step 2: Substitute $x = 2$ into the first equation.**

$$3(2) + 2y = 12 \implies 6 + 2y = 12 \implies 2y = 6 \implies y = 3$$

$$\boxed{x = 2, \quad y = 3}$$

**4.** How many solutions does a system of parallel lines have?

Parallel lines have the same slope but different $y$-intercepts, so they never intersect. A solution to a system is a point where both equations are satisfied at the same time — and no such point exists.

$$\boxed{\text{No solution}}$$

**5.** How many points of intersection can a line and a parabola have?

A line can miss a parabola entirely, be tangent to it (touching at exactly one point), or cross through it at two points. These are all three geometric possibilities.

$$\boxed{0,\ 1,\ \text{or } 2}$$

**6.** Solve the system: $y = x^2$ and $y = x + 2$.

**Step 1: Set the expressions equal to each other.**

$$x^2 = x + 2$$

**Step 2: Rearrange into standard form.**

$$x^2 - x - 2 = 0$$

**Step 3: Factor.**

$$(x - 2)(x + 1) = 0 \implies x = 2 \quad \text{or} \quad x = -1$$

**Step 4: Find the corresponding $y$-values using $y = x + 2$.**

$$x = 2: \quad y = 2 + 2 = 4$$

$$x = -1: \quad y = -1 + 2 = 1$$

$$\boxed{(-1,\ 1) \text{ and } (2,\ 4)}$$

**7.** Solve the system: $y = x^2 - 4$ and $y = -3$.

**Step 1: Substitute $y = -3$ into the first equation.**

$$-3 = x^2 - 4$$

**Step 2: Solve for $x$.**

$$x^2 = -3 + 4 = 1$$
$$x = \pm 1$$

**Step 3: Write the solution points.** Both have $y = -3$:

$$(-1,\ -3) \quad \text{and} \quad (1,\ -3)$$

$$\boxed{x = \pm 1 \quad \big((-1,\ -3) \text{ and } (1,\ -3)\big)}$$

**8.** How many solutions can two upward-opening parabolas have?

Two parabolas can miss each other entirely, touch at one point, cross at two points, or be identical (the same parabola), which gives infinitely many shared points.

$$\boxed{0,\ 1,\ 2,\ \text{or infinitely many}}$$

**9.** Solve the system: $y = x^2 + 3$ and $y = -x^2 + 5$.

**Step 1: Set the expressions equal.**

$$x^2 + 3 = -x^2 + 5$$

**Step 2: Collect like terms.**

$$2x^2 = 2 \implies x^2 = 1 \implies x = \pm 1$$

**Step 3: Find $y$ using $y = x^2 + 3$.**

$$x = 1: \quad y = 1 + 3 = 4$$

$$x = -1: \quad y = 1 + 3 = 4$$

$$\boxed{(1,\ 4) \text{ and } (-1,\ 4)}$$

**10.** Which point is in the solution region of $y > x + 1$ and $y < -x + 5$? Test $(0, 2)$.

**Check inequality 1:** $y > x + 1$

$$2 > 0 + 1 \implies 2 > 1 \quad \checkmark$$

**Check inequality 2:** $y < -x + 5$

$$2 < -0 + 5 \implies 2 < 5 \quad \checkmark$$

Both inequalities are satisfied.

$$\boxed{(0,\ 2)}$$

**11.** The solution to a system of inequalities is represented graphically by:

Each inequality shades a half-plane. A point satisfies the entire system only if it lies in *every* shaded region at once, which is their intersection.

$$\boxed{\text{The overlapping (intersecting) shaded region}}$$

**12.** The sum of two numbers is 20 and their difference is 2. Find the larger number.

**Step 1: Write the system.** Let the two numbers be $x$ (larger) and $y$ (smaller).

$$x + y = 20 \qquad x - y = 2$$

**Step 2: Add the equations to eliminate $y$.**

$$2x = 22 \implies x = 11$$

**Step 3: Find $y$.**

$$11 + y = 20 \implies y = 9$$

$$\boxed{\text{The larger number is 11}}$$

**13.** A rectangle has a perimeter of 30 cm and its length is twice the width. Find the width.

**Step 1: Write the equations.** Let $w$ = width, $l$ = length.

$$l = 2w \qquad 2(l + w) = 30$$

**Step 2: Substitute $l = 2w$ into the perimeter equation.**

$$2(2w + w) = 30 \implies 2(3w) = 30 \implies 6w = 30$$

**Step 3: Solve.**

$$w = 5 \text{ cm}, \qquad l = 2(5) = 10 \text{ cm}$$

$$\boxed{\text{Width} = 5 \text{ cm}}$$

**14.** Solve: $y = x + 4$ and $y = x^2 - 2$.

**Step 1: Set the expressions equal.**

$$x + 4 = x^2 - 2$$

**Step 2: Rearrange.**

$$0 = x^2 - x - 6$$

**Step 3: Factor.**

$$(x - 3)(x + 2) = 0 \implies x = 3 \quad \text{or} \quad x = -2$$

**Step 4: Find $y$-values using $y = x + 4$.**

$$x = 3: \quad y = 3 + 4 = 7$$

$$x = -2: \quad y = -2 + 4 = 2$$

$$\boxed{(-2,\ 2) \text{ and } (3,\ 7)}$$

**15.** Solve: $y = x^2 + 1$ and $y = 2x^2 - 3$.

**Step 1: Set the expressions equal.**

$$x^2 + 1 = 2x^2 - 3$$

**Step 2: Rearrange.**

$$0 = x^2 - 4 \implies x^2 = 4 \implies x = \pm 2$$

**Step 3: Find $y$ using $y = x^2 + 1$.**

$$x = 2: \quad y = 4 + 1 = 5$$

$$x = -2: \quad y = 4 + 1 = 5$$

$$\boxed{(2,\ 5) \text{ and } (-2,\ 5)}$$

**16.** If a linear-quadratic system has no real solutions, the line:

Setting the equations equal produces a quadratic with a negative discriminant, meaning no real intersection points. Geometrically, the line passes entirely above or below the parabola and never crosses it.

$$\boxed{\text{Does not intersect the parabola}}$$

**17.** Which inequality describes the region *below* the line $y = 2x - 3$?

Points below a line have $y$-values smaller than those on the line. Replacing $=$ with $<$ keeps the same boundary but selects everything underneath it.

$$\boxed{y < 2x - 3}$$

**18.** Is $(2, 5)$ a solution to $y = 2x + 1$ and $y = x^2 + 1$?

**Check equation 1:** $y = 2x + 1$

$$5 = 2(2) + 1 = 4 + 1 = 5 \quad \checkmark$$

**Check equation 2:** $y = x^2 + 1$

$$5 = (2)^2 + 1 = 4 + 1 = 5 \quad \checkmark$$

Both equations are satisfied.

$$\boxed{\text{Yes, } (2, 5) \text{ is a solution to both equations.}}$$

**19.** Solve: $2x + 3y = 6$ and $4x + 6y = 12$.

**Step 1: Compare the two equations.** Multiply the first equation by 2:

$$2(2x + 3y) = 2(6) \implies 4x + 6y = 12$$

This is identical to the second equation. The two equations represent the **same line**, so every point on that line is a solution.

$$\boxed{\text{Infinitely many solutions}}$$

**20.** Two angles are supplementary. One is 20 more than the other. Find the smaller angle.

**Step 1: Write the system.** Let the two angles be $a$ (larger) and $b$ (smaller).

$$a + b = 180 \qquad a = b + 20$$

**Step 2: Substitute $a = b + 20$.**

$$(b + 20) + b = 180 \implies 2b + 20 = 180 \implies 2b = 160 \implies b = 80$$

**Step 3: Find $a$.**

$$a = 80 + 20 = 100$$

$$\boxed{\text{The smaller angle is } 80}$$

**21.** A dashed boundary line on a graph indicates:

A *solid* line means points on the boundary satisfy the inequality ($\leq$ or $\geq$). A *dashed* line means the boundary itself is excluded ($<$ or $>$) — points on the line are not solutions.

$$\boxed{\text{Points on the line are NOT solutions (strict inequality)}}$$

**22.** Solve: $y = x^2$ and $y = -x^2$.

**Step 1: Set the expressions equal.**

$$x^2 = -x^2$$

**Step 2: Solve.**

$$2x^2 = 0 \implies x^2 = 0 \implies x = 0$$

**Step 3: Find $y$.**

$$y = 0^2 = 0$$

The only intersection point is the origin.

$$\boxed{(0,\ 0)}$$

**23.** Solve for $x$: $x + 2y = 8$ and $3x - 2y = 8$.

**Step 1: Add the equations.** The $y$-terms cancel.

$$(x + 2y) + (3x - 2y) = 8 + 8$$

$$4x = 16 \implies x = 4$$

**Step 2: Find $y$ to confirm (optional).**

$$4 + 2y = 8 \implies 2y = 4 \implies y = 2$$

$$\boxed{x = 4}$$

**24.** Mix A (10% salt) and Mix B (20% salt) to make 100 g of 15% salt solution.

**Step 1: Set up variables.** Let $a$ = grams of Mix A, $b$ = grams of Mix B.

**Step 2: Write the system.**

$$a + b = 100 \qquad \text{(total mass)}$$

$$0.10a + 0.20b = 0.15(100) = 15 \qquad \text{(salt content)}$$

**Step 3: Solve.** From the first equation, $a = 100 - b$. Substitute:

$$0.10(100 - b) + 0.20b = 15$$

$$10 - 0.10b + 0.20b = 15$$

$$10 + 0.10b = 15 \implies 0.10b = 5 \implies b = 50$$

$$a = 100 - 50 = 50$$

$$\boxed{\text{50 g of Mix A and 50 g of Mix B}}$$

**25.** A system of two lines with different slopes has:

Two lines with the same slope are parallel (or identical). Two lines with *different* slopes will always cross each other at exactly one point, regardless of where they are positioned.

$$\boxed{\text{Exactly one solution}}$$

---

# Thank you for learning with ApexMath!

If you found this lesson helpful, we'd love for you to share your feedback or leave a review.

*End of Lesson • ApexMath*

---