Trigonometry Quiz
Answer Key
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1. An angle of $150°$ is in standard position. Which quadrant is the terminal arm in?

Step 1: Recall that quadrants are defined by the following angle ranges:

$$\text{QI: } 0° < \theta < 90°, \quad \text{QII: } 90° < \theta < 180°, \quad \text{QIII: } 180° < \theta < 270°, \quad \text{QIV: } 270° < \theta < 360°$$

Step 2: Since $90° < 150° < 180°$, the terminal arm lies in Quadrant II.

$$\boxed{\text{Quadrant II}}$$

2. Find the reference angle for $\theta = 240°$.

Step 1: Determine the quadrant. Since $180° < 240° < 270°$, the angle is in Quadrant III.

Step 2: For an angle in Quadrant III, subtract $180°$ to find the reference angle.

$$\theta_R = 240° - 180° = 60°$$

$$\boxed{\theta_R = 60°}$$

3. Find the reference angle for $\theta = 315°$.

Step 1: Determine the quadrant. Since $270° < 315° < 360°$, the angle is in Quadrant IV.

Step 2: For an angle in Quadrant IV, subtract from $360°$ to find the reference angle.

$$\theta_R = 360° - 315° = 45°$$

$$\boxed{\theta_R = 45°}$$

4. If the point $(-3,\ 4)$ lies on the terminal arm of angle θ in standard position, what is $\sin(\theta)$?

Step 1: Identify the coordinates: $x = -3$ and $y = 4$.

Step 2: Find r, the distance from the origin to the point, using the Pythagorean theorem.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 3: Apply the definition $\sin(\theta) = \dfrac{y}{r}$.

$$\sin(\theta) = \frac{4}{5}$$

$$\boxed{\sin(\theta) = \frac{4}{5}}$$

5. If $\tan(\theta) = -1$ and θ is in Quadrant II, what is $\cos(\theta)$?

Step 1: Recall that $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} = -1$, which means $\sin(\theta) = -\cos(\theta)$.

Step 2: Substitute into the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

$$(-\cos\theta)^2 + \cos^2(\theta) = 1 \qquad \Rightarrow \qquad 2\cos^2(\theta) = 1 \qquad \Rightarrow \qquad \cos^2(\theta) = \frac{1}{2}$$

Step 3: Take the square root.

$$\cos(\theta) = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

Step 4: In Quadrant II, cosine is negative. Select the negative value.

$$\cos(\theta) = -\frac{\sqrt{2}}{2}$$

$$\boxed{\cos(\theta) = -\frac{\sqrt{2}}{2}}$$

6. In $\triangle ABC$, $a = 10$, $\angle A = 30^\circ$, and $\angle B = 45^\circ$. Find the length of side b.

Step 1: Two angles and a non-included side are known (AAS). Use the Sine Law.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Step 2: Substitute the known values.

$$\frac{10}{\sin 30^\circ} = \frac{b}{\sin 45^\circ}$$

Step 3: Use the exact values $\sin 30^\circ = \dfrac{1}{2}$ and $\sin 45^\circ = \dfrac{\sqrt{2}}{2}$, then solve for b.

$$b = 10 \times \frac{\sin 45^\circ}{\sin 30^\circ} = 10 \times \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = 10 \times \sqrt{2} = 10\sqrt{2}$$

Step 4: Evaluate numerically.

$$b = 10\sqrt{2} \approx 14.14$$

$$\boxed{b = 10\sqrt{2} \approx 14.14}$$

7. In $\triangle ABC$, $b = 5$, $c = 8$, and $\angle A = 60°$. Find the length of side a.

Step 1: Two sides and the contained angle are known (SAS). Use the Cosine Law.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Step 2: Substitute the known values. Recall $\cos 60° = \frac{1}{2}$.

$$a^2 = 5^2 + 8^2 - 2(5)(8)\cos 60° = 25 + 64 - 80 \times \frac{1}{2}$$

Step 3: Simplify step by step.

$$a^2 = 89 - 40 = 49$$

Step 4: Take the square root.

$$a = \sqrt{49} = 7$$

$$\boxed{a = 7}$$

8. In $\triangle ABC$, $a = 6$, $b = 10$, and $\angle A = 30°$. How many triangles are possible?

Step 1: This is the ambiguous case (SSA). The given angle $\angle A$ is opposite the given side a. Start by computing the height h of the triangle.

$$h = b\sin A = 10 \times \sin 30° = 10 \times 0.5 = 5$$

Step 2: Apply the ambiguous case rules (for an acute angle A):

- $a < h$: no triangle possible
- $a = h$: exactly one right triangle
- $h < a < b$: two distinct triangles
- $a \geq b$: exactly one triangle

Step 3: Compare the values. Here $h = 5$, $a = 6$, and $b = 10$.

$$h < a < b \qquad \Rightarrow \qquad 5 < 6 < 10$$

Two different triangles can be formed.

$$\boxed{\text{2 triangles are possible}}$$

9. What is the exact value of $\cos(210°)$?

Step 1: Find the reference angle. Since $180° < 210° < 270°$, the angle is in Quadrant III.

$$\theta_R = 210° - 180° = 30°$$

Step 2: Recall the exact value from the special triangle.

$$\cos(30°) = \frac{\sqrt{3}}{2}$$

Step 3: In Quadrant III, cosine is negative. Apply the sign.

$$\cos(210°) = -\frac{\sqrt{3}}{2}$$

$$\boxed{\cos(210°) = -\frac{\sqrt{3}}{2}}$$

10. In $\triangle ABC$, $a = 12$, $\angle A = 40°$, and $b = 15$. Find $\angle B$ (round to nearest degree).

Step 1: Two sides and an opposite angle are known (SSA). Use the Sine Law.

$$\sin B = \frac{b \sin A}{a}$$

Step 2: Substitute and compute.

$$\sin B = \frac{15 \times \sin 40°}{12} \approx \frac{15 \times 0.6428}{12} \approx \frac{9.642}{12} \approx 0.8035$$

Step 3: Find the principal value.

$$B_1 = \sin^{-1}(0.8035) \approx 53°$$

Step 4: Check the supplementary (obtuse) possibility: $B_2 = 180° - 53° = 127°$.

$$\angle A + B_2 = 40° + 127° = 167° < 180° \quad \checkmark$$

Both options keep the triangle valid, so there are two solutions.

$$\boxed{\angle B \approx 53° \quad \text{or} \quad \angle B \approx 127°}$$

11. In $\triangle ABC$, $a = 7$, $b = 9$, and $c = 12$. Find $\angle C$ (round to nearest degree).

Step 1: All three sides are known (SSS). Rearrange the Cosine Law to solve for the angle.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Step 2: Substitute the values.

$$\cos C = \frac{7^2 + 9^2 - 12^2}{2(7)(9)} = \frac{49 + 81 - 144}{126} = \frac{-14}{126} = -\frac{1}{9}$$

Step 3: Take the inverse cosine. The negative value confirms the angle is obtuse.

$$C = \cos^{-1}\left(-\frac{1}{9}\right) \approx 96^\circ$$

$$\boxed{\angle C \approx 96^\circ}$$

12. In $\triangle ABC$, $\angle A = 45^\circ$, $\angle B = 60^\circ$, and $c = 10$. Find $\angle C$.

Step 1: The three interior angles of any triangle always sum to 180°.

$$\angle A + \angle B + \angle C = 180^\circ$$

Step 2: Substitute the known angles and solve for $\angle C$.

$$45^\circ + 60^\circ + \angle C = 180^\circ \qquad \Rightarrow \qquad \angle C = 180^\circ - 105^\circ = 75^\circ$$

$$\boxed{\angle C = 75^\circ}$$

13. Which angle is coterminal with 120°?

Step 1: Coterminal angles share the same terminal arm. They are formed by adding or subtracting full rotations of 360°.

$$\theta_{\text{coterminal}} = 120^\circ + 360^\circ k, \quad k \in \mathbb{Z}$$

Step 2: Substitute $k = 1$ to find the next positive coterminal angle.

$$120^\circ + 360^\circ = 480^\circ$$

$$\boxed{480^\circ}$$

14. If $\sin(\theta) = -0.5$ and $\cos(\theta) > 0$, which quadrant is θ in?

Step 1: Use the CAST rule to determine which ratios are positive in each quadrant.

- $\sin(\theta) < 0 \Rightarrow \theta$ is in Quadrant III or Quadrant IV.
- $\cos(\theta) > 0 \Rightarrow \theta$ is in Quadrant I or Quadrant IV.

Step 2: The only quadrant satisfying both conditions simultaneously is Quadrant IV.

$$\boxed{\text{Quadrant IV}}$$

15. If you know all three side lengths of a triangle, which law should you use to find the first angle?

Explanation: The Sine Law requires at least one known angle to set up the ratio. When only side lengths are available (SSS), the Cosine Law must be used, rearranged to isolate the angle:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

This works for any angle once the three sides are known.

$$\boxed{\text{Cosine Law}}$$

16. The Sine Law can be used if you are given:

Explanation: The Sine Law relates sides to their opposite angles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To use this law, at least one complete side-angle pair must be known. This occurs in three cases:

- **AAS:** Two angles and a non-included side
- **ASA:** Two angles and the included side (find the third angle first)
- **SSA:** Two sides and an angle opposite one of them (the ambiguous case)

$$\boxed{\text{Two sides and an opposite angle (SSA), or two angles with any side}}$$

17. Find the exact value of $\tan(300°)$.

Step 1: Find the reference angle. Since $270° < 300° < 360°$, the angle is in Quadrant IV.

$$\theta_R = 360° - 300° = 60°$$

Step 2: Recall the exact value from the special triangle.

$$\tan(60°) = \sqrt{3}$$

Step 3: In Quadrant IV, sine is negative and cosine is positive, so tangent is negative.

$$\tan(300^\circ) = -\sqrt{3}$$

$$\boxed{\tan(300^\circ) = -\sqrt{3}}$$

18. In $\triangle ABC$, $a = 4$, $b = 10$, and $\angle A = 30^\circ$. How many triangles are possible?

Step 1: This is the ambiguous case (SSA). Compute the height h.

$$h = b \sin A = 10 \times \sin 30^\circ = 10 \times 0.5 = 5$$

Step 2: Compare a and h.

$$a = 4 \qquad h = 5 \qquad \Rightarrow \qquad a < h$$

Step 3: Since side a is too short to reach the base line, no triangle can be formed.

$$\boxed{\text{0 triangles are possible}}$$

19. Given $\theta = 180^\circ$, what is the value of $\sin(\theta)$?

Step 1: At $\theta = 180^\circ$, the terminal arm points in the negative x-direction. On the unit circle, this corresponds to the point $(-1,\ 0)$.
Step 2: Apply the definition $\sin(\theta) = y$.

$$\sin(180^\circ) = 0$$

$$\boxed{\sin(180^\circ) = 0}$$

20. In $\triangle ABC$, $a = 12$, $b = 12$, and $\angle C = 60^\circ$. What kind of triangle is this?

Step 1: Two sides are equal, so the triangle is at least isosceles. Use the Cosine Law to find the third side c.

$$c^2 = a^2 + b^2 - 2ab\cos C = 12^2 + 12^2 - 2(12)(12)\cos 60^\circ$$

Step 2: Recall $\cos 60^\circ = \dfrac{1}{2}$ and simplify.

$$c^2 = 144 + 144 - 288 \times \frac{1}{2} = 288 - 144 = 144$$

Step 3: Take the square root.

$$c = \sqrt{144} = 12$$

All three sides are equal ($a = b = c = 12$), and since all angles must therefore be 60°, the triangle is equilateral.

$$\boxed{\text{Equilateral triangle}}$$

21. A surveyor stands 50 m from the base of a building and measures the angle of elevation to the top as $40°$. How tall is the building?

Step 1: Sketch the situation. The horizontal distance (50 m), the building height h, and the line of sight form a right triangle. The angle of elevation is at ground level.

Step 2: Identify the ratio. The height h is opposite the $40°$ angle, and the 50 m distance is adjacent.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{h}{50}$$

Step 3: Solve for h.

$$h = 50 \times \tan(40°) \approx 50 \times 0.8391 \approx 41.95 \text{ m}$$

$$\boxed{h \approx 41.95 \text{ m}}$$

22. Reference angles are always:

Explanation: A reference angle is defined as the positive acute angle formed between the terminal arm of θ and the nearest part of the x-axis. By definition, it is always between $0°$ and $90°$. This means it is always acute, and it is always written as a positive value, regardless of which quadrant the original angle terminates in.

$$\boxed{\text{Acute and positive}}$$

23. Which ratio is positive in Quadrant III?

Step 1: In Quadrant III, both coordinates are negative: $x < 0$ and $y < 0$. Recall that r is always positive. Apply the definitions:

$$\sin(\theta) = \frac{y}{r} < 0 \qquad \cos(\theta) = \frac{x}{r} < 0 \qquad \tan(\theta) = \frac{y}{x} = \frac{(-)}{(-)} > 0$$

Step 2: Only tangent produces a positive result in Quadrant III. This is confirmed by the CAST rule: **T**angent is positive in Quadrant III.

$$\boxed{\text{Tangent}}$$

24. In $\triangle ABC$, $a = 8$, $b = 12$, and $\angle B = 100°$. Find $\angle A$.

Step 1: Two sides and an opposite angle are known (SSA). Use the Sine Law.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \Rightarrow \qquad \sin A = \frac{a \sin B}{b}$$

Step 2: Substitute and compute.

$$\sin A = \frac{8 \times \sin 100°}{12} \approx \frac{8 \times 0.9848}{12} \approx \frac{7.878}{12} \approx 0.6565$$

Step 3: Find the principal value.

$$A = \sin^{-1}(0.6565) \approx 41^\circ$$

Step 4: Check the obtuse possibility: $180^\circ - 41^\circ = 139^\circ$.

$$139^\circ + 100^\circ = 239^\circ > 180^\circ \quad \times$$

The obtuse option is impossible. There is only one valid answer.

$$\boxed{\angle A \approx 41^\circ}$$

25. If $\theta = -45^\circ$, what is its equivalent positive angle in $[0^\circ,\ 360^\circ)$?

Step 1: A negative angle represents a clockwise rotation from the positive x-axis. To convert it to an equivalent positive angle, add one full rotation of 360°.

$$\theta_{\text{positive}} = -45^\circ + 360^\circ = 315^\circ$$

Step 2: Confirm the result is within $[0^\circ, 360^\circ)$.

$$0^\circ \leq 315^\circ < 360^\circ \quad \checkmark$$

$$\boxed{315^\circ}$$

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