

Lesson 2: Basic Derivative Rules

ApexMath

Notes

- **Power Rule:** For any real number n ,

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

- **Constant Rule:**

$$\frac{d}{dx}[c] = 0$$

- **Sum/Difference Rule:**

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

- **Constant Multiple Rule:**

$$\frac{d}{dx}[a \cdot f(x)] = a \cdot f'(x)$$

Example:

$$\frac{d}{dx}[a \cdot x^n] = a \cdot nx^{n-1}$$

- These rules allow you to find derivatives quickly without using the limit definition.
- Remember: the derivative still represents the slope of the tangent line at a point.

Pro Tips

- Simplify powers before applying the power rule.
- Constants disappear when differentiating.
- Combine like terms first when possible before differentiating.
- Negative exponents and fractions: write as x^{-n} to apply the power rule easily.
- Radicals: always rewrite as exponents before applying the power rule. For example,

$$\sqrt{x} = x^{\frac{1}{2}}, \quad \sqrt[3]{x} = x^{\frac{1}{3}}$$

- **Put it all together now:** apply each rule step by step and double-check exponents.

Deep Walkthrough Examples

Example 1: $f(x) = 5x^3$

$$f'(x) = 5 \cdot 3x^{3-1} = 15x^2$$

$$\boxed{f'(x) = 15x^2}$$

Example 2: $f(x) = 7$

$$f'(x) = 0$$

$$\boxed{f'(x) = 0}$$

Example 3: $f(x) = x^3 + 2x^2 - 5$

$$f'(x) = 3x^2 + 4x + 0 = 3x^2 + 4x$$

$$\boxed{f'(x) = 3x^2 + 4x}$$

Example 4: $f(x) = \sqrt{x}$

Step 1: Rewrite the square root as an exponent:

$$\sqrt{x} = x^{\frac{1}{2}}$$

Step 2: Apply the power rule:

$$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$$

Step 3: Convert the negative exponent into a denominator.

$$\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

Common Mistakes

- Forgetting to reduce the exponent by 1 in the power rule.
- Differentiating constants incorrectly.
- Forgetting the sign when applying the sum/difference rule.
- Ignoring negative exponents or fractions.
- Forgetting to rewrite radicals as exponents first.

Worksheet Problems

1. $f(x) = x^5$
2. $f(x) = 3x^4 + 2x^2 - 7$
3. $f(x) = 10$
4. $f(x) = 4x^3 - x + 8$
5. $f(x) = x^{-2} + 5x$
6. $f(x) = \sqrt[3]{x}$

Detailed Answer Key

1. $f(x) = x^5$

$$f'(x) = 5x^{5-1} = 5x^4$$

$$\boxed{f'(x) = 5x^4}$$

2. $f(x) = 3x^4 + 2x^2 - 7$

$$f'(x) = 3 \cdot 4x^3 + 2 \cdot 2x^1 + 0 = 12x^3 + 4x$$

$$\boxed{f'(x) = 12x^3 + 4x}$$

3. $f(x) = 10$

$$f'(x) = 0$$

$$\boxed{f'(x) = 0}$$

4. $f(x) = 4x^3 - x + 8$

$$f'(x) = 12x^2 - 1 + 0 = 12x^2 - 1$$

$$\boxed{f'(x) = 12x^2 - 1}$$

5. $f(x) = x^{-2} + 5x$

$$f'(x) = -2x^{-3} + 5 = -\frac{2}{x^3} + 5$$

$$\boxed{f'(x) = -2\frac{1}{x^3} + 5}$$

6. $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \frac{1}{3\sqrt[3]{x^2}}$$

$$\boxed{f'(x) = \frac{1}{3\sqrt[3]{x^2}}}$$

Thank you for learning with ApexMath!

If you found this lesson helpful, we'd love for you to share your feedback or leave a review.

End of Lesson • ApexMath
