Lesson 13: Derivatives in Physics
ApexMath

Notes

- **Position/Distance** $d(t)$ gives the location of an object at time t. Initial distance is d_0.
- **Velocity** is the rate of change of distance:

$$v(t) = d'(t)$$

- **Acceleration** is the rate of change of velocity:

$$a(t) = v'(t) = d''(t)$$

- Units:
  - Distance: meters (m)
  - Velocity: meters/second (m/s)
  - Acceleration: meters/second^2 (m/s^2)
- Maximum or minimum in physics often occurs when

$$v(t) = 0$$

- Always check context: direction, signs, and units.
- **Fun fact:** The equations

$$v(t) = v_0 + at \quad \text{and} \quad d(t) = d_0 + v_0 t + \frac{1}{2}at^2$$

  come directly from derivatives:
  - Start with

$$d(t) = d_0 + v_0 t + \frac{1}{2}at^2$$

    where d_0 and v_0 are constants.
  - Differentiate once:

$$v(t) = d'(t) = 0 + v_0 + at = v_0 + at$$

    d_0 disappears because the derivative of a constant is 0.
  - Differentiate again:

$$a(t) = v'(t) = d''(t) = 0 + 0 + a = a$$

    v_0 disappears because the derivative of a constant is 0.
  - This shows acceleration is constant, velocity changes linearly, and distance follows a quadratic formula — all explained by derivatives.

Pro Tips

- Draw a diagram of motion and label position, velocity, acceleration.
- Always compute derivatives step by step and check units.
- Use the derivative = 0 condition to find extrema (max height, min distance, etc.).
- For word problems, translate words into functions carefully.

Examples

Example 1 Maximum Height

A ball is thrown upward with initial velocity $v_0 = 20$ m/s. Gravity is $g = 9.8$ m/s^2. Find the maximum height.

Step 1 Position function

$$d(t) = v_0 t - \frac{1}{2} g t^2 = 20t - 4.9t^2$$

Step 2 Velocity

$$v(t) = d'(t) = 20 - 9.8t$$

Step 3 Critical point

Maximum height occurs when velocity = 0:

$$20 - 9.8t = 0 \implies t = \frac{20}{9.8} \approx 2.041 \text{ s}$$

Step 4 Maximum height

$$d(2.041) = 20(2.041) - 4.9(2.041)^2 \approx 20.41 \text{ m}$$

Step 5 Answer

$$\boxed{\text{Max height } \approx 20.41 \text{ m at } t \approx 2.04 \text{ s}}$$

Example 2 Acceleration at a specific time

A particle moves along a line with distance function $d(t) = 5 + 3t - 0.5t^2$. Find the velocity and acceleration at $t = 4$ s.

Step 1 Velocity function

$$v(t) = d'(t) = 3 - t$$

Explanation: derivative of constant 5 is 0.

Step 2 Acceleration function

$$a(t) = v'(t) = -1$$

Explanation: derivative of constant 3 is 0.

Step 3 Evaluate at $t = 4$

$$v(4) = 3 - 4 = -1, \quad a(4) = -1$$

Step 4 Answer

$$\boxed{v(4) = -1 \text{ m/s}, \; a(4) = -1 \text{ m/s}^2}$$

Example 3 Maximum height for projectile

A projectile is launched with initial vertical velocity $v_0 = 15$ m/s. Gravity $g = 9.8$ m/s^2. Find the maximum height.

Step 1 Position function

$$d(t) = v_0 t - \frac{1}{2} g t^2 = 15t - 4.9t^2$$

Step 2 Velocity

$$v(t) = d'(t) = 15 - 9.8t$$

Step 3 Critical point

$$15 - 9.8t = 0 \implies t = \frac{15}{9.8} \approx 1.531 \text{ s}$$

Step 4 Maximum height

$$d(1.531) = 15(1.531) - 4.9(1.531)^2 \approx 11.48 \text{ m}$$

Step 5 Answer

$$\boxed{\text{Max height } \approx 11.48 \text{ m at } t \approx 1.53 \text{ s}}$$

Common Mistakes

- Forgetting units of velocity and acceleration.
- Confusing position, velocity, and acceleration.
- Forgetting to set derivative = 0 for max/min problems.
- Not checking physical context (negative height, impossible times).
- Not labeling diagram properly.

Worksheet

1. A particle moves along a line with distance given by $d(t) = 5 + 3t - 0.5t^2$. Find $v(t)$ and $a(t)$, then evaluate both at $t = 4$ s.
2. A particle has velocity $v(t) = 2t + 3$ with $d(0) = 0$. Find $a(t)$ and $d(t)$, then evaluate both at $t = 5$ s.
3. A particle moves with distance $d(t) = 10 + 20t - 5t^2$. Find $v(t)$ and $a(t)$, then evaluate at $t = 2$ s.
4. A particle has velocity $v(t) = 6 - 2t$ with $d(0) = 2$. Find $a(t)$ and $d(t)$, then evaluate at $t = 3$ s.
5. A particle moves with distance $d(t) = 4t^3 - 3t^2 + 2t$. Find $v(t)$ and $a(t)$, then evaluate at $t = 2$ s.

Answer Key – Physics Derivatives

1. **Question 1 – Distance given:** $d(t) = 5 + 3t - 0.5t^2$

   Step 1 – Find velocity:

   $$v(t) = \frac{d}{dt}[5 + 3t - 0.5t^2] = 0 + 3 - t = 3 - t$$

   Explanation: $d_0 = 5$ *is a constant, derivative = 0.*

   Step 2 – Find acceleration:

   $$a(t) = \frac{dv}{dt} = \frac{d}{dt}[3 - t] = -1$$

   Explanation: $v_0 = 3$ *is a constant, derivative = 0.*

   Step 3 – Evaluate at $t = 4$ s:

   $$v(4) = 3 - 4 = -1, \quad a(4) = -1$$

   Step 4 – Answer:

   $$\boxed{v(4) = -1 \text{ m/s}, \ a(4) = -1 \text{ m/s}^2}$$

Question 2 – Velocity given: $v(t) = 2t + 3$, $d(0) = 0$

Step 1 – Find acceleration:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}[2t + 3] = 2$$

Explanation: $v_0 = 3$ *is constant, derivative = 0.*

Step 2 – Find distance:

$$d(t) = \int v(t)dt = \int (2t+3)dt = t^2 + 3t + C$$

Step 3 – Solve for C using $d(0) = 0$:

$$0 = 0 + 0 + C \implies C = 0$$

$$d(t) = t^2 + 3t$$

Step 4 – Evaluate at $t = 5$ s:

$$a(5) = 2, \quad d(5) = 5^2 + 3(5) = 25 + 15 = 40$$

Step 5 – Answer:

$$\boxed{a(5) = 2 \text{ m/s}^2,\ d(5) = 40 \text{ m}}$$

Question 3 – Distance given: $d(t) = 10 + 20t - 5t^2$

Step 1 – Find velocity:

$$v(t) = \frac{d}{dt}[10 + 20t - 5t^2] = 0 + 20 - 10t = 20 - 10t$$

Explanation: constant term $d_0 = 10$ *derivative = 0.*

Step 2 – Find acceleration:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}[20 - 10t] = -10$$

Explanation: constant $v_0 = 20$ *derivative = 0.*

Step 3 – Evaluate at $t = 2$ s:

$$v(2) = 20 - 10(2) = 0, \quad a(2) = -10$$

Step 4 – Answer:

$$\boxed{v(2) = 0 \text{ m/s},\ a(2) = -10 \text{ m/s}^2}$$

Question 4 – Velocity given: $v(t) = 6 - 2t$, $d(0) = 2$

Step 1 – Find acceleration:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}[6 - 2t] = -2$$

Step 2 – Find distance:

$$d(t) = \int v(t)dt = \int (6 - 2t)dt = 6t - t^2 + C$$

Step 3 – Solve for C using $d(0) = 2$:

$$2 = 0 - 0 + C \implies C = 2$$

$$d(t) = 6t - t^2 + 2$$

Step 4 – Evaluate at $t = 3$ s:

$$a(3) = -2, \quad d(3) = 6(3) - 3^2 + 2 = 18 - 9 + 2 = 11$$

Step 5 – Answer:

$$\boxed{a(3) = -2 \text{ m/s}^2,\ d(3) = 11 \text{ m}}$$

Question 5 – Distance given: $d(t) = 4t^3 - 3t^2 + 2t$

Step 1 – Find velocity:

$$v(t) = \frac{d}{dt}[4t^3 - 3t^2 + 2t] = 12t^2 - 6t + 2$$

Step 2 – Find acceleration:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}[12t^2 - 6t + 2] = 24t - 6$$

Step 3 – Evaluate at $t = 2$ s:

$$v(2) = 12(4) - 6(2) + 2 = 48 - 12 + 2 = 38, \quad a(2) = 24(2) - 6 = 48 - 6 = 42$$

Step 4 – Answer:

$$\boxed{v(2) = 38 \text{ m/s},\ a(2) = 42 \text{ m/s}^2}$$

Thank you for learning with ApexMath!

If you found this lesson helpful, we'd love for you to share your feedback or leave a review.

End of Lesson • ApexMath
