Lesson 7: Derivatives of Cotangent, Secant, Cosecant, and Their Inverses

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Notes

- **Derivative of** $\cot(u)$**:**
$$\frac{d}{dx}\cot(u) = -\csc^2(u)\cdot\frac{du}{dx}$$

- **Derivative of** $\sec(u)$**:**
$$\frac{d}{dx}\sec(u) = \sec(u)\tan(u)\cdot\frac{du}{dx}$$

- **Derivative of** $\csc(u)$**:**
$$\frac{d}{dx}\csc(u) = -\csc(u)\cot(u)\cdot\frac{du}{dx}$$

- **Derivative of arccot**(u)**:**
$$\frac{d}{dx}\text{arccot}(u) = -\frac{1}{1+u^2}\cdot\frac{du}{dx}$$

- **Derivative of arcsec**(u)**:**
$$\frac{d}{dx}\text{arcsec}(u) = \frac{1}{\left|u\right|\sqrt{u^2-1}}\cdot\frac{du}{dx}$$

- **Derivative of arccsc**(u)**:**
$$\frac{d}{dx}\text{arccsc}(u) = -\frac{1}{\left|u\right|\sqrt{u^2-1}}\cdot\frac{du}{dx}$$

- **Pro Tip:** Always identify the inner function u and apply $\frac{du}{dx}$ when using the chain rule.

- **Pro Tip:** You can derive these using the definition of the derivative:
$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h},$$
but it takes a lot of time for trig and inverse trig functions, so it's best to memorize them.

Pro Tips

- Watch signs: cot, csc, arccot, arccsc have negatives.
- Remember absolute value in $\operatorname{arcsec}(u)$ and $\operatorname{arccsc}(u)$.
- Simplify radicals carefully in inverse trig derivatives.

Examples

Example 1: Differentiate $f(x) = \cot(4x)$

Step 1 – General Formula:

$$\frac{d}{dx}\cot(u) = -\csc^2(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 4x, \quad \frac{du}{dx} = 4$$

$$\frac{d}{dx}\cot(4x) = -\csc^2(4x) \cdot 4$$

Step 3 – Simplify:

$$-4\csc^2(4x)$$

$$\boxed{\frac{d}{dx}\cot(4x) = -4\csc^2(4x)}$$

Example 2: Differentiate $f(x) = \operatorname{arcsec}(2x)$

Step 1 – General Formula:

$$\frac{d}{dx}\operatorname{arcsec}(u) = \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 2x, \quad \frac{du}{dx} = 2$$

$$\frac{d}{dx}\operatorname{arcsec}(2x) = \frac{1}{|2x|\sqrt{(2x)^2-1}} \cdot 2$$

$$= \frac{2}{|2x|\sqrt{4x^2-1}}$$

$$= \frac{1}{|x|\sqrt{4x^2-1}}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}\text{arcsec}(2x) = \frac{1}{|x|\sqrt{4x^2 - 1}}}$$

Example 3: Differentiate $f(x) = \csc(x^2)$

Step 1 – General Formula:

$$\frac{d}{dx}\csc(u) = -\csc(u)\cot(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = x^2, \quad \frac{du}{dx} = 2x$$

$$\begin{aligned}\frac{d}{dx}\csc(x^2) &= -\csc(x^2)\cot(x^2) \cdot 2x \\ &= -2x\csc(x^2)\cot(x^2)\end{aligned}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}\csc(x^2) = -2x\csc(x^2)\cot(x^2)}$$

Common Mistakes

- Forgetting chain rule for inner function.
- Missing negative signs in cot, csc, arccot, arccsc.
- Forgetting absolute value for $\text{arcsec}(u)$ and $\text{arccsc}(u)$.
- Incorrect simplification of radicals in inverse trig derivatives.

Worksheet

Differentiate the following:

1. $f(x) = \cot(5x)$
2. $f(x) = \sec(3x^2)$
3. $f(x) = \csc(2x)$
4. $f(x) = \text{arccot}(x/3)$
5. $f(x) = \text{arcsec}(1 - 2x)$
6. $f(x) = \text{arccsc}(4x)$

Answer Key

1. Step 1 – General Formula:

$$\frac{d}{dx}\cot(u) = -\csc^2(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 5x, \quad \frac{du}{dx} = 5$$

$$\begin{aligned}\frac{d}{dx}\cot(5x) &= -\csc^2(5x) \cdot 5\\ &= -5\csc^2(5x)\end{aligned}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}\cot(5x) = -5\csc^2(5x)}$$

2. Step 1 – General Formula:

$$\frac{d}{dx}\sec(u) = \sec(u)\tan(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 3x^2, \quad \frac{du}{dx} = 6x$$

$$\begin{aligned}\frac{d}{dx}\sec(3x^2) &= \sec(3x^2)\tan(3x^2) \cdot 6x\\ &= 6x\sec(3x^2)\tan(3x^2)\end{aligned}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}\sec(3x^2) = 6x\sec(3x^2)\tan(3x^2)}$$

3. Step 1 – General Formula:

$$\frac{d}{dx}\csc(u) = -\csc(u)\cot(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 2x, \quad \frac{du}{dx} = 2$$

$$\begin{aligned}\frac{d}{dx}\csc(2x) &= -\csc(2x)\cot(2x)\cdot 2\\ &= -2\csc(2x)\cot(2x)\end{aligned}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}\csc(2x) = -2\csc(2x)\cot(2x)}$$

4. Step 1 – General Formula:

$$\frac{d}{dx}\text{arccot}(u) = -\frac{1}{1+u^2}\cdot\frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = \frac{x}{3}, \quad \frac{du}{dx} = \frac{1}{3}$$

$$\begin{aligned}\frac{d}{dx}\text{arccot}\left(\frac{x}{3}\right) &= -\frac{1}{1+(x/3)^2}\cdot\frac{1}{3}\\ &= -\frac{1}{3(1+x^2/9)}\\ &= -\frac{1}{3+x^2}\end{aligned}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}\text{arccot}\left(\frac{x}{3}\right) = -\frac{1}{3+x^2}}$$

5. Step 1 – General Formula:

$$\frac{d}{dx}\text{arcsec}(u) = \frac{1}{|u|\sqrt{u^2-1}}\cdot\frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 1-2x, \quad \frac{du}{dx} = -2$$

$$\frac{d}{dx}\text{arcsec}(1-2x) = \frac{-2}{|1-2x|\sqrt{(1-2x)^2-1}}$$

$$= -\frac{2}{\left|1-2x\right|\sqrt{1-4x+4x^2-1}}$$

$$= -\frac{2}{\left|1-2x\right|\sqrt{4x^2-4x}}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}\operatorname{arcsec}(1-2x) = -\frac{2}{\left|1-2x\right|\sqrt{(1-2x)^2-1}}}$$

6. Step 1 – General Formula:

$$\frac{d}{dx}\operatorname{arccsc}(u) = -\frac{1}{\left|u\right|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 4x, \quad \frac{du}{dx} = 4$$

$$\frac{d}{dx}\operatorname{arccsc}(4x) = -\frac{4}{|4x|\sqrt{(4x)^2-1}}$$

$$= -\frac{4}{4|x|\sqrt{16x^2-1}}$$

$$= -\frac{1}{|x|\sqrt{16x^2-1}}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}\operatorname{arccsc}(4x) = -\frac{1}{|x|\sqrt{16x^2-1}}}$$

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