

Lesson 5: Derivatives of Exponential and Logarithmic Functions

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Notes

- Derivative of e^u :

$$\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$$

- Derivative of a^u :

$$\frac{d}{dx}a^u = a^u \ln(a) \cdot \frac{du}{dx}$$

- Derivative of $\ln(u)$:

$$\frac{d}{dx}\ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$$

- Derivative of $\log_a(u)$:

$$\frac{d}{dx}\log_a(u) = \frac{1}{u \ln(a)} \cdot \frac{du}{dx}$$

- **Pro Tip:** Always identify the inner function u and apply $\frac{du}{dx}$ when needed.

Pro Tips

- Remember $\ln(a)$ when differentiating a^u .
- For complicated products, quotients, or powers, consider using logarithmic differentiation.
- Always simplify exponents and fractions in your final answer.

Examples

Example 1: Differentiate $f(x) = e^{3x}$

Step 1 – General Formula:

$$\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 3x, \quad \frac{du}{dx} = 3 \implies \frac{d}{dx}e^{3x} = e^{3x} \cdot 3$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}e^{3x} = 3e^{3x}}$$

Example 2: Differentiate $f(x) = 2^{x^2}$

Step 1 – General Formula:

$$\frac{d}{dx} a^u = a^u \ln(a) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = x^2, \quad \frac{du}{dx} = 2x \implies \frac{d}{dx} 2^{x^2} = 2^{x^2} \ln(2) \cdot 2x$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} 2^{x^2} = 2x \cdot 2^{x^2} \ln(2)}$$

Example 3: Differentiate $f(x) = \ln(5x)$

Step 1 – General Formula:

$$\frac{d}{dx} \ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 5x, \quad \frac{du}{dx} = 5 \implies \frac{d}{dx} \ln(5x) = \frac{1}{5x} \cdot 5$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \ln(5x) = \frac{5}{5x} = \frac{1}{x}}$$

Common Mistakes

- Forgetting the chain rule ($\frac{du}{dx}$).
- Forgetting the $\ln(a)$ factor for a^u .
- Confusing $\ln(u)$ with $\log_a(u)$.
- Missing parentheses in the argument of the exponential or log function.

Worksheet

Differentiate the following:

1. $f(x) = e^{2x}$
2. $f(x) = 3^{x^2}$
3. $f(x) = \ln(x^3)$
4. $f(x) = \log_5(4x)$
5. $f(x) = e^{x^2+1}$

Answer Key

1. Step 1 – General Formula:

$$\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 2x, \quad \frac{du}{dx} = 2 \implies \frac{d}{dx}e^{2x} = e^{2x} \cdot 2$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}e^{2x} = 2e^{2x}}$$

2. Step 1 – General Formula:

$$\frac{d}{dx}a^u = a^u \ln(a) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = x^2, \quad \frac{du}{dx} = 2x \implies \frac{d}{dx}3^{x^2} = 3^{x^2} \ln(3) \cdot 2x$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}3^{x^2} = 2x \cdot 3^{x^2} \ln(3)}$$

3. Step 1 – General Formula:

$$\frac{d}{dx} \ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = x^3, \quad \frac{du}{dx} = 3x^2 \implies \frac{d}{dx} \ln(x^3) = \frac{1}{x^3} \cdot 3x^2$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \ln(x^3) = \frac{3}{x}}$$

4. Step 1 – General Formula:

$$\frac{d}{dx} \log_a(u) = \frac{1}{u \ln(a)} \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 4x, \quad \frac{du}{dx} = 4 \implies \frac{d}{dx} \log_5(4x) = \frac{1}{4x \ln(5)} \cdot 4$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \log_5(4x) = \frac{1}{x \ln(5)}}$$

5. Step 1 – General Formula:

$$\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = x^2 + 1, \quad \frac{du}{dx} = 2x \implies \frac{d}{dx}e^{x^2+1} = e^{x^2+1} \cdot 2x$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx}e^{x^2+1} = 2x e^{x^2+1}}$$

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