

# Lesson 8: Higher-Order Derivatives

## ApexMath

### Notes

- **First derivative:** For a distance function  $d(t)$ , the first derivative  $d'(t)$  represents **velocity**.

$$d'(t) = \frac{d}{dt}d(t)$$

- **Second derivative:** The derivative of velocity is **acceleration**:

$$d''(t) = \frac{d}{dt}d'(t)$$

- **Third derivative:** The derivative of acceleration is sometimes called **jerk**:

$$d'''(t) = \frac{d}{dt}d''(t)$$

- **Power Rule:** For any term  $t^n$ :

$$\frac{d}{dt}t^n = nt^{n-1}$$

- **Sum/Difference Rule:**

$$\frac{d}{dt}[f(t) \pm g(t)] = f'(t) \pm g'(t)$$

- **Constant Rule:**

$$\frac{d}{dt}[c] = 0$$

### Pro Tips

- Always write down which derivative formula you are using.
- Work term by term; don't skip steps.
- Check the exponent: after differentiating  $t^n$ , the exponent decreases by 1.
- First derivative = velocity, second derivative = acceleration, third derivative = jerk.

## Examples

**Example 1:** Find  $d'(t)$  and  $d''(t)$  if  $d(t) = 5t^3 - 6t^2 + 2t$ .

Step 1: Differentiate each term to get velocity:

$$\begin{aligned}d'(t) &= \frac{d}{dt}(5t^3) - \frac{d}{dt}(6t^2) + \frac{d}{dt}(2t) \\&= 15t^2 - 12t + 2\end{aligned}$$

Step 2: Differentiate velocity to get acceleration:

$$\begin{aligned}d''(t) &= \frac{d}{dt}(15t^2) - \frac{d}{dt}(12t) + \frac{d}{dt}(2) \\&= 30t - 12\end{aligned}$$

**Example 2:** Find  $d'(t)$ ,  $d''(t)$ , and  $d'''(t)$  if  $d(t) = t^4 - 4t^3 + 3t$ .

$$\text{Step 1: } d'(t) = \frac{d}{dt}(t^4) - \frac{d}{dt}(4t^3) + \frac{d}{dt}(3t) = 4t^3 - 12t^2 + 3$$

$$\text{Step 2: } d''(t) = \frac{d}{dt}(4t^3) - \frac{d}{dt}(12t^2) + \frac{d}{dt}(3) = 12t^2 - 24t$$

$$\text{Step 3: } d'''(t) = \frac{d}{dt}(12t^2) - \frac{d}{dt}(24t) = 24t - 24$$

**Example 3:** Find  $d'(t)$  and  $d''(t)$  if  $d(t) = 2t^5 - 3t^3 + t - 4$ .

$$\begin{aligned}\text{Step 1: } d'(t) &= \frac{d}{dt}(2t^5) - \frac{d}{dt}(3t^3) + \frac{d}{dt}(t) - \frac{d}{dt}(4) \\&= 10t^4 - 9t^2 + 1\end{aligned}$$

$$\begin{aligned}\text{Step 2: } d''(t) &= \frac{d}{dt}(10t^4) - \frac{d}{dt}(9t^2) + \frac{d}{dt}(1) \\&= 40t^3 - 18t\end{aligned}$$

**Example 4:** Find  $d'(t)$ ,  $d''(t)$ , and  $d'''(t)$  if  $d(t) = t^6 - 2t^4 + 5t^2$ .

$$\text{Step 1: } d'(t) = 6t^5 - 8t^3 + 10t$$

$$\text{Step 2: } d''(t) = 30t^4 - 24t^2 + 10$$

$$\text{Step 3: } d'''(t) = 120t^3 - 48t$$

## Worksheet

Differentiate the following functions:

1.  $d(t) = 3t^3 - t^2 + 4t$  (find  $d'(t)$  and  $d''(t)$ )

2.  $d(t) = t^4 - 2t^2 + t$  (find  $d'(t)$ ,  $d''(t)$ , and  $d'''(t)$ )
3.  $d(t) = 5t^2 + 3t - 7$  (find  $d'(t)$  and  $d''(t)$ )
4.  $d(t) = 2t^5 - t^3 + 4$  (find  $d'(t)$ ,  $d''(t)$ , and  $d'''(t)$ )

## Answer Key with Steps

1.  $d(t) = 3t^3 - t^2 + 4t$

Step 1:  $d'(t) = \frac{d}{dt}(3t^3) - \frac{d}{dt}(t^2) + \frac{d}{dt}(4t) = 9t^2 - 2t + 4$

Step 2:  $d''(t) = \frac{d}{dt}(9t^2) - \frac{d}{dt}(2t) + \frac{d}{dt}(4) = 18t - 2$

2.  $d(t) = t^4 - 2t^2 + t$

Step 1:  $d'(t) = 4t^3 - 4t + 1$

Step 2:  $d''(t) = 12t^2 - 4$

Step 3:  $d'''(t) = 24t$

3.  $d(t) = 5t^2 + 3t - 7$

Step 1:  $d'(t) = 10t + 3$

Step 2:  $d''(t) = 10$

4.  $d(t) = 2t^5 - t^3 + 4$

Step 1:  $d'(t) = 10t^4 - 3t^2$

Step 2:  $d''(t) = 40t^3 - 6t$

Step 3:  $d'''(t) = 120t^2 - 6$

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*End of Lesson • ApexMath*

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