Lesson 6: Intermediate Value Theorem (IVT)

ApexMath

Notes on the Intermediate Value Theorem

- The Intermediate Value Theorem (IVT) states: If $f(x)$ is continuous on a closed interval $[a, b]$, and N is any number between $f(a)$ and $f(b)$, then there exists at least one $c \in [a, b]$ such that $f(c) = N$.

- Key IVT conditions:

  - Function must be continuous on the closed interval $[a, b]$
  - Value N must lie between $f(a)$ and $f(b)$
  - IVT guarantees existence of at least one solution, but does not give exact number or quantity

- Reminder: A function is continuous at $x = a$ if:

$$\lim_{x \to a} f(x) = f(a)$$

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Common Mistakes

- Forgetting to check that the function is continuous on the entire interval [a,b].
- Assuming IVT works even if N is outside the range of $f(a)$ and $f(b)$.
- Confusing "exists at least one solution" with "finding the exact solution."
- Using IVT on a function with discontinuities, like division by zero.
- Not carefully evaluating endpoint values to see if N lies strictly between them.

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Deep Walkthrough Examples

Example A. Determine if the IVT applies to $f(x) = x^3$ on $[-1, 2]$ for $N = 1$.

Step 1: Check continuity:

$$f(x) = x^3 \text{ is a polynomial, continuous everywhere.}$$

Step 2: Check if $N = 1$ is between $f(-1) = -1$ and $f(2) = 8$.

$$-1 < 1 < 8$$

Step 3: Conclude:

$$\text{There exists a value at } N = 1 \text{ in the interval } [-1, 2]$$

$$\boxed{\text{There exists a value at } N = 1 \text{ in the interval } [-1, 2]}$$

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Example B. Does IVT apply to $f(x) = 1/x$ on [1,2] for $N = 0.5$?

Step 1: Continuous? $f(x) = 1/x$ is continuous on [1,2]

Step 2: N between endpoints? $f(1) = 1$, $f(2) = 0.5 \rightarrow$ yes, $0.5 \leq 0.5 \leq 1$

Step 3: Conclude:

$$\text{There exists a value at } N = 0.5 \text{ in the interval } [1, 2]$$

$$\boxed{\text{There exists a value at } N = 0.5 \text{ in the interval } [1, 2]}$$

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Worksheet Problems

1. Determine if the IVT applies to $f(x) = x^2$ on [1,3] for $N = 5$
2. Determine if the IVT applies to $f(x) = 1/(x - 2)$ on [3,4] for $N = 1$
3. Determine if the IVT applies to $f(x) = \sin x$ on [0, π]$for N = 0.5$
4. Determine if the IVT applies to $f(x) = |x|$ on [-2,1] for $N = 1$
5. Determine if the IVT applies to $f(x) = \sqrt{x}$ on [0,4] for $N = 3$
6. Determine if the IVT applies to $f(x) = x^3 - x$ on [-2,2] for $N = 0$

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Answer Key

1. $f(x) = x^2$ on [1,3], $N = 5$

Step 1: Check continuity: $f(x) = x^2$ is continuous everywhere.

Step 2: Check if N is between $f(1) = 1$ and $f(3) = 9$:

$$1 < 5 < 9$$

Step 3: Conclude: IVT applies.

$$\boxed{\text{There exists a value at } N = 5 \text{ in the interval } [1, 3]}$$

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2. $f(x) = 1/(x - 2)$ on [3,4], $N = 1$

Step 1: Continuous? $f(x) = 1/(x-2)$ is continuous on [3,4] (no discontinuities).

Step 2: Check if $N = 1$ is between $f(3) = 1$ and $f(4) = 1/2$:

$$1/2 < 1 \leq 1$$

Step 3: Conclude: IVT applies.

$$\boxed{\text{There exists a value at } N = 1 \text{ in the interval } [3, 4]}$$

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3. $f(x) = \sin x$ on $[0, \pi], N = 0.5$

Step 1: Continuous? $\sin x$ is continuous everywhere.

Step 2: Check if $N = 0.5$ is between $f(0) = 0$ and $f(\pi) = 0$:

$$0.5 \notin [0, 0]$$

Step 3: Conclude: IVT does not apply.

$$\boxed{\text{No, IVT does not apply; } N \text{ is not between } f(0) \text{ and } f(\pi)}$$

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4. $f(x) = |x|$ on [-2,1], $N = 1$

Step 1: Continuous? $|x|$ is continuous everywhere.

Step 2: Check if $N = 1$ is between $f(-2) = 2$ and $f(1) = 1$:

$$1 \in [1, 2]$$

Step 3: Conclude: IVT applies.

$$\boxed{\text{There exists a value at } N = 1 \text{ in the interval } [-2, 1]}$$

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5. $f(x) = \sqrt{x}$ on [0,4], $N = 3$

Step 1: Continuous? $\sqrt{x}$ is continuous on [0,4].

Step 2: Check if $N = 3$ is between $f(0) = 0$ and $f(4) = 2$:

$$3 \notin [0, 2]$$

Step 3: Conclude: IVT does not apply.

$$\boxed{\text{No, IVT does not apply; } N \text{ is not between } f(0) \text{ and } f(4)}$$

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6. $f(x) = x^3 - x$ on [-2,2], $N = 0$

Step 1: Continuous? Polynomial, continuous everywhere.

Step 2: Check if $N = 0$ is between $f(-2) = -6$ and $f(2) = 6$:

$$-6 < 0 < 6$$

Step 3: Conclude: IVT applies.

$$\boxed{\text{There exists a value at } N = 0 \text{ in the interval } [-2, 2]}$$

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