

Lesson 1: Limits

ApexMath

Notes on Limits

- A limit describes the value that a function approaches as the input approaches a certain value.
- Standard limits to memorize:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

- Limits can be evaluated by:
 - Direct substitution
 - Factoring and simplifying (removable discontinuities)
 - Recognizing standard limits
 - Using tables or graphs for visual understanding
- A hole occurs when the function is undefined at a point but the limit exists.

Deep Walkthrough Examples

Example A. Evaluate $\lim_{x \rightarrow 5} (3x - 2)$

Step 1. Check if direct substitution is possible (linear function, continuous everywhere).

Step 2. Substitute $x = 5$:

$$3(5) - 2$$

Step 3. Simplify:

$$15 - 2 = 13$$

13

Example B. Evaluate $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

Step 1. Recognize this as a standard limit to memorize.

Step 2. Apply the memorized result:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

1

Example C. Evaluate $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$

Step 1. Factor the numerator:

$$x^2 - 1 = (x + 1)(x - 1)$$

Step 2. Simplify the fraction (removable discontinuity):

$$\frac{(x + 1)(x - 1)}{x + 1} = x - 1, \quad x \neq -1$$

Step 3. Substitute $x = -1$:

$$-1 - 1 = -2$$

$$\boxed{-2}$$

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Example D. Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$ (hole)

Step 1. Factor the numerator:

$$x^2 - 9 = (x - 3)(x + 3)$$

Step 2. Simplify the fraction:

$$\frac{(x - 3)(x + 3)}{x - 3} = x + 3, \quad x \neq 3$$

Step 3. Substitute $x = 3$ or observe nearby values:

$$f(2.9) = 5.9, \quad f(3.0 \text{ hole}) \approx 6$$

Step 4. Limit value:

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$$

$$\boxed{6}$$

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Example E. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$

Step 1. Recognize as a standard limit to memorize.

Step 2. Apply the memorized result:

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\boxed{0}$$

Worksheet Problems

1. $\lim_{x \rightarrow 2} (4x + 1)$
2. $\lim_{x \rightarrow 0} \frac{\sin x}{x}$
3. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$
4. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$ (hole)
5. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$

Answer Key (Step-by-Step)

1. $\lim_{x \rightarrow 2} (4x + 1)$

Step 1. Linear function, direct substitution applies.

Step 2. Substitute $x = 2$:

$$4(2) + 1$$

Step 3. Simplify:

$$8 + 1 = 9$$

9

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2. $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

Step 1. Recognize standard limit.

Step 2. Apply memorized result:

$$1$$

1

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3. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

Step 1. Factor numerator:

$$x^2 - 1 = (x - 1)(x + 1)$$

Step 2. Simplify fraction:

$$x + 1, \quad x \neq 1$$

Step 3. Substitute $x = 1$:

$$1 + 1 = 2$$

2

4. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

Step 1. Factor numerator:

$$x^2 - 9 = (x - 3)(x + 3)$$

Step 2. Simplify fraction:

$$x + 3, \quad x \neq 3$$

Step 3. Substitute $x = 3$ or use nearby values:

$$3 + 3 = 6$$

$$\boxed{6}$$

5. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$

Step 1. Standard limit.

Step 2. Apply memorized result:

$$0$$

$$\boxed{0}$$

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End of Lesson • ApexMath
