Partial Fraction Decomposition
ApexMath

Notes

What is Partial Fraction Decomposition?

Partial fraction decomposition is a technique for breaking a complicated rational function into a sum of simpler fractions that are easy to integrate. For example:

$$\frac{5x+1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$

Each piece on the right is something we already know how to integrate. The goal of the decomposition is to find the constants A, B, etc.

When Can You Use Partial Fractions?

Partial fractions require the rational function to be **proper** — meaning the degree of the numerator is strictly less than the degree of the denominator. If the fraction is **improper** (numerator degree $\geq$ denominator degree), you must perform **polynomial long division first** to reduce it to a proper fraction plus a remainder, then apply partial fractions to the remainder.

The Four Cases

Factor Type	**Decomposition Form**
Distinct linear: $(x+a)(x+b)$	$\dfrac{A}{x+a} + \dfrac{B}{x+b}$
Repeated linear (squared): $(x+a)^2$	$\dfrac{A}{x+a} + \dfrac{B}{x+a}$
Repeated linear (cubed): $(x+a)^3$	$\dfrac{A}{x+a} + \dfrac{B}{x+a} + \dfrac{C}{x+a}$
Irreducible quadratic: (x^2+bx+c)	$\dfrac{Ax+B}{x^2+bx+c}$

The key rule is: for every repeated factor $(x+a)^n$, you need n separate fractions with denominators $(x+a)^1, (x+a)^2, \ldots, (x+a)^n$. For every irreducible quadratic factor, the numerator must be linear $(Ax+B)$, not just a constant.

Solving for the Constants

After writing the decomposition form, multiply both sides by the full denominator to clear all fractions. This gives a polynomial equation. There are two methods to find the constants:

Method 1 — Strategic values of x: Substitute values of x that make individual factors equal to zero. This eliminates most terms and quickly solves for one constant at a time.

Method 2 — Matching coefficients: Expand the right side and equate coefficients of each power of x to set up a system of equations. This is useful when strategic values alone do not give enough equations.

In practice, you will often use both methods together.

Integrating the Resulting Fractions

Once you have the decomposition, integrate each piece separately:

- $\displaystyle\int \frac{A}{x+a}\,dx = A\ln|x+a| + C$
- $\displaystyle\int \frac{A}{(x+a)^n}\,dx = \frac{A}{(1-n)(x+a)^{n-1}} + C \quad \text{for } n \geq 2$
- $\displaystyle\int \frac{Ax+B}{x^2+bx+c}\,dx$: split into a ln part and an arctan part by completing the square or rewriting the numerator

Pro Tip

- Always factor the denominator completely before writing any decomposition. You cannot set up the partial fractions correctly until every factor is identified.
- When using strategic x values, choose values that zero out factors one at a time. If the denominator has factor $(x-3)$, plug in $x=3$ to wipe out every term containing that factor.
- After finding all constants, always verify by recombining the partial fractions over a common denominator and checking you get the original expression back.
- For a repeated linear factor $(x+a)^3$, you need three separate fractions. Do not try to combine them into one — each power gets its own fraction with its own constant.
- For an irreducible quadratic factor like x^2+4, the numerator of that fraction must be $Ax+B$, not just A. Using a constant numerator on a quadratic denominator is one of the most common setup errors.

Examples

Example 1 (Distinct Linear Factors): Find $\displaystyle\int \frac{5x+1}{(x+1)(x-2)}\,dx$.

Step 1: Set up the decomposition.

$$\frac{5x+1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$

Step 2: Multiply both sides by $(x+1)(x-2)$.

$$5x+1 = A(x-2) + B(x+1)$$

Step 3: Use strategic values of x.
Let $x=2$: $\quad 5(2)+1 = A(0)+B(3) \implies 11 = 3B \implies B = \dfrac{11}{3}$
Let $x=-1$: $\quad 5(-1)+1 = A(-3)+B(0) \implies -4 = -3A \implies A = \dfrac{4}{3}$
Step 4: Write the decomposition.

$$\frac{5x+1}{(x+1)(x-2)} = \frac{4/3}{x+1} + \frac{11/3}{x-2}$$

Step 5: Integrate.

$$\int \frac{5x+1}{(x+1)(x-2)}\,dx = \frac{4}{3}\ln|x+1| + \frac{11}{3}\ln|x-2| + C$$

$$\boxed{\int \frac{5x+1}{(x+1)(x-2)}\,dx = \frac{4}{3}\ln|x+1| + \frac{11}{3}\ln|x-2| + C}$$

Example 2 (Repeated Linear Factor — Squared): Find $\displaystyle\int \frac{3x+5}{(x+1)^2}\,dx$.

Step 1: Set up the decomposition for a repeated linear factor.

$$\frac{3x+5}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

Step 2: Multiply both sides by $(x+1)^2$.

$$3x+5 = A(x+1) + B$$

Step 3: Find the constants.
Let $x=-1$: $\quad 3(-1)+5 = A(0)+B \implies 2 = B$
Match coefficients of x: $\quad 3 = A$
Step 4: Write the decomposition.

$$\frac{3x+5}{(x+1)^2} = \frac{3}{x+1} + \frac{2}{(x+1)^2}$$

Step 5: Integrate. For $\dfrac{2}{(x+1)^2} = 2(x+1)^{-2}$, use the power rule.

$$\int \frac{3x+5}{(x+1)^2}\,dx = 3\ln|x+1| + \frac{2(x+1)^{-1}}{-1} + C = 3\ln|x+1| - \frac{2}{x+1} + C$$

$$\boxed{\int \frac{3x+5}{(x+1)^2}\,dx = 3\ln|x+1| - \frac{2}{x+1} + C}$$

Example 3 (Repeated Linear Factor — Cubed): Find $\displaystyle\int \frac{2x^2+3x+1}{x^3}\,dx.$

Step 1: The denominator is $x^3 = (x)^3$, a repeated linear factor of degree 3. Set up three fractions.

$$\frac{2x^2+3x+1}{x^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3}$$

Step 2: Multiply both sides by x^3.

$$2x^2+3x+1 = Ax^2 + Bx + C$$

Step 3: Match coefficients of each power of x.
x^2: $A = 2$ x^1: $B = 3$ x^0: $C = 1$
Step 4: Integrate each piece.

$$\int \left(\frac{2}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right) dx = 2\ln|x| + \frac{3x^{-1}}{-1} + \frac{x^{-2}}{-2} + C$$

$$\boxed{\int \frac{2x^2+3x+1}{x^3}\,dx = 2\ln|x| - \frac{3}{x} - \frac{1}{2x^2} + C}$$

Example 4 (Irreducible Quadratic Factor): Find $\displaystyle\int \frac{3x^2+2x+4}{(x-1)(x^2+4)}\,dx.$

Step 1: x^2+4 cannot be factored over the reals (discriminant < 0), so it is irreducible. Set up:

$$\frac{3x^2+2x+4}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$$

Step 2: Multiply both sides by $(x-1)(x^2+4)$.

$$3x^2+2x+4 = A(x^2+4) + (Bx+C)(x-1)$$

Step 3: Find A using $x = 1$.

$$3(1) + 2(1) + 4 = A(1+4) + 0 \implies 9 = 5A \implies A = \frac{9}{5}$$

Step 4: Expand the right side to find B and C.

$$A(x^2+4)+(Bx+C)(x-1) = Ax^2+4A+Bx^2-Bx+Cx-C$$

$$= (A+B)x^2+(-B+C)x+(4A-C)$$

Match coefficients:
x^2: $A+B=3 \implies B = 3-\frac{9}{5} = \frac{6}{5}$
x^0: $4A-C=4 \implies C = 4A-4 = \frac{36}{5}-4 = \frac{16}{5}$
Step 5: Integrate. For the quadratic piece, split the numerator:

$$\frac{Bx+C}{x^2+4} = \frac{(6/5)x}{x^2+4} + \frac{16/5}{x^2+4}$$

$$\int \frac{(6/5)x}{x^2+4}\,dx = \frac{6}{5}\cdot\frac{1}{2}\ln(x^2+4) = \frac{3}{5}\ln(x^2+4)$$

$$\int \frac{16/5}{x^2+4}\,dx = \frac{16}{5}\cdot\frac{1}{2}\arctan\left(\frac{x}{2}\right) = \frac{8}{5}\arctan\left(\frac{x}{2}\right)$$

$$\int \frac{3x^2+2x+4}{(x-1)(x^2+4)}\,dx = \frac{9}{5}\ln|x-1| + \frac{3}{5}\ln(x^2+4) + \frac{8}{5}\arctan\left(\frac{x}{2}\right) + C$$

$$\boxed{\frac{9}{5}\ln|x-1| + \frac{3}{5}\ln(x^2+4) + \frac{8}{5}\arctan\left(\frac{x}{2}\right) + C}$$

Example 5 (Long Division First): Find $\int \frac{x^3+2x}{x^2-1}\,dx$.

Step 1: The degree of the numerator (3) is greater than the degree of the denominator (2), so this is improper. Perform long division.
Divide x^3+2x by x^2-1:
$x^3+2x \div (x^2-1)$: the first term is x. Multiply: $x(x^2-1) = x^3-x$.
Subtract: $(x^3+2x)-(x^3-x) = 3x$.
So:

$$\frac{x^3+2x}{x^2-1} = x + \frac{3x}{x^2-1}$$

Step 2: Apply partial fractions to $\frac{3x}{x^2-1} = \frac{3x}{(x-1)(x+1)}$.

$$\frac{3x}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$3x = A(x+1)+B(x-1)$$

$x=1$: $3=2A \implies A = \frac{3}{2}$

$x = -1$: $-3 = -2B \implies B = \dfrac{3}{2}$

Step 3: Integrate.

$$\int \frac{x^3 + 2x}{x^2 - 1}\,dx = \int x\,dx + \frac{3}{2}\int \frac{1}{x-1}\,dx + \frac{3}{2}\int \frac{1}{x+1}\,dx$$

$$= \frac{x^2}{2} + \frac{3}{2}\ln|x-1| + \frac{3}{2}\ln|x+1| + C$$

$$\boxed{\int \frac{x^3 + 2x}{x^2 - 1}\,dx = \frac{x^2}{2} + \frac{3}{2}\ln|x-1| + \frac{3}{2}\ln|x+1| + C}$$

Example 6 (Definite Integral): Evaluate $\displaystyle\int_2^4 \frac{4}{x^2 - 4}\,dx$.

Step 1: Factor the denominator and set up the decomposition.

$$x^2 - 4 = (x-2)(x+2), \qquad \frac{4}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$4 = A(x+2) + B(x-2)$$

$x = 2$: $4 = 4A \implies A = 1$

$x = -2$: $4 = -4B \implies B = -1$

Step 2: Integrate.

$$\int_2^4 \frac{4}{x^2 - 4}\,dx = \int_2^4 \frac{1}{x-2}\,dx - \int_2^4 \frac{1}{x+2}\,dx = \Big[\ln|x-2| - \ln|x+2|\Big]_2^4$$

Step 3: Evaluate.

At $x = 4$: $\ln|2| - \ln|6| = \ln\left(\dfrac{2}{6}\right) = \ln\left(\dfrac{1}{3}\right)$

At $x = 2$: $\ln|0| - \ln|4|$

The term $\ln|x-2|$ approaches $-\infty$ as $x \to 2^+$, so this integral **diverges** (is an improper integral). A note to students: always check whether the integrand has a vertical asymptote inside or at the boundary of the interval before evaluating a definite integral. Here $x = 2$ is a vertical asymptote of the integrand, making this an improper integral — a topic covered in the next lesson.

For a well-defined definite integral, use limits that avoid the singularity. Let us instead evaluate $\displaystyle\int_3^5 \frac{4}{x^2 - 4}\,dx$.

At $x = 5$: $\ln|3| - \ln|7|$

At $x = 3$: $\ln|1| - \ln|5| = 0 - \ln 5 = -\ln 5$

$$(\ln 3 - \ln 7) - (-\ln 5) = \ln 3 - \ln 7 + \ln 5 = \ln\left(\frac{15}{7}\right)$$

$$\boxed{\int_3^5 \frac{4}{x^2 - 4}\,dx = \ln\left(\frac{15}{7}\right)}$$

Common Mistakes

- **Not checking if the fraction is proper first:** If the numerator degree is greater than or equal to the denominator degree, you must do long division before any partial fractions. Skipping this step leads to an incorrect decomposition.

- **Using a constant numerator on a quadratic denominator:** For an irreducible quadratic factor $x^2 + bx + c$, the numerator must be $Ax + B$, not just A. Writing $\frac{A}{x^2+4}$ instead of $\frac{Ax+B}{x^2+4}$ is a setup error that makes it impossible to find the correct constants.

- **Missing fractions for repeated factors:** A factor $(x+a)^3$ requires three separate fractions: $\frac{A}{x+a}$, $\frac{B}{(x+a)^2}$, and $\frac{C}{(x+a)^3}$. Writing only one or two fractions gives an incomplete decomposition.

- **Forgetting to verify the decomposition:** After finding A, B, C, recombine the fractions over a common denominator and confirm you recover the original expression. This takes thirty seconds and catches algebra errors.

- **Integrating $\frac{1}{(x+a)^2}$ as a logarithm:** The antiderivative of $\frac{1}{(x+a)^2}$ is $-\frac{1}{x+a}$, not $\ln|(x+a)^2|$. The ln rule only applies to the first power $\frac{1}{x+a}$.

Worksheet

1. Find $\displaystyle\int \frac{1}{(x+3)(x-1)}\,dx$.

2. Find $\displaystyle\int \frac{7x-3}{(x-3)(x+2)}\,dx$.

3. Find $\displaystyle\int \frac{4x+1}{(x+2)^2}\,dx$.

4. Find $\displaystyle\int \frac{x^2+2x+3}{x^3+3x^2}\,dx$.

   (Factor the denominator as $x^2(x+3)$, which has a repeated factor.)

5. Find $\displaystyle\int \frac{5x^2+20x+6}{x(x+1)^3}\,dx$.

   (The denominator has a cubed repeated factor.)

6. Find $\displaystyle\int \frac{2x+1}{(x+1)(x^2+1)}\,dx$.

7. Find $\displaystyle\int \frac{3x^2+2}{x^3+2x}\,dx$.

   (Factor as $x(x^2+2)$.)

8. Find $\displaystyle\int \frac{x^3+x+2}{x^2+x-2}\,dx$.

   (Improper fraction — do long division first.)

9. Evaluate $\displaystyle\int_0^1 \frac{3}{(x+1)(x+2)}\,dx$.

10. (Challenge) Find $\displaystyle\int \frac{x^3-x+3}{x^2+x-2}\,dx$.

(Long division first, then partial fractions on the remainder.)

11. (Challenge) Find $\displaystyle\int \frac{2x^2 - x + 4}{x(x^2 + 4)}\,dx$.

Answer Key

1. $\int \frac{1}{(x+3)(x-1)}\,dx$

$\frac{1}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$, so $1 = A(x-1) + B(x+3)$.

$x = 1$: $1 = 4B \implies B = \frac{1}{4}$. $\quad x = -3$: $1 = -4A \implies A = -\frac{1}{4}$.

$$\int \frac{1}{(x+3)(x-1)}\,dx = -\frac{1}{4}\ln|x+3| + \frac{1}{4}\ln|x-1| + C$$

$$\boxed{-\frac{1}{4}\ln|x+3| + \frac{1}{4}\ln|x-1| + C}$$

2. $\int \frac{7x-3}{(x-3)(x+2)}\,dx$

$7x - 3 = A(x+2) + B(x-3)$.

$x = 3$: $18 = 5A \implies A = \frac{18}{5}$. $\quad x = -2$: $-17 = -5B \implies B = \frac{17}{5}$.

$$\boxed{\frac{18}{5}\ln|x-3| + \frac{17}{5}\ln|x+2| + C}$$

3. $\int \frac{4x+1}{(x+2)^2}\,dx$

$\frac{4x+1}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$, so $4x+1 = A(x+2) + B$.

$x = -2$: $-7 = B$. Match x-coefficients: $A = 4$.

$$= 4\ln|x+2| - \frac{7(x+2)^{-1}}{-1} \cdot (-1) + C$$

Wait — integrate $\frac{B}{(x+2)^2} = -7(x+2)^{-2}$:

$$\int -7(x+2)^{-2}\,dx = \frac{-7(x+2)^{-1}}{-1} = \frac{7}{x+2}$$

Hmm, but $B = -7$, so:

$$\int \frac{-7}{(x+2)^2}\,dx = \frac{7}{x+2}$$

$$\boxed{\int \frac{4x+1}{(x+2)^2}\,dx = 4\ln|x+2| + \frac{7}{x+2} + C}$$

Verify: $\frac{d}{dx}\left[4\ln|x+2| + \frac{7}{x+2}\right] = \frac{4}{x+2} - \frac{7}{(x+2)^2} = \frac{4(x+2)-7}{(x+2)^2} = \frac{4x+1}{(x+2)^2}$ ✓

4. $\displaystyle\int \frac{x^2+2x+3}{x^2(x+3)}\,dx$

$\dfrac{x^2+2x+3}{x^2(x+3)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3}$

$x^2+2x+3 = Ax(x+3) + B(x+3) + Cx^2$

$x=0$: $3 = 3B \implies B = 1.$

$x=-3$: $9-6+3 = 9C \implies 6 = 9C \implies C = \dfrac{2}{3}.$

Match x^2: $1 = A + C \implies A = 1 - \dfrac{2}{3} = \dfrac{1}{3}.$

$$\int \left(\frac{1/3}{x} + \frac{1}{x^2} + \frac{2/3}{x+3} \right) dx = \frac{1}{3}\ln|x| - \frac{1}{x} + \frac{2}{3}\ln|x+3| + C$$

$$\boxed{\frac{1}{3}\ln|x| - \frac{1}{x} + \frac{2}{3}\ln|x+3| + C}$$

5. $\displaystyle\int \frac{5x^2+20x+6}{x(x+1)^3}\,dx$

Set up: $\dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2} + \dfrac{D}{(x+1)^3}$

$5x^2+20x+6 = A(x+1)^3 + Bx(x+1)^2 + Cx(x+1) + Dx$

$x=0$: $6 = A.$

$x=-1$: $5-20+6 = -D \implies -9 = -D \implies D = 9.$

Expand and match x^3: $0 = A + B \implies B = -6.$

Match x^2: $5 = 3A + 2B + C = 18 - 12 + C \implies C = -1.$

$$\int \left(\frac{6}{x} - \frac{6}{x+1} - \frac{1}{(x+1)^2} + \frac{9}{(x+1)^3} \right) dx$$

$$= 6\ln|x| - 6\ln|x+1| + \frac{1}{x+1} - \frac{9}{2(x+1)^2} + C$$

$$\boxed{6\ln|x| - 6\ln|x+1| + \frac{1}{x+1} - \frac{9}{2(x+1)^2} + C}$$

6. $\displaystyle\int \frac{2x+1}{(x+1)(x^2+1)}\,dx$

$\dfrac{2x+1}{(x+1)(x^2+1)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1}$

$2x+1 = A(x^2+1) + (Bx+C)(x+1)$

$x=-1$: $-1 = 2A \implies A = -\dfrac{1}{2}.$

Match x^2: $0 = A + B \implies B = \dfrac{1}{2}.$

Match x^0: $1 = A + C \implies C = 1 - A = \dfrac{3}{2}.$

$$\int \left(\frac{-1/2}{x+1} + \frac{(1/2)x+3/2}{x^2+1} \right) dx$$
$$= -\frac{1}{2}\ln|x+1| + \frac{1}{4}\ln(x^2+1) + \frac{3}{2}\arctan(x) + C$$
$$\boxed{-\frac{1}{2}\ln|x+1| + \frac{1}{4}\ln(x^2+1) + \frac{3}{2}\arctan(x) + C}$$

7. $\displaystyle\int \frac{3x^2+2}{x(x^2+2)}\,dx$
$\displaystyle\frac{3x^2+2}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$
$3x^2+2 = A(x^2+2) + (Bx+C)x$
$x=0$: $2 = 2A \implies A = 1$.
Match x^2: $3 = A + B \implies B = 2$.
Match x^1: $0 = C$.

$$\int \left(\frac{1}{x} + \frac{2x}{x^2+2} \right) dx = \ln|x| + \ln(x^2+2) + C$$
$$\boxed{\ln|x| + \ln(x^2+2) + C}$$

8. $\displaystyle\int \frac{x^3+x+2}{x^2+x-2}\,dx$
Step 1: Improper (degree 3 > degree 2). Divide x^3+x+2 by x^2+x-2.
$x^3 \div x^2 = x$. Multiply: $x(x^2+x-2) = x^3+x^2-2x$. Subtract: $-x^2+3x+2$.
$-x^2 \div x^2 = -1$. Multiply: $-1(x^2+x-2) = -x^2-x+2$. Subtract: $4x$.
So $\displaystyle\frac{x^3+x+2}{x^2+x-2} = x - 1 + \frac{4x}{x^2+x-2}$.
Step 2: Factor $x^2+x-2 = (x+2)(x-1)$.
$\displaystyle\frac{4x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$, so $4x = A(x-1) + B(x+2)$.
$x=1$: $4 = 3B \implies B = \frac{4}{3}$. $\quad x=-2$: $-8 = -3A \implies A = \frac{8}{3}$.
Step 3: Integrate.

$$\int \left(x - 1 + \frac{8/3}{x+2} + \frac{4/3}{x-1} \right) dx = \frac{x^2}{2} - x + \frac{8}{3}\ln|x+2| + \frac{4}{3}\ln|x-1| + C$$
$$\boxed{\frac{x^2}{2} - x + \frac{8}{3}\ln|x+2| + \frac{4}{3}\ln|x-1| + C}$$

9. $\displaystyle\int_0^1 \frac{3}{(x+1)(x+2)}\,dx$

$\dfrac{3}{(x+1)(x+2)} = \dfrac{A}{x+1} + \dfrac{B}{x+2}$, so $3 = A(x+2) + B(x+1)$.
$x = -1$: $3 = A$. $\quad x = -2$: $3 = -B \implies B = -3$.

$$\int_0^1 \frac{3}{(x+1)(x+2)}\,dx = \Big[3\ln|x+1| - 3\ln|x+2|\Big]_0^1 = \left[3\ln\left(\frac{x+1}{x+2}\right)\right]_0^1$$

At $x = 1$: $3\ln\left(\dfrac{2}{3}\right)$. $\quad$ At $x = 0$: $3\ln\left(\dfrac{1}{2}\right)$.

$$= 3\ln\left(\frac{2}{3}\right) - 3\ln\left(\frac{1}{2}\right) = 3\ln\left(\frac{2/3}{1/2}\right) = 3\ln\left(\frac{4}{3}\right)$$

$$\boxed{\int_0^1 \frac{3}{(x+1)(x+2)}\,dx = 3\ln\left(\frac{4}{3}\right)}$$

10. (Challenge) $\displaystyle\int \frac{x^3 - x + 3}{x^2 + x - 2}\,dx$
Step 1: Long division. $x^3 \div x^2 = x$. Multiply: $x^3 + x^2 - 2x$. Subtract: $-x^2 + x + 3$.
$-x^2 \div x^2 = -1$. Multiply: $-x^2 - x + 2$. Subtract: $2x + 1$.
So $\dfrac{x^3 - x + 3}{x^2 + x - 2} = x - 1 + \dfrac{2x+1}{x^2 + x - 2}$.
Step 2: $x^2 + x - 2 = (x+2)(x-1)$.
$\dfrac{2x+1}{(x+2)(x-1)} = \dfrac{A}{x+2} + \dfrac{B}{x-1}$
$x = 1$: $3 = 3B \implies B = 1$. $\quad x = -2$: $-3 = -3A \implies A = 1$.
Step 3: Integrate.

$$\int \left(x - 1 + \frac{1}{x+2} + \frac{1}{x-1}\right) dx = \frac{x^2}{2} - x + \ln|x+2| + \ln|x-1| + C$$

$$\boxed{\frac{x^2}{2} - x + \ln|x+2| + \ln|x-1| + C}$$

11. (Challenge) $\displaystyle\int \frac{2x^2 - x + 4}{x(x^2+4)}\,dx$
$\dfrac{2x^2 - x + 4}{x(x^2+4)} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 4}$
$2x^2 - x + 4 = A(x^2+4) + (Bx + C)x$
$x = 0$: $4 = 4A \implies A = 1$.
Match x^2: $2 = A + B \implies B = 1$.
Match x^1: $-1 = C$.

$$\int \left(\frac{1}{x} + \frac{x-1}{x^2+4}\right) dx = \ln|x| + \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

$$\boxed{\ln|x| + \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}$$

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