

Lesson 3: Product and Quotient Rule

ApexMath

Notes

- **Product Rule:** If $f(x) = u \cdot v$, then

$$f'(x) = u'v + uv'$$

- **Quotient Rule:** If $f(x) = \frac{u}{v}$, then

$$f'(x) = \frac{u'v - uv'}{v^2}$$

- Label each part clearly as u and v before differentiating.
- Simplify after applying the rule.
- Radicals: rewrite as exponents first.

Pro Tips

- Take it slow: differentiate one term at a time.
- For products, apply the rule once per problem.
- For quotients, write the numerator and denominator clearly and square the denominator.
- Simplify radicals first: $\sqrt{x} = x^{1/2}$, $\sqrt[3]{x} = x^{1/3}$.

Deep Walkthrough Examples

Example 1: $f(x) = (x + 2)(x - 3)$

Step 1: Let $u = x + 2$, $v = x - 3$.

Step 2: Derivatives: $u' = 1$, $v' = 1$.

Step 3: Apply product rule:

$$f'(x) = u'v + uv' = (1)(x - 3) + (x + 2)(1) = 2x - 1$$

$f'(x) = 2x - 1$

Example 2: $f(x) = \frac{x^2 + 1}{x + 1}$

Step 1: Let $u = x^2 + 1$, $v = x + 1$.

Step 2: Derivatives: $u' = 2x$, $v' = 1$.

Step 3: Apply quotient rule:

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{(2x)(x+1) - (x^2+1)(1)}{(x+1)^2} = \frac{x^2+2x-1}{(x+1)^2}$$

$$f'(x) = (x^2+2x-1)/(x+1)^2$$

Example 3: $f(x) = (x+1)\sqrt{x}$

Step 1: Rewrite $\sqrt{x} = x^{1/2}$.

Step 2: Apply product rule:

$$f'(x) = (1)(x^{1/2}) + (x+1)(1/2)x^{-1/2} = x^{1/2} + \frac{x+1}{2\sqrt{x}} = \frac{3x+1}{2\sqrt{x}}$$

$$f'(x) = (3x+1)/(2\sqrt{x})$$

Common Mistakes

- Forgetting to apply the correct rule.
- Forgetting to square the denominator for quotient rule.
- Skipping steps when expanding products.
- Forgetting to rewrite radicals as exponents.

Worksheet Problems

1. $f(x) = (x+3)(x-5)$
2. $f(x) = (2x-1)(x+4)$
3. $f(x) = \frac{x^2+2}{x+3}$
4. $f(x) = \frac{x^2-1}{x-2}$
5. $f(x) = (x+1)\sqrt{x}$
6. Bonus: $f(x) = (x+2)(x-1)(x+3)$

Detailed Answer Key

1. $f(x) = (x+3)(x-5)$

$$f'(x) = 1 \cdot (x-5) + (x+3) \cdot 1 = 2x-2$$

$$f'(x) = 2x-2$$

2. $f(x) = (2x-1)(x+4)$

$$f'(x) = 2 \cdot (x+4) + (2x-1) \cdot 1 = 4x+7$$

$$f'(x) = 4x+7$$

3. $f(x) = \frac{x^2+2}{x+3}$

$$f'(x) = \frac{2x(x+3) - (x^2+2) \cdot 1}{(x+3)^2} = \frac{x^2+6x-2}{(x+3)^2}$$

$$f'(x) = (x^2+6x-2)/(x+3)^2$$

4. $f(x) = \frac{x^2-1}{x-2}$

$$f'(x) = \frac{2x(x-2) - (x^2-1) \cdot 1}{(x-2)^2} = \frac{x^2-4x+1}{(x-2)^2}$$

$$f'(x) = (x^2-4x+1)/(x-2)^2$$

5. $f(x) = (x+1)\sqrt{x}$

$$f'(x) = x^{1/2} + \frac{x+1}{2\sqrt{x}} = \frac{3x+1}{2\sqrt{x}}$$

$$f'(x) = (3x+1)/(2\sqrt{x})$$

Bonus. $f(x) = (x+2)(x-1)(x+3)$

Step 1: Let

$$u = (x+2)(x-1), \quad v = (x+3)$$

Step 2: Compute derivatives:

$$u' = 1 \cdot (x-1) + (x+2) \cdot 1 = 2x+1, \quad v' = 1$$

Step 3: Apply product rule:

$$f'(x) = u'v + uv' = (2x+1)(x+3) + (x+2)(x-1)$$

Step 4: Expand and simplify:

$$(2x+1)(x+3) + (x+2)(x-1) = 3x^2 + 8x + 1$$

$$f'(x) = 3x^2 + 8x + 1$$

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