

Lesson 6: Trigonometric and Inverse Trigonometric Derivatives

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Notes

- Derivative of $\sin(u)$:

$$\frac{d}{dx} \sin(u) = \cos(u) \cdot \frac{du}{dx}$$

- Derivative of $\cos(u)$:

$$\frac{d}{dx} \cos(u) = -\sin(u) \cdot \frac{du}{dx}$$

- Derivative of $\tan(u)$:

$$\frac{d}{dx} \tan(u) = \sec^2(u) \cdot \frac{du}{dx}$$

- Derivative of $\arcsin(u)$:

$$\frac{d}{dx} \arcsin(u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

- Derivative of $\arccos(u)$:

$$\frac{d}{dx} \arccos(u) = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

- Derivative of $\arctan(u)$:

$$\frac{d}{dx} \arctan(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

- **Pro Tip:** Always identify the inner function u and apply $\frac{du}{dx}$ for the chain rule when needed.

Pro Tips

- Remember $\sec^2(x)$ for $\tan(x)$ derivative.
- Watch signs for $\cos$ (negative) and $\arccos$ (negative).
- Always simplify fractions and radicals in inverse trig derivatives.

Examples

Example 1: Differentiate $f(x) = \sin(3x)$

Step 1 – General Formula:

$$\frac{d}{dx} \sin(u) = \cos(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 3x, \quad \frac{du}{dx} = 3 \implies \frac{d}{dx} \sin(3x) = \cos(3x) \cdot 3$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \sin(3x) = 3 \cos(3x)}$$

Example 2: Differentiate $f(x) = \arctan(2x)$

Step 1 – General Formula:

$$\frac{d}{dx} \arctan(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 2x, \quad \frac{du}{dx} = 2 \implies \frac{d}{dx} \arctan(2x) = \frac{2}{1+(2x)^2}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \arctan(2x) = \frac{2}{1+4x^2}}$$

Example 3: Differentiate $f(x) = \cos(x^2)$

Step 1 – General Formula:

$$\frac{d}{dx} \cos(u) = -\sin(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = x^2, \quad \frac{du}{dx} = 2x \implies \frac{d}{dx} \cos(x^2) = -\sin(x^2) \cdot 2x$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \cos(x^2) = -2x \sin(x^2)}$$

Common Mistakes

- Forgetting chain rule for the inner function.
- Forgetting negative sign for cos and arccos.
- Miswriting $\sec^2(x)$ for $\tan(x)$ derivative.
- Forgetting to square the argument in arctan denominator.

Worksheet

Differentiate the following:

1. $f(x) = \sin(5x)$
2. $f(x) = \cos(3x^2)$
3. $f(x) = \tan(2x)$
4. $f(x) = \arcsin(x/2)$
5. $f(x) = \arccos(1 - 2x)$
6. $f(x) = \arctan(3x)$

Answer Key

1. Step 1 – General Formula:

$$\frac{d}{dx} \sin(u) = \cos(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 5x, \quad \frac{du}{dx} = 5 \implies \frac{d}{dx} \sin(5x) = \cos(5x) \cdot 5$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \sin(5x) = 5 \cos(5x)}$$

2. Step 1 – General Formula:

$$\frac{d}{dx} \cos(u) = -\sin(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 3x^2, \quad \frac{du}{dx} = 6x \implies \frac{d}{dx} \cos(3x^2) = -\sin(3x^2) \cdot 6x$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \cos(3x^2) = -6x \sin(3x^2)}$$

3. Step 1 – General Formula:

$$\frac{d}{dx} \tan(u) = \sec^2(u) \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 2x, \quad \frac{du}{dx} = 2 \implies \frac{d}{dx} \tan(2x) = \sec^2(2x) \cdot 2$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \tan(2x) = 2 \sec^2(2x)}$$

4. Step 1 – General Formula:

$$\frac{d}{dx} \arcsin(u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = x/2, \quad \frac{du}{dx} = 1/2 \implies \frac{d}{dx} \arcsin(x/2) = \frac{1/2}{\sqrt{1-(x/2)^2}}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \arcsin(x/2) = \frac{1}{2\sqrt{1-x^2/4}}}$$

5. Step 1 – General Formula:

$$\frac{d}{dx} \arccos(u) = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 1 - 2x, \quad \frac{du}{dx} = -2 \implies \frac{d}{dx} \arccos(1 - 2x) = -\frac{-2}{\sqrt{1-(1-2x)^2}}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \arccos(1 - 2x) = \frac{2}{\sqrt{1-(1-2x)^2}}}$$

6. Step 1 – General Formula:

$$\frac{d}{dx} \arctan(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

Step 2 – Plug in Function:

$$u = 3x, \quad \frac{du}{dx} = 3 \implies \frac{d}{dx} \arctan(3x) = \frac{3}{1 + (3x)^2}$$

Step 3 – Simplify:

$$\boxed{\frac{d}{dx} \arctan(3x) = \frac{3}{1 + 9x^2}}$$

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