

Functions & Relations Mastery Pack: Logarithmic Functions

Tutoring Notes

Lesson: Introduction to Logarithms and Laws of Logarithms

What is a Logarithm?

Definition: A logarithm tells us the power to which a base must be raised to get a number:

$$y = \log_b(x) \quad \text{means that} \quad b^y = x$$

Example:

$$\log_2(8) = 3 \quad \text{because} \quad 2^3 = 8$$

Important Notes:

- The base b must be positive and not equal to 1.
- The argument x must be positive.
- Logs are inverses of exponential functions.
- Special logarithms:
 - $\log(x) = \log_{10}(x)$ (common logarithm)
 - $\ln(x) = \log_e(x)$ (natural logarithm)
- Euler's number $e \approx 2.718$ is the base of natural logarithms and appears often in science and finance.

Laws of Logarithms

1. Product Rule:

$$\log_b(M \cdot N) = \log_b(M) + \log_b(N)$$

2. Quotient Rule:

$$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$$

3. Power Rule:

$$\log_b(M^p) = p \cdot \log_b(M)$$

4. Change of Base Formula:

$$\log_b(M) = \frac{\log_c(M)}{\log_c(b)}$$

where c can be any base, commonly $\log_{10}$ or $\ln$.

Walkthrough Examples

Example 1: Solving a Simple Log Equation

Solve $\log_3(x) = 4$

Step 1: Convert logarithm to exponential form:

$$3^4 = x$$

Step 2: Calculate: $3^4 = 81$

Answer: 81

Example 2: Using Logarithm Laws

Simplify $\log_2(8) + \log_2(4)$

Step 1: Apply product rule: $\log_2(8 \cdot 4) = \log_2(32)$

Step 2: Evaluate: $2^5 = 32$, so $\log_2(32) = 5$

Answer: 5

Example 3: Evaluating a Logarithm of a Fraction

Simplify $\log_5\left(\frac{25}{5}\right)$

Step 1: Apply quotient rule: $\log_5(25/5) = \log_5(25) - \log_5(5)$

Step 2: Calculate individually: $\log_5(25) = 2$, $\log_5(5) = 1$

Step 3: Subtract: $2 - 1 = 1$

Answer: 1

Example 4: Using the Power Rule

Simplify $\log_3(9^2)$

Step 1: Apply power rule: $\log_3(9^2) = 2 \cdot \log_3(9)$

Step 2: Simplify $\log_3(9)$: $3^2 = 9$, so $\log_3(9) = 2$

Step 3: Multiply: $2 \cdot 2 = 4$

Answer: 4

Example 5: Real-Life Application - pH Calculation

The pH of a solution is given by:

$$pH = -\log_{10}([H^+])$$

If $[H^+] = 10^{-5}$, find the pH.

Step 1: Substitute the value:

$$pH = -\log_{10}(10^{-5})$$

Step 2: Use the power rule of logs: $\log_{10}(10^{-5}) = -5$

Step 3: Multiply by the negative sign: $-(-5) = 5$

Answer: 5

Worksheet

Problem 1: Solve $\log_5(x) = 2$

Problem 2: Solve $\log_2(16)$

Problem 3: Solve $2\log_3(x) = 4$

Problem 4: Simplify $\log_7(49) + \log_7(7)$

Problem 5: Simplify $\log_4(64) - \log_4(4)$

Problem 6: Derive t from $P = 1000 \cdot 2^t$ using logs

Problem 7: Real-life: The pH of a solution is $pH = -\log_{10}([H^+])$. If $[H^+] = 10^{-5}$, find the pH.

Problem 8: Solve $\log_2(x) + \log_2(x - 2) = 3$

Answer Key (Walkthrough Style)

Problem 1: Solve $\log_5(x) = 2$

Step 1: Convert to exponential: $5^2 = x$

Step 2: Calculate: $5^2 = 25$

Answer: 25

Problem 2: Solve $\log_2(16)$

Step 1: Find x such that $2^x = 16$

Step 2: $2^4 = 16$

Answer: 4

Problem 3: Solve $2\log_3(x) = 4$

Step 1: Divide both sides by 2: $\log_3(x) = 2$

Step 2: Convert to exponential: $3^2 = x$

Step 3: Calculate: $3^2 = 9$

Answer: 9

Problem 4: Simplify $\log_7(49) + \log_7(7)$

Step 1: Product rule: $\log_7(49 \cdot 7) = \log_7(343)$

Step 2: $7^3 = 343$, so $\log_7(343) = 3$

Answer: 3

Problem 5: Simplify $\log_4(64) - \log_4(4)$

Step 1: Quotient rule: $\log_4(64/4) = \log_4(16)$

Step 2: $4^2 = 16$

Answer: 2

Problem 6: Derive t from $P = 1000 \cdot 2^t$

Step 1: Divide by 1000: $P/1000 = 2^t$

Step 2: Apply log base 2: $t = \log_2(P/1000)$

Answer: $t = \log_2(P/1000)$

Problem 7: pH calculation

Step 1: Substitute: $pH = -\log_{10}(10^{-5})$

Step 2: Use power rule: $\log_{10}(10^{-5}) = -5$

Step 3: Multiply by negative: $-(-5) = 5$

Answer: 5

Problem 8: Solve $\log_2(x) + \log_2(x - 2) = 3$

Step 1: Product rule: $\log_2(x(x - 2)) = 3$

Step 2: Convert to exponential: $x(x - 2) = 2^3 = 8$

Step 3: Solve quadratic: $x^2 - 2x - 8 = 0$

Step 4: Factor: $(x - 4)(x + 2) = 0 \implies x = 4$ or $x = -2$

Step 5: Check domain: $x > 0$ and $x - 2 > 0$, only $x = 4$ valid

Answer: 4

Thank You!

If you found this lesson helpful, please consider writing a review. Your feedback helps us improve our tutoring materials and reach more students like you. We appreciate your support!