

Functions & Relations Mastery Pack: Properties of Logarithms

ApexMath

Lesson: Properties of Logarithms

Notes (read slowly, then model)

If $b > 0$, $b \neq 1$, and $x > 0$, then

$$\log_b(x) = y \iff b^y = x.$$

Core Properties:

$$\log_b(MN) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b(M^k) = k \log_b M$$

$$\log_b M = \frac{\ln M}{\ln b}.$$

Always: input > 0 , base $b > 0$, $b \neq 1$.

Deep Walkthrough Examples

Example A. Expand

$$\log_3\left(\frac{81x^{5/2}\sqrt{y}}{2(1-3x)^3z^{1/4}}\right)$$

Step 1. Split the quotient:

$$\log_3(81x^{5/2}\sqrt{y}) - \log_3(2(1-3x)^3z^{1/4})$$

Step 2. Expand each product:

$$(\log_3 81 + \frac{5}{2} \log_3 x + \frac{1}{2} \log_3 y) - (\log_3 2 + 3 \log_3 |1-3x| + \frac{1}{4} \log_3 z)$$

Step 3. Simplify constant:

$$\log_3 81 = 4$$

Final Simplified Expression:

$$4 + \frac{5}{2} \log_3 x + \frac{1}{2} \log_3 y - \log_3 2 - 3 \log_3 |1 - 3x| - \frac{1}{4} \log_3 z$$

Domain: $x > 0, y > 0, z > 0, x \neq \frac{1}{3}$

Example B. Condense

$$3 \log_5(x + 1) + 2 \log_5 y - \frac{1}{2} \log_5 25 - \log_5(x - 2)$$

Step 1. Apply power rule in reverse:

$$\log_5(x + 1)^3 + \log_5(y^2) - \log_5 5 - \log_5(x - 2)$$

Step 2. Simplify constant:

$$\log_5 5 = 1$$

Step 3. Combine into a single log:

$$\log_5 \left(\frac{(x+1)^3 y^2}{5(x-2)} \right)$$

Final Simplified Expression:

$$\log_5 \left(\frac{(x+1)^3 y^2}{5(x-2)} \right)$$

Domain: $x > 2, y > 0$

Example C. Change of Base

$$\log_{\sqrt{3}} 27 = \frac{\ln 27}{\ln \sqrt{3}} = \frac{3 \ln 3}{\frac{1}{2} \ln 3} = 6$$

$$\log_3 50 = \frac{\ln 50}{\ln 3} \approx \frac{3.912}{1.099} = 3.562$$

Final Answers: $\boxed{6}, \boxed{3.562}$

Example D. Express in terms of p, q

If $\log_a 2 = p$, $\log_a 5 = q$, find $\log_a(50\sqrt{8})$.

Step 1. Factorize:

$$50\sqrt{8} = 5^2 \cdot 2^{3/2}$$

Step 2. Apply log rules:

$$\log_a(5^2) + \log_a(2^{3/2}) = 2q + \frac{3}{2}p$$

Final Expression:

$2q + \frac{3}{2}p$

Worksheet Problems

Work carefully. Simplify constants and state domain when needed.

1. Expand fully: $\log_2\left(\frac{32x^3y^{5/4}}{(x-1)^2\sqrt{z}}\right)$
 2. Expand: $\ln\left(\frac{9a^2b^3}{\sqrt[3]{c(a-4)^2}}\right)$
 3. Expand: $\log_7\left(\sqrt{\frac{49m^5}{n^2(m-3)}}\right)$
 4. Condense: $3\log_3(x+1) - \frac{1}{2}\log_3(9x) + 2\log_3 y - \log_3(x-2)$
 5. Condense: $\ln u + 2\ln v - \frac{1}{3}\ln w - \ln 4$
 6. Evaluate exactly: $\log_{\sqrt{5}} 125 - \log_{25} 5$
 7. Approximate: $\log_3 50$, $\log_{10} 7$ (round to 3 decimal places)
 8. Express with $p = \log_a 2$, $q = \log_a 3$: $\log_a\left(\frac{108\sqrt{12}}{9}\right)$
 9. Combine into one log: $\log(x-1) + \frac{1}{2}\log(x+3) - \log(\sqrt{x})$
 10. Simplify: $\log_b\left(\frac{x\sqrt{b}}{b^2\sqrt[3]{y}}\right)$
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Detailed Answer Key

1. Expand $\log_2\left(\frac{32x^3y^{5/4}}{(x-1)^2\sqrt{z}}\right)$

Step 1. Split the quotient:

$$\log_2(32x^3y^{5/4}) - \log_2((x-1)^2\sqrt{z})$$

Step 2. Expand numerator:

$$\log_2 32 + 3\log_2 x + \frac{5}{4}\log_2 y$$

Step 3. Expand denominator:

$$2\log_2 |x-1| + \frac{1}{2}\log_2 z$$

Step 4. Simplify constant:

$$\log_2 32 = 5$$

Final Expression:

$$5 + 3\log_2 x + \frac{5}{4}\log_2 y - 2\log_2 |x-1| - \frac{1}{2}\log_2 z$$

Domain: $x > 0, x \neq 1, y > 0, z > 0$

2. Expand $\ln\left(\frac{9a^2b^3}{\sqrt[3]{c(a-4)^2}}\right)$

Step 1. Split the quotient:

$$\ln(9a^2b^3) - \ln(\sqrt[3]{c(a-4)^2})$$

Step 2. Expand numerator:

$$\ln 9 + 2\ln a + 3\ln b$$

Step 3. Expand denominator:

$$\frac{1}{3}\ln c + \frac{2}{3}\ln |a-4|$$

Step 4. Simplify constant:

$$\ln 9 = 2\ln 3$$

Final Expression:

$$\boxed{2 \ln 3 + 2 \ln a + 3 \ln b - \frac{1}{3} \ln c - \frac{2}{3} \ln |a - 4|}$$

Domain: $a > 0, b > 0, c > 0, a \neq 4$

3. Expand $\log_7 \left(\sqrt{\frac{49m^5}{n^2(m-3)}} \right)$

Step 1. Pull out square root:

$$\frac{1}{2} \log_7 \left(\frac{49m^5}{n^2(m-3)} \right)$$

Step 2. Split quotient:

$$\frac{1}{2} [\log_7(49m^5) - \log_7(n^2(m-3))]$$

Step 3. Expand numerator:

$$\log_7 49 + 5 \log_7 m$$

Step 4. Expand denominator:

$$2 \log_7 n + \log_7 |m - 3|$$

Step 5. Simplify constant:

$$\log_7 49 = 2$$

Final Expression:

$$\boxed{1 + \frac{5}{2} \log_7 m - \log_7 n - \frac{1}{2} \log_7 |m - 3|}$$

Domain: $m > 0, n > 0, m \neq 3$

4. Condense $3 \log_3(x+1) - \frac{1}{2} \log_3(9x) + 2 \log_3 y - \log_3(x-2)$

Step 1. Apply power rule in reverse:

$$\log_3(x+1)^3 - \log_3(3\sqrt{x}) + \log_3 y^2 - \log_3(x-2)$$

Step 2. Combine into one log:

$$\log_3 \left(\frac{(x+1)^3 y^2}{3\sqrt{x}(x-2)} \right)$$

Final Expression:

$$\log_3 \left(\frac{(x+1)^3 y^2}{3\sqrt{x(x-2)}} \right)$$

Domain: $x > 2, y > 0$

5. Condense $\ln u + 2 \ln v - \frac{1}{3} \ln w - \ln 4$

Step 1. Apply power rule in reverse:

$$\ln u + \ln v^2 - \ln w^{1/3} - \ln 4$$

Step 2. Combine logs:

$$\ln \left(\frac{uv^2}{4w^{1/3}} \right)$$

Final Expression:

$$\ln \left(\frac{uv^2}{4w^{1/3}} \right)$$

Domain: $u, v, w > 0$

6. Evaluate $\log_{\sqrt{5}} 125 - \log_{25} 5$

Step 1. Compute first log:

$$\log_{\sqrt{5}} 125 = \frac{\ln 125}{\ln \sqrt{5}} = \frac{3 \ln 5}{\frac{1}{2} \ln 5} = 6$$

Step 2. Compute second log:

$$\log_{25} 5 = \frac{\ln 5}{\ln 25} = \frac{\ln 5}{2 \ln 5} = \frac{1}{2}$$

Step 3. Subtract:

$$6 - \frac{1}{2} = \frac{11}{2}$$

Final Answer: $\boxed{\frac{11}{2}}$

7. Approximate $\log_3 50, \log_{10} 7$

Step 1. Change of base:

$$\log_3 50 = \frac{\ln 50}{\ln 3} \approx \frac{3.912}{1.099} = 3.562$$

Step 2. Change of base:

$$\log_{10} 7 = \frac{\ln 7}{\ln 10} \approx \frac{1.946}{2.303} = 0.845$$

Final Answers: $\boxed{3.562}$, $\boxed{0.845}$

8. Express with $p = \log_a 2$, $q = \log_a 3$: $\log_a\left(\frac{108\sqrt{12}}{9}\right)$

Step 1. Simplify inside:

$$\frac{108\sqrt{12}}{9} = 12\sqrt{12} = 12 \cdot 2\sqrt{3} = 24\sqrt{3}$$

$$24\sqrt{3} = 2^3 \cdot 3^{3/2}$$

Step 2. Apply logs:

$$3p + \frac{3}{2}q$$

Final Expression: $\boxed{3p + \frac{3}{2}q}$

9. Combine $\log(x-1) + \frac{1}{2}\log(x+3) - \log(\sqrt{x})$

Step 1. Rewrite half log:

$$\frac{1}{2}\log(x+3) = \log(\sqrt{x+3})$$

Step 2. Rewrite $\log(\sqrt{x})$:

$$\log(\sqrt{x}) = \frac{1}{2}\log x$$

Step 3. Combine:

$$\log(x-1) + \log(\sqrt{x+3}) - \frac{1}{2}\log x$$

$$= \log\left(\frac{(x-1)\sqrt{x+3}}{\sqrt{x}}\right)$$

Final Expression:

$$\boxed{\log\left(\frac{(x-1)\sqrt{x+3}}{\sqrt{x}}\right)}$$

Domain: $x > 1$

10. Simplify $\log_b \left(\frac{x\sqrt{b}}{b^2 \sqrt[3]{y}} \right)$

Step 1. Split:

$$\log_b x + \log_b \sqrt{b} - \log_b (b^2) - \log_b (\sqrt[3]{y})$$

Step 2. Simplify terms:

$$\log_b \sqrt{b} = \frac{1}{2}, \quad \log_b (b^2) = 2, \quad \log_b (\sqrt[3]{y}) = \frac{1}{3} \log_b y$$

Step 3. Collect terms:

$$\log_b x + \frac{1}{2} - 2 - \frac{1}{3} \log_b y$$

$$= \log_b x - \frac{3}{2} - \frac{1}{3} \log_b y$$

Final Expression: $\boxed{\log_b x - \frac{3}{2} - \frac{1}{3} \log_b y}$

Domain: $x, y > 0, b > 0, b \neq 1$

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