Functions & Relations Mastery Pack: Solving Exponential & Logarithmic Equations

ApexMath

Lesson: Solving Exponential and Logarithmic Equations

Notes

Exponential Equations Strategy:

- If bases can be made the same, equate exponents.
- Otherwise, apply ln or log to both sides and solve for the variable.
- Watch for quadratic forms in disguise (e.g. e^{2x} terms).

Logarithmic Equations Strategy:

- Combine logs into one expression if possible.
- Rewrite the log equation in exponential form.
- Always check solutions against the domain (log arguments > 0).

—

Deep Walkthrough Examples

Example A. Solve $\quad 7^{2x-1} = 4^{x+2}$

Step 1. Take natural log of both sides: $\ln(7^{2x-1}) = \ln(4^{x+2})$

Step 2. Expand exponents: $(2x-1)\ln 7 = (x+2)\ln 4$

Step 3. Rearrange: $2x\ln 7 - \ln 7 = x\ln 4 + 2\ln 4$

$2x\ln 7 - x\ln 4 = \ln 7 + 2\ln 4$

Step 4. Factor x: $x(2\ln 7 - \ln 4) = \ln 7 + 2\ln 4$

Step 5. Solve: $x = \frac{\ln 7 + 2\ln 4}{2\ln 7 - \ln 4} \approx 1.883$

Final Answer: $\boxed{\frac{\ln 7 + 2\ln 4}{2\ln 7 - \ln 4} \approx 1.883}$

—

Example B. Solve $3^x + 3^{x+1} = 108$

Step 1. Factor 3^x: $3^x(1 + 3) = 108$

$3^x \cdot 4 = 108$

Step 2. Simplify: $3^x = 27$

Step 3. Solve: $3^x = 3^3 \implies x = 3$

Final Answer: $\boxed{3}$

—

Example C. Solve $e^{2x} - 5e^x + 6 = 0$

Step 1. Substitution: Let $y = e^x$, then equation is $y^2 - 5y + 6 = 0$

Step 2. Factor: $(y - 2)(y - 3) = 0$

So $y = 2$ or $y = 3$

Step 3. Back-substitute: $e^x = 2 \Rightarrow x = \ln 2 \; e^x = 3 \Rightarrow x = \ln 3$

Final Answer: $\boxed{\ln 2,\ \ln 3}$

—

Example D. Solve $\log_2(x + 1) + \log_2(x - 3) = 4$

Step 1. Combine logs: $\log_2((x + 1)(x - 3)) = 4$

Step 2. Exponential form: $(x + 1)(x - 3) = 2^4 = 16$

Step 3. Expand: $x^2 - 2x - 3 = 16$

$x^2 - 2x - 19 = 0$

Step 4. Solve quadratic: $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-19)}}{2}$

$x = \frac{2 \pm \sqrt{80}}{2}$

$x = \frac{2 \pm 4\sqrt{5}}{2} = 1 \pm 2\sqrt{5}$

Step 5. Domain check: $x - 3 > 0 \implies x > 3$.

So only $x = 1 + 2\sqrt{5} \approx 5.47$ is valid.

Final Answer: $\boxed{1 + 2\sqrt{5} \ \approx 5.47}$

Common Mistakes to Avoid

- **Forgetting domain restrictions:** Always check that log arguments are > 0 and denominators are not 0. Extra solutions often appear after squaring or manipulating logs.
- **Mixing log properties:** Remember $\log_b(MN) = \log_b M + \log_b N$ $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$ *but not* $\log_b(M + N)$.
- **Dropping parentheses:** For example, $\ln(2x^2 - 5) \neq \ln(2x^2) - \ln(5)$.
- **Rounding too early:** Keep answers exact until the last step (e.g. $\ln 2$ or $\frac{\ln 7}{2 \ln 5}$), then approximate to 3 d.p.
- **Not isolating the exponential:** Before applying ln, try to write the equation in the form $a^{f(x)} = k$ to make logs cleaner.

—

Worksheet Problems

1. Solve $7^{2x-1} = 4^{x+2}$

2. Solve $3^x + 3^{x+1} = 108$

3. Solve $5^{2x} = 7$

4. Solve $\ln(2x^2 - 5) = 3$

5. Solve $\log_2(x + 1) + \log_2(x - 3) = 4$

6. Solve $\log_5(x - 1) - \log_5(2) = 2$

7. Solve $e^{2x} - 5e^x + 6 = 0$

8. Solve $\ln(x + 2) - \ln(x) = \ln 3$

9. Solve $4^{x+1} = 7^{x-2}$

10. Solve $\ln(3x) - \ln(x - 2) = 2$

—

Detailed Answer Key

1. $7^{2x-1} = 4^{x+2}$

$\ln(7^{2x-1}) = \ln(4^{x+2})$

$(2x-1)\ln 7 = (x+2)\ln 4$

$2x\ln 7 - x\ln 4 = \ln 7 + 2\ln 4$

$x = \frac{\ln 7 + 2\ln 4}{2\ln 7 - \ln 4} \approx 1.883$

$$\text{Answer: } \boxed{\frac{\ln 7+2\ln 4}{2\ln 7-\ln 4} \approx 1.883}$$

—

2. $3^x + 3^{x+1} = 108$

$3^x(1+3) = 108$

$3^x = 27$

$x = 3$

$$\text{Answer: } \boxed{3}$$

—

3. $5^{2x} = 7$

Take ln: $2x\ln 5 = \ln 7$

$x = \frac{\ln 7}{2\ln 5} \approx 0.605$

$$\text{Answer: } \boxed{\frac{\ln 7}{2\ln 5} \approx 0.605}$$

—

4. $\ln(2x^2 - 5) = 3$

$2x^2 - 5 = e^3$

$2x^2 = e^3 + 5$

$x^2 = \frac{e^3+5}{2}$

$x = \pm\sqrt{\frac{e^3+5}{2}} \approx \pm 3.542$

Domain: $2x^2 - 5 > 0 \to$ both valid.

$$\text{Answer: } \boxed{\pm 3.542}$$

—

5. $\log_2(x+1) + \log_2(x-3) = 4$

$\log_2((x+1)(x-3)) = 4$

$(x+1)(x-3) = 16$

$x^2 - 2x - 3 = 16$

$x^2 - 2x - 19 = 0$

$x = 1 \pm 2\sqrt{5}$

Domain: $x > 3$ so $x = 1 + 2\sqrt{5} \approx 5.47$

$$\text{Answer: } \boxed{1 + 2\sqrt{5} \ \approx 5.47}$$

—

6. $\log_5(x-1) - \log_5(2) = 2$

$\log_5\left(\frac{x-1}{2}\right) = 2$

$\frac{x-1}{2} = 5^2 = 25$

$x - 1 = 50$

$x = 51$

$$\text{Answer: } \boxed{51}$$

—

7. $e^{2x} - 5e^x + 6 = 0$

Let $y = e^x$: $y^2 - 5y + 6 = 0$

$(y-2)(y-3) = 0$

$y = 2, \ y = 3$

$x = \ln 2, \ \ln 3$

Answer: $\boxed{\ln 2,\ \ln 3}$

—

8. $\ln(x+2) - \ln(x) = \ln 3$

$\ln\left(\frac{x+2}{x}\right) = \ln 3$

$\frac{x+2}{x} = 3$

$x + 2 = 3x$

$2x = 2$

$x = 1$

Domain: $x > 0$, valid.

Answer: $\boxed{1}$

—

9. $4^{x+1} = 7^{x-2}$

Take ln: $(x+1)\ln 4 = (x-2)\ln 7$

$x\ln 4 + \ln 4 = x\ln 7 - 2\ln 7$

$x(\ln 4 - \ln 7) = -2\ln 7 - \ln 4$

$x = \frac{-2\ln 7 - \ln 4}{\ln 4 - \ln 7} \approx 9.432$

Answer: $\boxed{9.432}$

—

10. $\ln(3x) - \ln(x-2) = 2$

$\ln\left(\frac{3x}{x-2}\right) = 2$

$\frac{3x}{x-2} = e^2$

$3x = e^2(x-2)$

$3x = e^2x - 2e^2$

$3x - e^2x = -2e^2$

$x = \frac{-2e^2}{3-e^2} \approx 3.367$

Domain: $x > 2$, valid.

Answer: $\boxed{3.367}$

—

Thank you for learning with ApexMath!

If you found this lesson helpful, we'd love for you to share your feedback or leave a review.
