# Lesson 8: Applications of Trigonometric, Exponential, and Sequential Functions

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## Notes

### 1) Trigonometric Functions

- Identify amplitude, period, phase shift, and vertical shift.
- Formula: $y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$.
- Solve equations by isolating the trig function and using reference angles.

### 2) Exponential Functions

- General form: $y = a \cdot b^x$.
- Continuous growth/decay: $y = ae^{kx}$.
- Solve for $x$ using logarithms.

### 3) Sequences and Series

- Arithmetic: $a_n = a_1 + (n-1)d$, sum $S_n = \frac{n}{2}(a_1 + a_n)$.
- Geometric: $a_n = a_1 r^{n-1}$, sum $S_n = a_1 \frac{1-r^n}{1-r}$ if $r \neq 1$.

# Walkthrough Questions

**1. Trigonometric Function Walkthrough:**

A Ferris wheel has radius 10 meters and completes a full rotation every 20 seconds. The lowest point is 2 meters above the ground.

1. Step 1: Determine amplitude $A = 10$ and vertical shift $D = 12$.
2. Step 2: Calculate $B = \frac{2\pi}{20} = \frac{\pi}{10}$ for period.
3. Step 3: Write equation: $h(t) = 10\cos(\frac{\pi}{10}t) + 12$.
4. Step 4: Solve for $t$ when $h(t) = 20$.

**2. Exponential Function Walkthrough:**

A bacteria culture grows as $P(t) = 200e^{0.3t}$.

1. Step 1: Set $P(t) = 1000$ and isolate $e^{0.3t}$.
2. Step 2: Apply natural logarithm: $0.3t = \ln(5)$.
3. Step 3: Solve for $t$: $t = \frac{\ln(5)}{0.3} \approx 5.36$ hours.
4. Step 4: Find $P(6) = 200e^{0.3 \cdot 6} \approx 1,206$.

**3. Arithmetic Sequence Walkthrough:**

Sequence: $a_1 = 5$, $d = 7$.

1. Step 1: Find 15th term: $a_{15} = 5 + 14 \cdot 7 = 103$.
2. Step 2: Sum first 15 terms: $S_{15} = \frac{15}{2}(5 + 103) = 810$.

**4. Geometric Sequence Walkthrough:**

Sequence: $a_1 = 3$, $r = 2$.

1. Step 1: Find 8th term: $a_8 = 3 \cdot 2^7 = 384$.
2. Step 2: Sum first 8 terms: $S_8 = 3\frac{1-2^8}{1-2} = 765$.

# Worksheet

1. A Ferris wheel has radius 10 meters and completes a full rotation every 20 seconds. The lowest point is 2 meters above the ground. Write the equation for height $h(t)$ and determine $t$ when $h(t) = 20$.

2. A bacteria culture grows according to $P(t) = 200e^{0.3t}$. How long will it take for the population to reach 1000? What is the population after 6 hours?

3. An arithmetic sequence starts with $a_1 = 5$ and has common difference $d = 7$. Find the 15th term and the sum of the first 15 terms.

4. A geometric sequence has first term $a_1 = 3$ and common ratio $r = 2$. Find the 8th term and the sum of the first 8 terms.

# Answer Key

## 1. Trigonometric Function

$$h(t) = 10\cos\left(\frac{\pi}{10}t\right) + 12, \quad t \approx 2.25 \text{ seconds when } h(t) = 20$$

## 2. Exponential Function

$$200e^{0.3t} = 1000 \implies t = \frac{\ln(5)}{0.3} \approx 5.36$$
$$P(6) = 200e^{1.8} \approx 1,206$$

## 3. Arithmetic Sequence

$$a_{15} = 5 + 14 \cdot 7 = 103$$
$$S_{15} = \frac{15}{2}(5 + 103) = 810$$

## 4. Geometric Sequence

$$a_8 = 3 \cdot 2^7 = 384$$
$$S_8 = 3\frac{1 - 2^8}{1 - 2} = 765$$