

Lesson 2: Function Operations

ApexMath

Notes

1) Overview of Function Operations

- Functions can be combined through addition, subtraction, multiplication, and division.
- Notation:

$$(f + g)(x) = f(x) + g(x), \quad (f - g)(x) = f(x) - g(x),$$
$$(f \cdot g)(x) = f(x)g(x), \quad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

- Composition of functions: $(f \circ g)(x) = f(g(x))$

2) Domain Considerations

- For combined functions, the domain is the set of all x values common to both functions where the operations are defined.
- For division, $g(x) \neq 0$.
- For composition, the input x must be in the domain of g and $g(x)$ must be in the domain of f .

3) Example Functions

$$f(x) = 2x + 3, \quad g(x) = x^2 - 1$$

Walkthrough of Example Questions

Example 1: Addition and Subtraction

Compute $(f + g)(x)$ and $(f - g)(x)$.

$$(f + g)(x) = f(x) + g(x) = (2x + 3) + (x^2 - 1) = x^2 + 2x + 2$$

$$(f - g)(x) = f(x) - g(x) = (2x + 3) - (x^2 - 1) = -x^2 + 2x + 4$$

Example 2: Multiplication and Division

$$(f \cdot g)(x) = (2x + 3)(x^2 - 1) = 2x^3 + 3x^2 - 2x - 3$$

$$\left(\frac{f}{g}\right)(x) = \frac{2x + 3}{x^2 - 1}, \quad x \neq \pm 1$$

Example 3: Composition

$$(f \circ g)(x) = f(g(x)) = 2(x^2 - 1) + 3 = 2x^2 + 1$$

$$(g \circ f)(x) = g(f(x)) = (2x + 3)^2 - 1 = 4x^2 + 12x + 8$$

Worksheet

A. Function Operations

Let $f(x) = x^2 + 2$ and $g(x) = 3x - 1$.

1. Compute $(f + g)(x)$
2. Compute $(f - g)(x)$
3. Compute $(f \cdot g)(x)$
4. Compute $\left(\frac{f}{g}\right)(x)$
5. Find $(f \circ g)(x)$ and $(g \circ f)(x)$

B. Domain Practice

1. $f(x) = \sqrt{x+1}$, $g(x) = x - 3$. Find the domain of $(f + g)(x)$
2. $f(x) = x^2 - 4$, $g(x) = \sqrt{x-1}$. Find the domain of $(f \cdot g)(x)$
3. $f(x) = 1/(x-2)$, $g(x) = x + 5$. Find the domain of $(f \circ g)(x)$

Answer Key

A. Function Operations

1. $(f + g)(x) = (x^2 + 2) + (3x - 1) = x^2 + 3x + 1$

$$(f + g)(x) = x^2 + 3x + 1$$

2. $(f - g)(x) = (x^2 + 2) - (3x - 1) = x^2 - 3x + 3$

$$(f - g)(x) = x^2 - 3x + 3$$

$$3. (f \cdot g)(x) = (x^2 + 2)(3x - 1) = 3x^3 - x^2 + 6x - 2$$

$$(f \cdot g)(x) = 3x^3 - x^2 + 6x - 2$$

$$4. \left(\frac{f}{g}\right)(x) = \frac{x^2+2}{3x-1}, \quad x \neq 1/3$$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2+2}{3x-1}, \quad x \neq 1/3$$

$$5. (f \circ g)(x) = f(g(x)) = f(3x - 1) = (3x - 1)^2 + 2 = 9x^2 - 6x + 3$$

$$(f \circ g)(x) = 9x^2 - 6x + 3$$

$$6. (g \circ f)(x) = g(f(x)) = g(x^2 + 2) = 3(x^2 + 2) - 1 = 3x^2 + 5$$

$$(g \circ f)(x) = 3x^2 + 5$$

B. Domain Practice

$$1. (f + g)(x) = \sqrt{x+1} + (x-3) \rightarrow x+1 \geq 0 \implies x \geq -1$$

$$\text{Domain: all real numbers } x \geq -1$$

$$2. (f \cdot g)(x) = (x^2 - 4)\sqrt{x-1} \rightarrow x-1 \geq 0 \implies x \geq 1$$

$$\text{Domain: all real numbers } x \geq 1$$

$$3. (f \circ g)(x) = f(g(x)) = 1/(x+3) \rightarrow x \neq -3$$

$$\text{Domain: all real numbers except } -3$$