Lesson 6: Graph Interpretation Using Multiplicity

ApexMath

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Notes

1) Trends of Even vs. Odd Degree Polynomials

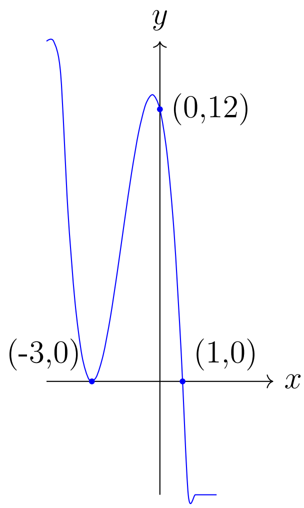
- Even degree polynomials: Both ends of the graph point in the same direction (up-up if leading coefficient positive, down-down if negative)
- Odd degree polynomials: Ends of the graph point in opposite directions (up-down if leading coefficient positive, down-up if negative)
- Multiplicity at zero affects the shape at that zero:
  - Even multiplicity: graph touches the x-axis and turns around
  - Odd multiplicity: graph crosses the x-axis

2) Domain and Range

- Domain: all x values for which the function is defined
- Range: all possible output values $f(x)$

Walkthrough Example

Step 1: Observe the graph



Step 2: Identify key points from the graph

- Zero at $x = -3$ with multiplicity 2 (graph touches and turns)
- Zero at $x = 1$ with multiplicity 1 (graph crosses x-axis)
- y-intercept at $(0, 12)$

Step 3: Write the general polynomial using zeros and multiplicities

$$P(x) = k(x+3)^2(x-1)$$

Step 4: Solve for k using the y-intercept $(0, 12)$

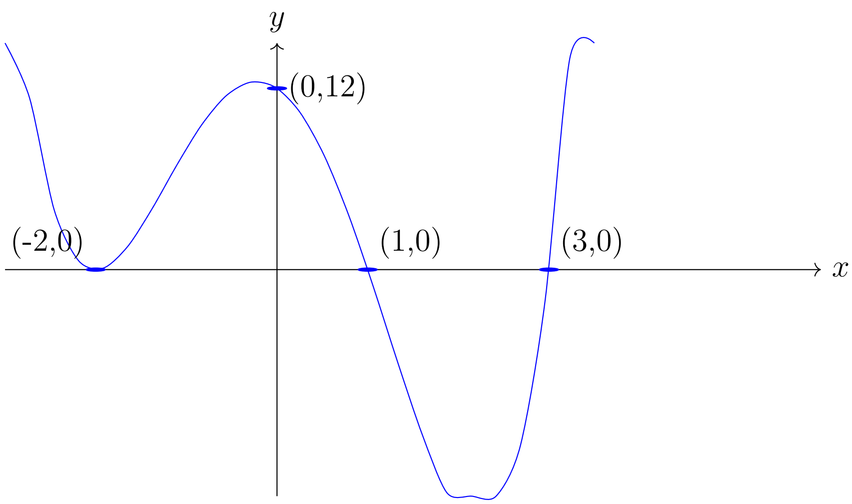
$$\begin{aligned}
P(0) &= k(0+3)^2(0-1) \\
12 &= k(9)(-1) \\
12 &= -9k \\
k &= -\frac{12}{9} = -\frac{4}{3}
\end{aligned}$$

Step 5: Final polynomial equation

$$\boxed{P(x) = -\frac{4}{3}(x+3)^2(x-1)}$$

Worksheet

Question 1: Interpret the following quartic graph and derive its polynomial equation.



Other Questions:

1. A polynomial graph shows zeros at $x = -2$ (multiplicity 3) and $x = 4$ (multiplicity 1), passing through $(0, 16)$. Derive $P(x)$.

2. A polynomial has zeros at $x = -1$ (multiplicity 1), $x = 2$ (multiplicity 2), $x = 3$ (multiplicity 1), and y-intercept at $(0, -24)$. Find $Q(x)$.

3. A polynomial of degree 4 passes through zeros $x = -2$ (multiplicity 2) and $x = 1$ (multiplicity 2), with y-intercept at $(0, 8)$. Find $R(x)$.

4. Graph a polynomial with zeros $x = -3$ (multiplicity 1), $x = 0$ (multiplicity 2), and y-intercept at $(0, 0)$. Then write its polynomial equation.

5. A polynomial has zeros at $x = -1$ (multiplicity 2), $x = 2$ (multiplicity 1), $x = 3$ (multiplicity 1), passing through $(0, 6)$. Derive the polynomial.

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Answer Key

1.

Zeros: $x = -2$ (multiplicity 2), $x = 1$ (multiplicity 1), $x = 3$ (multiplicity 1)
General form:

$$P(x) = k(x+2)^2(x-1)(x-3)$$

Use y-intercept $(0, 12)$:

$$12 = k(2^2)(-1)(-3) = k(4)(3) = 12k \Rightarrow k = 1$$

$$\boxed{P(x) = (x+2)^2(x-1)(x-3)}$$

2.

$$P(x) = k(x+2)^3(x-4)$$

$(0, 16)$:

$$16 = k(8)(-4) = -32k \Rightarrow k = -\frac{1}{2}$$

$$\boxed{P(x) = -\frac{1}{2}(x+2)^3(x-4)}$$

3.

$$Q(x) = k(x+1)(x-2)^2(x-3)$$

$(0, -24)$:

$$-24 = k(1)(4)(-3) = -12k \Rightarrow k = 2$$

$$\boxed{Q(x) = 2(x+1)(x-2)^2(x-3)}$$

4.

$$R(x) = k(x+2)^2(x-1)^2$$

$(0, 8)$:

$$8 = k(4)(1) = 4k \Rightarrow k = 2$$

$$\boxed{R(x) = 2(x+2)^2(x-1)^2}$$

5.

Zeros: $x = -3$ (1), $x = 0$ (2), y-intercept $(0, 0)$:

$$P(x) = k(x + 3)x^2$$

Since $P(0) = 0$, simplest form $k = 1$:

$$\boxed{P(x) = (x + 3)x^2}$$