Lesson 1: Introduction to Functions

ApexMath

Notes

1) What is a function?

- A function is a relation where each input has exactly one output.
- Notation: $f(x)$ represents the output of function f for input x.
- **Vertical line test:** If any vertical line crosses the graph more than once, it is **not** a function, since for every x value it produces more than one output value on the graph.
- Domain: the set of all possible input values (x) for which the function is defined.
- Range: the set of all possible output values ($f(x)$).

2) Domain and Range

- The domain determines the allowable x values, often restricted to avoid undefined expressions such as division by zero or negative numbers under a square root.
- The range is found by examining the outputs produced by all inputs in the domain.
- Example: $h(x) = \sqrt{x-2}$
  There is a domain restriction on $h(x)$ because otherwise it would not pass the vertical line test. To avoid imaginary numbers, $x - 2 \geq 0 \implies x \geq 2$. The range is all non-negative numbers: $[0, \infty)$.

3) Identifying Functions

- Use tables, graphs, or equations to check if each input has exactly one output.
- Apply the vertical line test to graphs.

4) Examples of Functions

$$\begin{aligned} f(x) &= 2x + 3 \quad \text{(linear function)} \\ g(x) &= x^2 - 1 \quad \text{(quadratic function)} \\ h(x) &= \sqrt{x - 2} \quad \text{(radical function with restricted domain } x \geq 2) \end{aligned}$$

Walkthrough of Example Questions

Example 1: Identify Domain and Range

$f(x) = x^2 - 4$

Step 1: Domain All real numbers are allowed since you can square any real number.

$$\text{Domain: } (-\infty, \infty)$$

Step 2: Range The minimum occurs at $x = 0$, $f(0) = -4$, and $f(x)$ increases without bound as $|x|$ increases.

$$\text{Range: } [-4, \infty)$$

Example 2: Determine if Relation is a Function

Given: $(1, 2), (2, 3), (3, 3), (2, 4)$

- Step 1: Check if any input appears more than once with different outputs. - Here, input 2 maps to 3 and 4 $\rightarrow$ violates function rule.

$$\text{Not a function.}$$

Example 3: Radical Function Domain

$h(x) = \sqrt{x - 2}$

- Step 1: Ensure the expression under the square root is non-negative.

$$x - 2 \geq 0 \implies x \geq 2$$

- Step 2: Determine range. Square root always outputs non-negative numbers:

$$\text{Range: } [0, \infty)$$

Worksheet

A. Identify Domain and Range

1. $f(x) = 3x + 1$
2. $g(x) = x^2 - 5x + 6$

3. $h(x) = \sqrt{2x-4}$
4. $k(x) = \frac{1}{x-3}$
5. $m(x) = \sqrt{5-x}$
6. $n(x) = x^3 - 2x$

B. Identify Functions

1. Relation: $(0,1),(1,2),(2,3),(3,4)$
2. Relation: $(1,2),(2,3),(1,5),(3,6)$
3. Equation: $y = x^3 - 2x$
4. Equation: $y^2 = x + 1$
5. Graph with multiple outputs for some x-values (draw example)

Answer Key

A. Domain and Range

1. $f(x) = 3x + 1$: Domain $(-\infty, \infty)$, Range $(-\infty, \infty)$

   $\boxed{\text{Domain: } (-\infty, \infty),\ \text{Range: } (-\infty, \infty)}$

2. $g(x) = x^2 - 5x + 6$: Vertex at $x_v = 5/2$, $y_v = -1/4$

   $\boxed{\text{Domain: } (-\infty, \infty),\ \text{Range: } [-1/4, \infty)}$

3. $h(x) = \sqrt{2x-4}$: $2x - 4 \geq 0 \implies x \geq 2$, Range $[0, \infty)$

   $\boxed{\text{Domain: } [2, \infty),\ \text{Range: } [0, \infty)}$

4. $k(x) = 1/(x-3)$: $x \neq 3$, Range $(-\infty, 0) \cup (0, \infty)$

   $\boxed{\text{Domain: } (-\infty, 3) \cup (3, \infty),\ \text{Range: } (-\infty, 0) \cup (0, \infty)}$

5. $m(x) = \sqrt{5-x}$: $5 - x \geq 0 \implies x \leq 5$, Range $[0, \infty)$

   $\boxed{\text{Domain: } (-\infty, 5],\ \text{Range: } [0, \infty)}$

6. $n(x) = x^3 - 2x$: Domain all real numbers, Range all real numbers

   $\boxed{\text{Domain: } (-\infty, \infty),\ \text{Range: } (-\infty, \infty)}$

B. Identify Functions

1. Each input unique $\rightarrow$ Function

   $\boxed{\text{Function}}$

2. Input 1 maps to 2 and 5 $\rightarrow$ Not a function

   $\boxed{\text{Not a function}}$

3. $y = x^3 - 2x$: one output per input $\rightarrow$ Function

   $\boxed{\text{Function}}$

4. $y^2 = x + 1$: for $x = 0$, $y = \pm 1$ $\rightarrow$ Not a function

   $\boxed{\text{Not a function}}$

5. Graph with multiple outputs for same x $\rightarrow$ Not a function

   $\boxed{\text{Not a function}}$