Absolute Value — Definition and Graph
ApexMath

Notes

What Is Absolute Value?

The **absolute value** of a number is its **distance from zero** on the number line. Distance is always non-negative, so the absolute value of any number is always greater than or equal to zero.

We write the absolute value of x as $|x|$, read as "the absolute value of x."

Key examples:

- $|7| = 7$ (7 is already 7 units from zero)
- $|-5| = 5$ (-5 is 5 units from zero)
- $|0| = 0$ (zero is zero units from zero)

The Piecewise Definition

The formal definition of absolute value splits into two cases depending on the sign of x:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Read this carefully: when x is negative, we take $-x$, which makes the result positive. For example, if $x = -4$, then $-x = -(-4) = 4$.

Absolute Value Inside an Expression

You must simplify the expression *inside* the absolute value bars first, then apply the absolute value.

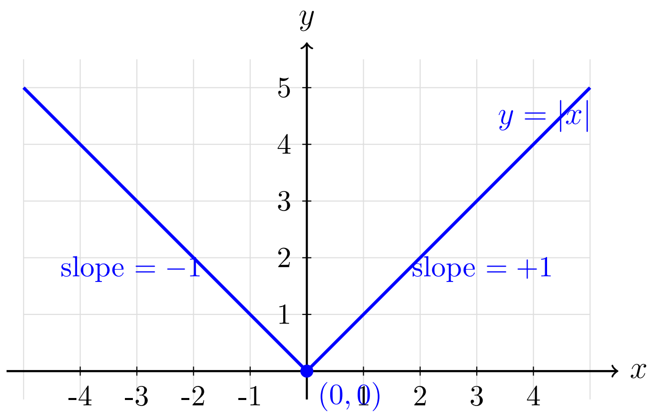
- $|3 - 8| = |-5| = 5$
- $|-4 + 1| = |-3| = 3$
- $|2(-3)| = |-6| = 6$

The Graph of $y = |x|$

When we plot $y = |x|$ for all real values of x, we get a **V-shape**. This is because:

- For $x \geq 0$, the equation becomes $y = x$ — a straight line with slope $+1$ going up to the right.
- For $x < 0$, the equation becomes $y = -x$ — a straight line with slope -1 going up to the left.

The two arms meet at the **vertex** $(0, 0)$.



Key Features of $y = |x|$

- **Vertex:** $(0, 0)$ — the lowest point of the graph
- **Axis of symmetry:** $x = 0$ (the y-axis) — the left and right arms are mirror images
- **Slopes:** $+1$ on the right arm, -1 on the left arm
- **Domain:** all real numbers, $(-\infty, \infty)$
- **Range:** $y \geq 0$, written $[0, \infty)$ — the graph never goes below the x-axis
- **y-intercept:** $(0, 0)$
- **x-intercept:** $(0, 0)$

Pro Tip

- Always simplify the expression *inside* the absolute value bars before applying the absolute value. Treat the bars like parentheses — do the inside first.
- The result of an absolute value is *never negative*. If you get a negative answer, you made an arithmetic error somewhere inside the bars.
- When using the piecewise definition, the case $|x| = -x$ does *not* mean the answer is negative. When x is negative, $-x$ is positive. For example, $x = -3$ gives $-x = -(-3) = 3$.
- To plot $y = |x|$ quickly: find the vertex, then go one unit right and one unit up (slope $+1$), and one unit left and one unit up (slope -1). Connect those points to form the V.
- The absolute value of a number equals the absolute value of its negative: $|x| = |-x|$ for any x. This means $|5| = |-5|$ and $|-3| = |3|$.

Examples

Example 1 (Evaluating Absolute Value): Evaluate each expression.

(a) $|-12|$

The number -12 is 12 units from zero.

$$|-12| = 12$$

(b) $|3-8|$

Simplify inside the bars first:

$$|3-8| = |-5| = 5$$

(c) $|-4+1|$

Simplify inside first:

$$|-4+1| = |-3| = 3$$

(d) $|2(-3)|$

Simplify inside first:

$$|2(-3)| = |-6| = 6$$

Example 2 (Using the Piecewise Definition): Use the piecewise definition to evaluate $|-9|$.

Step 1: Identify which case applies. Since $-9 < 0$, we use the second case: $|x| = -x$.

Step 2: Apply the definition.

$$|-9| = -(-9) = 9$$

The two negatives cancel and give a positive result. This confirms that $|-9| = 9$.

Example 3 (Building the Table for $y = |x|$): Complete the table of values and explain what you notice.

x	$y = \|x\|$
-4	4
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3
4	4

Notice that the y-values are *symmetric*: $x = -3$ and $x = 3$ both give $y = 3$, $x = -2$ and $x = 2$ both give $y = 2$, and so on. This symmetry is what creates the V-shape on the graph. The minimum y-value is 0, reached only at $x = 0$.

Example 4 (Identifying Graph Features): State the vertex, slopes, domain, and range of $y = |x|$.

- **Vertex:** The V meets at $(0, 0)$.
- **Right arm:** For $x \geq 0$, $y = x$. Slope $= +1$.
- **Left arm:** For $x < 0$, $y = -x$. Slope $= -1$.
- **Domain:** Any real number can be substituted for x. Domain: $(-\infty, \infty)$.
- **Range:** The output $y = |x|$ is always ≥ 0. Range: $[0, \infty)$.

Example 5 (Comparing Absolute Values): Without a calculator, determine which is larger: $|-11|$ or $|9|$.

Step 1: Evaluate each.

$$|-11| = 11 \qquad |9| = 9$$

Step 2: Compare.

$$11 > 9 \implies |-11| > |9|$$

Even though -11 is *less than* 9 on the number line, $|-11|$ is *greater than* $|9|$ because -11 is farther from zero.

Common Mistakes

- **Thinking $|-x|$ is always negative:** The expression $-x$ is not automatically negative. If $x = -4$, then $-x = 4$. The piecewise definition uses $-x$ precisely to flip a negative input into a positive output.
- **Applying absolute value before simplifying inside:** $|3 - 8| \neq |3| - |8|$. Always simplify the expression inside the bars first. $|3 - 8| = |-5| = 5$, not $3 - 8 = -5$.
- **Saying the range includes negative numbers:** The range of $y = |x|$ is $y \geq 0$. The graph never dips below the x-axis. Every output is non-negative.
- **Confusing $|-x| = x$:** This is only true when $x > 0$. If $x = -3$, then $|-x| = |-(-3)| = |3| = 3$, not -3. Always substitute the actual number.
- **Forgetting that $|0| = 0$:** Zero is its own absolute value. It is neither positive nor negative, and its distance from zero is zero.

Worksheet

Part 1 — Evaluate each expression.

1. $|9|$

2. $|-6|$

3. $|0|$

4. $|-15|$

5. $|4-11|$

6. $|11-4|$

7. $|-3+8|$

8. $|2-2|$

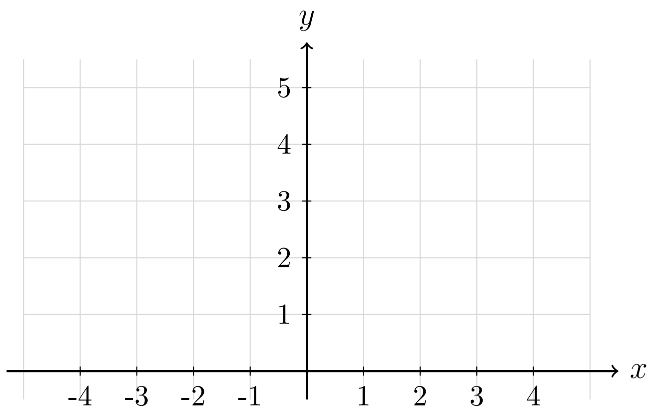
9. $3\,|-4|$

10. $-|-7|$

Part 2 — Complete the table of values for $y = |x|$.

x	$y = \|x\|$
-3	
-2	
-1	
0	
1	
2	
3	

Part 3 — Graph $y = |x|$ using your completed table.



Part 4 — Answer the following questions about $y = |x|$.

11. What are the coordinates of the vertex?

12. What is the slope of the right arm (for $x > 0$)?

13. What is the slope of the left arm (for $x < 0$)?

14. State the domain of $y = |x|$.

15. State the range of $y = |x|$.

Part 5 — Challenge.

16. Find all values of x such that $|x| = 5$.

17. Find all values of x such that $|x| = 0$.

18. Explain in one sentence why $|x| = -3$ has no solution.

19. Without evaluating, explain which is larger: $|-10|$ or $|7|$, and why.

20. If $|x| = |y|$, does that mean $x = y$? Explain with an example.

Answer Key

Part 1

1.

$$|9| = 9$$

$$\boxed{9}$$

2.

$$|-6| = 6$$

$$\boxed{6}$$

3.

$$|0| = 0$$

$$\boxed{0}$$

4.

$$|-15| = 15$$

$$\boxed{15}$$

5. Simplify inside first.

$$|4 - 11| = |-7| = 7$$

$$\boxed{7}$$

6. Simplify inside first.

$$|11 - 4| = |7| = 7$$

$$\boxed{7}$$

7. Simplify inside first.

$$|-3 + 8| = |5| = 5$$

$$\boxed{5}$$

8. Simplify inside first.

$$|2 - 2| = |0| = 0$$

$$\boxed{0}$$

9. Evaluate the absolute value first, then multiply.

$$3\,|-4| = 3 \cdot 4 = 12$$

$$\boxed{12}$$

10. Evaluate the absolute value first, then apply the negative sign outside.

$$-|-7| = -(7) = -7$$

Note: the negative sign is *outside* the bars, so it is applied after.

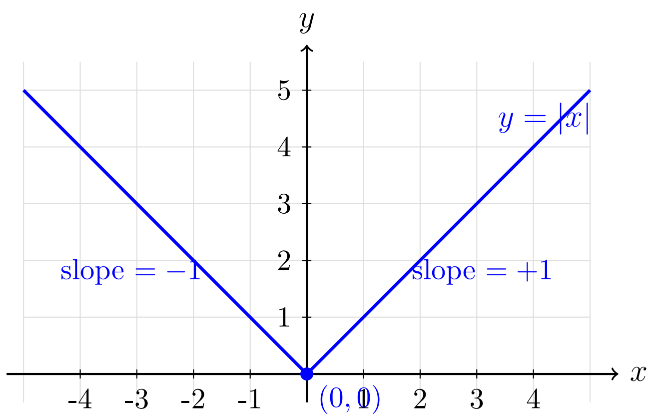
$$\boxed{-7}$$

Part 2 — Completed table

x	$y = \lvert x \rvert$
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

Part 3 — Graph

The completed graph should show a V-shape with vertex at $(0, 0)$, right arm along $y = x$ (slope $+1$), and left arm along $y = -x$ (slope -1).



Part 4

11.

$$\boxed{\text{Vertex: } (0,\ 0)}$$

12.

$$\boxed{\text{Slope of right arm} = +1}$$

13.

$$\boxed{\text{Slope of left arm} = -1}$$

14.

$$\boxed{\text{Domain: all real numbers, } (-\infty,\ \infty)}$$

15.

$$\boxed{\text{Range: } y \geq 0, \text{ written } [0, \infty)}$$

Part 5 — Challenge

16.

$|x| = 5$ means x is exactly 5 units from zero. There are two such values:

$$x = 5 \qquad \text{or} \qquad x = -5$$

Check: $|5| = 5$ ✓ $\quad |-5| = 5$ ✓.

$$\boxed{x = 5 \text{ or } x = -5}$$

17.

$|x| = 0$ means x is zero units from zero. Only one number is that close to zero:

$$x = 0$$

Check: $|0| = 0$ ✓.

$$\boxed{x = 0}$$

18.

Absolute value measures distance, and distance can never be negative. No number is -3 units from zero, so $|x| = -3$ has no solution.

$$\boxed{\text{Absolute value is always } \geq 0, \text{ so it can never equal a negative number.}}$$

19.

$$|-10| = 10 \qquad |7| = 7 \qquad 10 > 7$$

$|-10|$ is larger because -10 is farther from zero than 7 is.

$$\boxed{|-10| = 10 > 7 = |7|, \text{ so } |-10| \text{ is larger}}$$

20.

No. $|x| = |y|$ means x and y are the same distance from zero, but they could be on opposite sides. For example, $x = 3$ and $y = -3$: $|3| = |-3| = 3$, but $3 \neq -3$.

$$\boxed{x = y \text{ or } x = -y. \text{ Example: } |3| = |-3| = 3, \text{ but } 3 \neq -3.}$$

Thank you for learning with ApexMath!

If you found this lesson helpful, we'd love for you to share your feedback or leave a review.

End of Lesson · ApexMath
