Unit 6 Practice Test
Sequences and Series
ApexMath

Name: ______________________________ **Date:** ____________

This test covers all topics from Unit 6: arithmetic and geometric sequences, arithmetic and geometric series, infinite geometric series, and sigma notation. Show all steps for full marks.

Section 1 Sequences /24

1. The arithmetic sequence $6,\ 11,\ 16,\ 21,\ \ldots$ has $t_1 = 6$ and $d = 5$. Find t_{20}. **[2]**

2. The geometric sequence $4,\ 12,\ 36,\ \ldots$ has $t_1 = 4$ and $r = 3$. Find t_7. **[2]**

3. An arithmetic sequence has $t_3 = 14$ and $t_8 = 34$. Find t_1, d, and t_{15}. **[4]**

4. A geometric sequence has $t_2 = 10$ and $t_5 = 1250$. Find t_1, r, and t_6. **[4]**

5. Is 200 a term of the arithmetic sequence 8, 14, 20, 26, ...? Show your work and justify your answer. **[3]**

6. Is 486 a term of the geometric sequence 2, 6, 18, 54, ...? If so, which term? Show your work. **[3]**

7. Classify each sequence as **arithmetic**, **geometric**, or **neither**. State d or r where applicable. **[6]**

   (a) 100, 50, 25, 12.5, ...

   (b) 1, 4, 9, 16, 25, ...

(c) $-3,\ 1,\ 5,\ 9,\ \ldots$

Section 2 Series /20

8. Find S_{20} for the arithmetic sequence $3,\ 8,\ 13,\ 18,\ \ldots$ **[3]**

9. Find S_7 for the geometric sequence with $t_1 = 2$ and $r = 3$. **[3]**

10. Find the sum $7 + 10 + 13 + \cdots + 52$.
(Find n first, then apply a sum formula.) **[4]**

11. An arithmetic sequence has $t_1 = 2$ and $d = 4$. How many terms must be added to reach a sum of 128? **[4]**

12. A geometric series has $t_1 = 2$ and $r = 2$. How many terms are needed for $S_n = 1022$? **[3]**

13. Evaluate $\sum_{k=1}^{6}(4k-3)$. Show each substitution. **[3]**

Section 3 Infinite Geometric Series /15

14. Find S_∞ for the geometric series with $t_1 = 10$ and $r = \frac{1}{2}$. **[2]**

15. Find S_∞ for the geometric series with $t_1 = 6$ and $r = -\frac{1}{3}$. **[3]**

16. Find S_∞ for the series $15 + 12 + 9.6 + \cdots$
(Find r first.) **[3]**

17. An infinite geometric series has $S_\infty = 40$ and $r = \dfrac{3}{4}$. Find t_1. **[3]**

18. An infinite geometric series has $t_1 = 9$ and $S_\infty = 36$. Find r. **[3]**

19. Does the series with $t_1 = 5$ and $r = 3$ converge? Justify your answer. **[1]**

Section 4 Sigma Notation /11

20. Evaluate $\displaystyle\sum_{k=1}^{5} 3 \cdot 2^{k-1}$. **[3]**

21. Evaluate $\displaystyle\sum_{k=2}^{5} (k^2 - 1)$. **[3]**

22. Write the series $4 + 8 + 12 + 16 + 20 + 24$ in sigma notation. **[2]**

23. Write the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ in sigma notation. **[3]**

Total: _______ / 70

Answer Key

Unit 6 Practice Test — Sequences and Series

Section 1 Sequences

1. $t_1 = 6$, $d = 5$, find t_{20}.

$$t_n = t_1 + (n-1)\,d$$

$$t_{20} = 6 + (20-1)(5) = 6 + 19 \times 5 = 6 + 95$$

$$\boxed{t_{20} = 101}$$

2. $t_1 = 4$, $r = 3$, find t_7.

$$t_7 = 4 \cdot 3^{6} = 4 \times 729$$

$$\boxed{t_7 = 2916}$$

3. $t_3 = 14$, $t_8 = 34$. Find t_1, d, t_{15}.

Step 1: Write two equations.

$$t_1 + 2d = 14 \quad \cdots(1) \qquad t_1 + 7d = 34 \quad \cdots(2)$$

Step 2: Subtract (1) from (2).

$$5d = 20 \qquad \Rightarrow \qquad d = 4$$

Step 3: Substitute into (1).

$$t_1 + 8 = 14 \qquad \Rightarrow \qquad t_1 = 6$$

Step 4: Find t_{15}.

$$t_{15} = 6 + 14 \times 4 = 6 + 56$$

$$\boxed{t_1 = 6, \quad d = 4, \quad t_{15} = 62}$$

4. $t_2 = 10$, $t_5 = 1250$. Find t_1, r, t_6.
Step 1: Write two equations.

$$t_1 \cdot r = 10 \quad \cdots(1) \qquad t_1 \cdot r^4 = 1250 \quad \cdots(2)$$

Step 2: Divide (2) by (1).

$$r^3 = \frac{1250}{10} = 125 \qquad \Rightarrow \qquad r = 5$$

Step 3: Substitute into (1).

$$t_1 \times 5 = 10 \qquad \Rightarrow \qquad t_1 = 2$$

Step 4: Find t_6.

$$t_6 = 2 \cdot 5^5 = 2 \times 3125$$

$$\boxed{t_1 = 2, \quad r = 5, \quad t_6 = 6250}$$

5. Sequence $8,\ 14,\ 20,\ 26,\ \ldots$ Is 200 a term?
Step 1: $t_1 = 8$, $d = 6$. General term: $t_n = 8 + (n-1)(6) = 6n + 2$.
Step 2: Set $t_n = 200$.

$$6n + 2 = 200 \qquad \Rightarrow \qquad 6n = 198 \qquad \Rightarrow \qquad n = 33$$

Since $n = 33$ is a positive whole number, 200 is a term.

$$\boxed{\text{200 is the 33rd term.}}$$

6. Sequence $2,\ 6,\ 18,\ 54,\ \ldots$ Is 486 a term?
Step 1: $t_1 = 2$, $r = 3$. Set $t_n = 486$.

$$2 \cdot 3^{n-1} = 486 \qquad \Rightarrow \qquad 3^{n-1} = 243 = 3^5 \qquad \Rightarrow \qquad n = 6$$

Verify: $t_6 = 2 \cdot 3^5 = 2 \times 243 = 486$ ✓

$$\boxed{\text{486 is the 6th term.}}$$

7.

(a) $100,\ 50,\ 25,\ 12.5,\ \ldots$ Ratios: $\dfrac{50}{100} = \dfrac{1}{2}, \dfrac{25}{50} = \dfrac{1}{2}$. Constant ratio.

$$\boxed{\textbf{Geometric},\ r = \frac{1}{2}}$$

(b) $1,\ 4,\ 9,\ 16,\ 25,\ \ldots$ Differences: $3, 5, 7, 9$ (not constant). Ratios: $4, 2.25, \ldots$ (not constant).

$$\boxed{\textbf{Neither} \text{ — these are perfect squares, } t_n = n^2}$$

(c) $-3,\ 1,\ 5,\ 9,\ \ldots$ Differences: $1 - (-3) = 4$, $5 - 1 = 4$. Constant difference.

$$\boxed{\textbf{Arithmetic},\ d = 4}$$

Section 2 Series

8. S_{20} for $3,\ 8,\ 13,\ \ldots$ $(t_1 = 3,\ d = 5)$
Step 1: Find $t_{20} = 3 + 19(5) = 3 + 95 = 98$.
Step 2: Apply Formula 2.

$$S_{20} = \frac{20}{2}(3 + 98) = 10 \times 101$$

$$\boxed{S_{20} = 1010}$$

9. S_7 for $t_1 = 2$, $r = 3$.

$$S_7 = \frac{2\,(3^7 - 1)}{3 - 1} = \frac{2\,(2187 - 1)}{2} = 2186$$

$$\boxed{S_7 = 2186}$$

10. Sum $7 + 10 + 13 + \cdots + 52$. $\quad (t_1 = 7,\ d = 3)$
Step 1: Find n.

$$7 + (n-1)(3) = 52 \qquad \Rightarrow \qquad (n-1)(3) = 45 \qquad \Rightarrow \qquad n = 16$$

Step 2: Apply Formula 2.

$$S_{16} = \frac{16}{2}(7 + 52) = 8 \times 59$$

$$\boxed{S_{16} = 472}$$

11. $t_1 = 2$, $d = 4$, $S_n = 128$. Find n.
Step 1: Set up Formula 1 and expand.

$$\frac{n}{2}\big(2(2) + (n-1)(4)\big) = 128$$
$$\frac{n}{2}(4n) = 128$$
$$2n^2 = 128$$

Step 2: Solve.

$$n^2 = 64 \qquad \Rightarrow \qquad n = 8$$

Step 3: Verify.

$$S_8 = \frac{8}{2}\big(2(2) + 7(4)\big) = 4(4 + 28) = 4 \times 32 = 128\checkmark$$

$$\boxed{n = 8 \text{ terms}}$$

12. $t_1 = 2$, $r = 2$, $S_n = 1022$. Find n.

$$\frac{2\left(2^n - 1\right)}{2 - 1} = 1022 \qquad \Rightarrow \qquad 2\left(2^n - 1\right) = 1022$$
$$2^n - 1 = 511 \qquad \Rightarrow \qquad 2^n = 512 = 2^9 \qquad \Rightarrow \qquad n = 9$$

Verify: $S_9 = 2(2^9 - 1) = 2(511) = 1022$ ✓

$$\boxed{n = 9 \text{ terms}}$$

13. $\sum_{k=1}^{6}(4k-3)$

$k=1:\ 1 \qquad k=2:\ 5 \qquad k=3:\ 9 \qquad k=4:\ 13 \qquad k=5:\ 17 \qquad k=6:\ 21$

$$1+5+9+13+17+21=66$$

$$\boxed{\sum_{k=1}^{6}(4k-3)=66}$$

Section 3 Infinite Geometric Series

14. $t_1=10$, $r=\frac{1}{2}$. $\quad \left|\frac{1}{2}\right|<1\ \checkmark$

$$S_\infty=\frac{10}{1-\frac{1}{2}}=\frac{10}{\frac{1}{2}}=10\times 2$$

$$\boxed{S_\infty=20}$$

15. $t_1=6$, $r=-\frac{1}{3}$. $\quad \left|-\frac{1}{3}\right|=\frac{1}{3}<1\ \checkmark$

$$S_\infty=\frac{6}{1-\left(-\frac{1}{3}\right)}=\frac{6}{\frac{4}{3}}=6\times\frac{3}{4}=\frac{18}{4}$$

$$\boxed{S_\infty=\frac{9}{2}}$$

16. Series $15+12+9.6+\cdots$

Step 1: Find $r=\frac{12}{15}=\frac{4}{5}$. $\quad \left|\frac{4}{5}\right|<1\ \checkmark$

Step 2:

$$S_\infty=\frac{15}{1-\frac{4}{5}}=\frac{15}{\frac{1}{5}}=15\times 5$$

$$\boxed{S_\infty=75}$$

17. $S_\infty = 40$, $r = \dfrac{3}{4}$. Find t_1.

$$t_1 = S_\infty(1-r) = 40\left(1 - \frac{3}{4}\right) = 40 \times \frac{1}{4}$$

$$\boxed{t_1 = 10}$$

18. $t_1 = 9$, $S_\infty = 36$. Find r.

$$1 - r = \frac{t_1}{S_\infty} = \frac{9}{36} = \frac{1}{4} \qquad \Rightarrow \qquad r = 1 - \frac{1}{4}$$

Verify: $S_\infty = \dfrac{9}{1-\frac{3}{4}} = \dfrac{9}{\frac{1}{4}} = 36$ ✓

$$\boxed{r = \frac{3}{4}}$$

19. $t_1 = 5$, $r = 3$. $\quad |r| = 3 \geq 1$.

$$\boxed{\text{The series \textbf{diverges} — no finite sum exists.}}$$

Section 4 Sigma Notation

20. $\displaystyle\sum_{k=1}^{5} 3 \cdot 2^{k-1}$

$$k=1:\ 3 \qquad k=2:\ 6 \qquad k=3:\ 12 \qquad k=4:\ 24 \qquad k=5:\ 48$$

$$3 + 6 + 12 + 24 + 48 = 93$$

$$\boxed{\sum_{k=1}^{5} 3 \cdot 2^{k-1} = 93}$$

21. $\sum_{k=2}^{5}(k^2-1)$

$$k=2:\ 4-1=3 \qquad k=3:\ 9-1=8 \qquad k=4:\ 16-1=15 \qquad k=5:\ 25-1=24$$

$$3+8+15+24=50$$

$$\boxed{\sum_{k=2}^{5}(k^2-1)=50}$$

22. $4+8+12+16+20+24$
Step 1: Arithmetic with $t_1=4$, $d=4$. General term: $f(k)=4k$.
Step 2: Verify: $k=1 \to 4$ ✓, $k=6 \to 24$ ✓. There are 6 terms.

$$\boxed{\sum_{k=1}^{6} 4k}$$

23. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$
Step 1: Geometric with $t_1=\frac{1}{2}$, $r=\frac{1}{2}$. General term: $\left(\frac{1}{2}\right)^k$.
Step 2: Verify: $k=1 \to \frac{1}{2}$ ✓, $k=4 \to \frac{1}{16}$ ✓. There are 4 terms.

$$\boxed{\sum_{k=1}^{4}\left(\frac{1}{2}\right)^k}$$

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End of Unit 6 Practice Test • ApexMath
