Annuities: Future Value
ApexMath

Notes

What Is an Annuity?

In the previous lessons, we assumed that one lump sum of money was deposited at the start and left to grow. An **annuity** is different: instead of one large deposit, you make *regular, equal payments* over time. Each payment is made at the end of a compounding period, earns interest, and combines with all previous payments to build up a total.

Common real-life examples include monthly contributions to a savings account, regular deposits into a retirement fund, or any situation where the same amount is set aside on a fixed schedule.

Key Vocabulary

- **Payment** (R): The fixed amount deposited at the end of each period. This stays the same every single period.
- **Interest Rate per Period** (i): The annual rate divided by the number of compounding periods per year.
$$i = \frac{r}{n}$$
- **Number of Payments** (n): The total number of payments made over the entire investment. If you pay monthly for 3 years, $n = 12 \times 3 = 36$.
- **Future Value** (FV): The total amount in the account at the end of all payments, including all interest earned.

The Future Value Annuity Formula

$$FV = R \cdot \frac{(1+i)^n - 1}{i}$$

This formula comes directly from summing a geometric series. Each payment grows by the factor $(1+i)$ for a different number of periods, and the formula adds all of those results together at once.

How to Use the Formula — A Clear Process

- **Step 1:** Identify R, the annual rate r, the compounding frequency n_{freq}, and the total time t in years.
- **Step 2:** Calculate the interest rate per period: $i = r \div n_{\text{freq}}$
- **Step 3:** Calculate the total number of payments: $n = n_{\text{freq}} \times t$
- **Step 4:** Substitute R, i, and n into the formula and evaluate.

Connection to Previous Lessons

Notice that the term $(1+i)^n$ inside the formula is the same growth factor from compound interest. The annuity formula is essentially adding up many compound interest calculations — one for each payment — all at once.

Pro Tip

- Write out i and n explicitly before substituting into the formula. These two values are where most errors happen. Confusing the annual rate with the rate per period is the single most common mistake in annuity problems.
- The quantity $\dfrac{(1+i)^n - 1}{i}$ is sometimes called the **annuity factor**. You can compute it separately on your calculator first, then multiply by R. This reduces bracket errors.
- Always check that your answer is *larger* than the total amount you deposited $(R \times n)$. The difference between FV and your total deposits is the interest earned — a useful sanity check.
- Do not round i mid-calculation. Keep all decimal places in i until the very final step, then round the answer to two decimal places.
- The payments in this lesson are assumed to be made at the *end* of each period. This is the most common type of annuity and is the standard assumption unless told otherwise.

Examples

Example 1 (Monthly Deposits): Keisha deposits \$200 at the end of every month into a savings account that earns 6% per year, compounded monthly. What is the total amount after 3 years?

Step 1: Identify the values.

$$R = 200, \qquad r = 6\% = 0.06, \qquad n_{\text{freq}} = 12, \qquad t = 3 \text{ years}$$

Step 2: Calculate the interest rate per period.

$$i = \frac{r}{n_{\text{freq}}} = \frac{0.06}{12} = 0.005$$

Step 3: Calculate the total number of payments.

$$n = n_{\text{freq}} \times t = 12 \times 3 = 36$$

Step 4: Substitute into the future value formula.

$$FV = 200 \cdot \frac{(1.005)^{36} - 1}{0.005}$$

Step 5: Evaluate the power, then simplify.

$$(1.005)^{36} \approx 1.196681$$

$$FV = 200 \cdot \frac{1.196681 - 1}{0.005} = 200 \cdot \frac{0.196681}{0.005} = 200 \times 39.336$$

$$\boxed{FV \approx \$7{,}867.22}$$

Example 2 (Quarterly Deposits): David contributes \$500 at the end of every quarter into an investment account earning 4% per year, compounded quarterly, for 5 years. Find the future value.

Step 1: Identify the values.

$$R = 500, \qquad r = 4\% = 0.04, \qquad n_{\text{freq}} = 4, \qquad t = 5 \text{ years}$$

Step 2: Calculate the interest rate per period.

$$i = \frac{0.04}{4} = 0.01$$

Step 3: Calculate the total number of payments.

$$n = 4 \times 5 = 20$$

Step 4: Substitute into the formula.

$$FV = 500 \cdot \frac{(1.01)^{20} - 1}{0.01}$$

Step 5: Evaluate.

$$(1.01)^{20} \approx 1.220190$$

$$FV = 500 \cdot \frac{0.220190}{0.01} = 500 \times 22.019$$

$$\boxed{FV \approx \$11{,}009.50}$$

Example 3 (Semi-Annual Deposits): Ana deposits \$1,000 at the end of every 6 months into an account earning 6% per year, compounded semi-annually, for 5 years. Find the future value.

Step 1: Identify the values.

$$R = 1000, \qquad r = 6\% = 0.06, \qquad n_{\text{freq}} = 2, \qquad t = 5 \text{ years}$$

Step 2: Calculate the interest rate per period.

$$i = \frac{0.06}{2} = 0.03$$

Step 3: Calculate the total number of payments.

$$n = 2 \times 5 = 10$$

Step 4: Substitute into the formula.

$$FV = 1000 \cdot \frac{(1.03)^{10} - 1}{0.03}$$

Step 5: Evaluate.

$$(1.03)^{10} \approx 1.343916$$

$$FV = 1000 \cdot \frac{0.343916}{0.03} = 1000 \times 11.464$$

$$\boxed{FV \approx \$11{,}463.88}$$

Example 4 (Monthly Deposits, Longer Term): Marcus saves \$300 per month in an account earning 5% per year, compounded monthly, for 5 years. Find the future value.

Step 1: Identify the values.

$$R = 300, \qquad r = 5\% = 0.05, \qquad n_{\text{freq}} = 12, \qquad t = 5 \text{ years}$$

Step 2: Calculate the interest rate per period.

$$i = \frac{0.05}{12} \approx 0.004167$$

Step 3: Calculate the total number of payments.

$$n = 12 \times 5 = 60$$

Step 4: Substitute into the formula.

$$FV = 300 \cdot \frac{(1.004167)^{60} - 1}{0.004167}$$

Step 5: Evaluate.

$$(1.004167)^{60} \approx 1.283359$$

$$FV = 300 \cdot \frac{0.283359}{0.004167} = 300 \times 68.006$$

$$\boxed{FV \approx \$20{,}401.82}$$

Example 5 (Monthly Deposits, Find Total Interest): Priya deposits \$150 per month into an account earning 8% per year, compounded monthly, for 2 years. Find the future value and the total interest earned.

Step 1: Identify the values.

$$R = 150, \qquad r = 8\% = 0.08, \qquad n_{\text{freq}} = 12, \qquad t = 2 \text{ years}$$

Step 2: Calculate the interest rate per period and total payments.

$$i = \frac{0.08}{12} \approx 0.006667 \qquad \text{and} \qquad n = 12 \times 2 = 24$$

Step 3: Substitute into the formula.

$$FV = 150 \cdot \frac{(1.006667)^{24} - 1}{0.006667}$$

Step 4: Evaluate.

$$(1.006667)^{24} \approx 1.172888$$

$$FV = 150 \cdot \frac{0.172888}{0.006667} = 150 \times 25.933$$

$$FV \approx \$3{,}889.98$$

Step 5: Find the total interest earned.

$$\text{Total deposited} = R \times n = 150 \times 24 = \$3{,}600$$

$$\text{Interest earned} = FV - \text{Total deposited} = 3889.98 - 3600$$

$$\boxed{FV \approx \$3{,}889.98, \quad \text{Interest earned} \approx \$289.98}$$

Common Mistakes

- **Using the annual rate r instead of the rate per period i.** You must always divide r by the compounding frequency before substituting. Using r directly gives a drastically wrong answer.
- **Using the number of years instead of the total number of payments.** The formula requires $n = n_{\text{freq}} \times t$, not just t. For monthly payments over 3 years, $n = 36$, not 3.
- **Subtracting 1 from the wrong part of the formula.** The subtraction applies only to the power: $(1+i)^n - 1$. The denominator is just i alone. A common error is writing $(1+i)^{n-1}$ instead.

- **Rounding i too early.** For rates like $0.05/12$, keep at least 6 decimal places in i throughout the calculation. Early rounding compounds into a significant error by the final step.
- **Confusing future value with total deposits.** The future value FV includes interest. The total amount you personally put in is $R \times n$. These are two different numbers.

Worksheet

1. You deposit \$100 at the end of every month into an account earning 6% per year, compounded monthly, for 2 years. What is the future value?

2. \$250 is deposited at the end of every quarter into an account earning 8% per year, compounded quarterly, for 4 years. Find the future value.

3. \$400 is deposited at the end of every 6 months into an account earning 5% per year, compounded semi-annually, for 6 years. Find the future value.

4. You save \$500 per month in an account earning 4% per year, compounded monthly, for 5 years. What is the future value?

5. \$200 is deposited at the end of every quarter into an account earning 6% per year, compounded quarterly, for 5 years. Find the future value.

6. \$1,000 is deposited every 6 months into an account earning 5% per year, compounded semi-annually, for 10 years. Find the future value.

7. You deposit \$75 per month into an account earning 9% per year, compounded monthly, for 4 years. Find the future value.

8. (Challenge) You deposit \$200 per month into an account earning 6% per year, compounded monthly, for 30 years. Find the future value. How much of that total did you deposit yourself, and how much came from interest?

Answer Key

1. $R = \$100$, $r = 0.06$, $n_{\text{freq}} = 12$, $t = 2$ years.

Step 1: Find i and n.

$$i = \frac{0.06}{12} = 0.005 \qquad \text{and} \qquad n = 12 \times 2 = 24$$

Step 2: Substitute into the formula.

$$FV = 100 \cdot \frac{(1.005)^{24} - 1}{0.005}$$

Step 3: Evaluate.

$$(1.005)^{24} \approx 1.127160$$

$$FV = 100 \cdot \frac{0.127160}{0.005} = 100 \times 25.432$$

$$\boxed{FV \approx \$2{,}543.20}$$

2. $R = \$250$, $r = 0.08$, $n_{\text{freq}} = 4$, $t = 4$ years.

Step 1: Find i and n.

$$i = \frac{0.08}{4} = 0.02 \qquad \text{and} \qquad n = 4 \times 4 = 16$$

Step 2: Substitute into the formula.

$$FV = 250 \cdot \frac{(1.02)^{16} - 1}{0.02}$$

Step 3: Evaluate.

$$(1.02)^{16} \approx 1.372786$$

$$FV = 250 \cdot \frac{0.372786}{0.02} = 250 \times 18.639$$

$$\boxed{FV \approx \$4{,}659.82}$$

3. $R = \$400$, $r = 0.05$, $n_{\text{freq}} = 2$, $t = 6$ years.

Step 1: Find i and n.

$$i = \frac{0.05}{2} = 0.025 \qquad \text{and} \qquad n = 2 \times 6 = 12$$

Step 2: Substitute into the formula.

$$FV = 400 \cdot \frac{(1.025)^{12} - 1}{0.025}$$

Step 3: Evaluate.

$$(1.025)^{12} \approx 1.344889$$

$$FV = 400 \cdot \frac{0.344889}{0.025} = 400 \times 13.796$$

$$\boxed{FV \approx \$5{,}518.22}$$

4. $R = \$500$, $r = 0.04$, $n_{\text{freq}} = 12$, $t = 5$ years.

Step 1: Find i and n.

$$i = \frac{0.04}{12} \approx 0.003333 \qquad \text{and} \qquad n = 12 \times 5 = 60$$

Step 2: Substitute into the formula.

$$FV = 500 \cdot \frac{(1.003333)^{60} - 1}{0.003333}$$

Step 3: Evaluate.

$$(1.003333)^{60} \approx 1.221234$$

$$FV = 500 \cdot \frac{0.221234}{0.003333} = 500 \times 66.368$$

$$\boxed{FV \approx \$33{,}149.49}$$

5. $R = \$200$, $r = 0.06$, $n_{\text{freq}} = 4$, $t = 5$ years.

Step 1: Find i and n.

$$i = \frac{0.06}{4} = 0.015 \qquad \text{and} \qquad n = 4 \times 5 = 20$$

Step 2: Substitute into the formula.

$$FV = 200 \cdot \frac{(1.015)^{20} - 1}{0.015}$$

Step 3: Evaluate.

$$(1.015)^{20} \approx 1.346855$$

$$FV = 200 \cdot \frac{0.346855}{0.015} = 200 \times 23.124$$

$$\boxed{FV \approx \$4{,}624.73}$$

6. $R = \$1{,}000$, $r = 0.05$, $n_{\text{freq}} = 2$, $t = 10$ years.

Step 1: Find i and n.

$$i = \frac{0.05}{2} = 0.025 \qquad \text{and} \qquad n = 2 \times 10 = 20$$

Step 2: Substitute into the formula.

$$FV = 1000 \cdot \frac{(1.025)^{20} - 1}{0.025}$$

Step 3: Evaluate.

$$(1.025)^{20} \approx 1.638616$$

$$FV = 1000 \cdot \frac{0.638616}{0.025} = 1000 \times 25.545$$

$$\boxed{FV \approx \$25{,}544.66}$$

7. $R = \$75$, $r = 0.09$, $n_{\text{freq}} = 12$, $t = 4$ years.

Step 1: Find i and n.

$$i = \frac{0.09}{12} = 0.0075 \qquad \text{and} \qquad n = 12 \times 4 = 48$$

Step 2: Substitute into the formula.

$$FV = 75 \cdot \frac{(1.0075)^{48} - 1}{0.0075}$$

Step 3: Evaluate.

$$(1.0075)^{48} \approx 1.431405$$

$$FV = 75 \cdot \frac{0.431405}{0.0075} = 75 \times 57.521$$

$$\boxed{FV \approx \$4{,}314.05}$$

8. (Challenge) $R = \$200$, $r = 0.06$, $n_{\text{freq}} = 12$, $t = 30$ years.

Step 1: Find i and n.

$$i = \frac{0.06}{12} = 0.005 \qquad \text{and} \qquad n = 12 \times 30 = 360$$

Step 2: Substitute into the formula.

$$FV = 200 \cdot \frac{(1.005)^{360} - 1}{0.005}$$

Step 3: Evaluate.

$$(1.005)^{360} \approx 6.022575$$

$$FV = 200 \cdot \frac{5.022575}{0.005} = 200 \times 1004.515$$

$$FV \approx \$200{,}903.01$$

Step 4: Find total deposited and interest earned.

$$\text{Total deposited} = 200 \times 360 = \$72{,}000$$

$$\text{Interest earned} = 200{,}903.01 - 72{,}000$$

$$\boxed{FV \approx \$200{,}903.01 \quad \text{Deposited: } \$72{,}000 \quad \text{Interest: } \$128{,}903.01}$$

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