Compound Interest
ApexMath

Notes

What Is Compound Interest?

In the previous lesson, simple interest was always calculated on the original principal only. Compound interest works differently: after each compounding period, the interest earned is *added to the principal*, and the next period's interest is calculated on this new, larger amount. This means your money grows faster and faster over time.

Key Vocabulary

- **Principal** (P): The original amount of money invested or borrowed.
- **Annual Interest Rate** (r): The yearly interest rate written as a decimal. Convert a percentage before using it: $6\% = 0.06$.
- **Compounding Frequency** (n): How many times interest is applied per year.
  - Annually: $n = 1$
  - Semi-annually: $n = 2$
  - Quarterly: $n = 4$
  - Monthly: $n = 12$
- **Time** (t): The total number of years the money is invested or owed.
- **Amount** (A): The total value at the end, including all accumulated interest.

The Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Each part of this formula has a specific meaning:

- $\frac{r}{n}$ is the interest rate per compounding period.
- nt is the total number of compounding periods over the entire investment.
- $\left(1 + \frac{r}{n}\right)^{nt}$ is the growth factor — how many times larger the investment becomes.

Solving for the Principal (Present Value)

If you know the final amount A and want to find the starting principal P, divide both sides of the formula by the growth factor:

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

This is called finding the **present value** — how much money you need *right now* to reach a target amount in the future.

Connection to Geometric Sequences

Each compounding period multiplies the current amount by the same number, $\left(1 + \frac{r}{n}\right)$. This is exactly the structure of a geometric sequence, where each term is the previous term times a constant ratio. The compound interest formula is the general term of that sequence.

Pro Tip

- Always identify all four values — P, r, n, t — before touching the formula. Write them out explicitly. Missing one is the most common source of errors.
- The exponent is nt, not $n + t$. Multiplying n and t gives the *total number of compounding periods*, which is what the formula needs.
- Do not round r/n in the middle of the calculation. Keep all decimal places until you reach the final answer, then round to two decimal places for money.
- If the question only gives an annual rate and says "compounded annually," then $n = 1$ and the formula simplifies to $A = P(1 + r)^t$.
- To check your answer makes sense: compound interest always gives a *larger* result than simple interest for the same P, r, and t. If your compound answer is smaller, something went wrong.

Examples

Example 1 (Compounded Annually): Amir invests \$1,000 at an annual interest rate of 6%, compounded annually, for 3 years. What is the total amount at the end?

Step 1: Identify all values and convert the rate.

$$P = 1000, \qquad r = 6\% = 0.06, \qquad n = 1, \qquad t = 3$$

Step 2: Find the rate per period and the total number of periods.

$$\frac{r}{n} = \frac{0.06}{1} = 0.06 \qquad \text{and} \qquad nt = 1 \times 3 = 3$$

Step 3: Substitute into the formula.

$$A = 1000\,(1 + 0.06)^3$$

$$A = 1000\,(1.06)^3$$

Step 4: Evaluate the power, then multiply.

$$A = 1000 \times 1.191016$$

$$\boxed{A \approx \$1{,}191.02}$$

Example 2 (Compounded Semi-Annually): Sofia deposits \$2,000 into an account earning 8% per year, compounded semi-annually, for 5 years. Find the total amount.

Step 1: Identify all values. Semi-annually means $n = 2$.

$$P = 2000, \qquad r = 8\% = 0.08, \qquad n = 2, \qquad t = 5$$

Step 2: Find the rate per period and the total number of periods.

$$\frac{r}{n} = \frac{0.08}{2} = 0.04 \qquad \text{and} \qquad nt = 2 \times 5 = 10$$

Step 3: Substitute into the formula.

$$A = 2000\,(1 + 0.04)^{10}$$

$$A = 2000\,(1.04)^{10}$$

Step 4: Evaluate. Use a calculator for the power.

$$A = 2000 \times 1.480244$$

$$\boxed{A \approx \$2{,}960.49}$$

Example 3 (Compounded Monthly): Elena invests \$5,000 at 12% per year, compounded monthly, for 2 years. Find the total amount.

Step 1: Identify all values. Monthly means $n = 12$.

$$P = 5000, \qquad r = 12\% = 0.12, \qquad n = 12, \qquad t = 2$$

Step 2: Find the rate per period and the total number of periods.

$$\frac{r}{n} = \frac{0.12}{12} = 0.01 \qquad \text{and} \qquad nt = 12 \times 2 = 24$$

Step 3: Substitute into the formula.

$$A = 5000\,(1 + 0.01)^{24}$$

$$A = 5000\,(1.01)^{24}$$

Step 4: Evaluate.

$$A = 5000 \times 1.269735$$

$$\boxed{A \approx \$6{,}348.67}$$

Example 4 (Compounded Quarterly): A \$3,000 investment earns 5% per year, compounded quarterly, for 10 years. Find the total amount.

Step 1: Identify all values. Quarterly means $n = 4$.

$$P = 3000, \qquad r = 5\% = 0.05, \qquad n = 4, \qquad t = 10$$

Step 2: Find the rate per period and the total number of periods.

$$\frac{r}{n} = \frac{0.05}{4} = 0.0125 \qquad \text{and} \qquad nt = 4 \times 10 = 40$$

Step 3: Substitute into the formula.

$$A = 3000\,(1 + 0.0125)^{40}$$

$$A = 3000\,(1.0125)^{40}$$

Step 4: Evaluate.

$$A = 3000 \times 1.643619$$

$$\boxed{A \approx \$4{,}930.86}$$

Example 5 (Solve for Present Value): How much money must be invested today at 6% per year, compounded monthly, so that the account grows to \$5,000 in 4 years?

Step 1: Identify all values. P is the unknown.

$$A = 5000, \qquad r = 6\% = 0.06, \qquad n = 12, \qquad t = 4$$

Step 2: Use the present value formula.

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Step 3: Compute the rate per period and the total number of periods.

$$\frac{r}{n} = \frac{0.06}{12} = 0.005 \qquad \text{and} \qquad nt = 12 \times 4 = 48$$

Step 4: Substitute and evaluate.

$$P = \frac{5000}{(1.005)^{48}}$$

$$P = \frac{5000}{1.270489}$$

$$\boxed{P \approx \$3{,}935.49}$$

Common Mistakes

- **Using the annual rate instead of the rate per period.** The formula requires r/n, not r alone. If money is compounded monthly, the rate used in the base must be $r \div 12$.
- **Using $n + t$ instead of nt as the exponent.** The exponent is the *total number of compounding periods*, which is found by *multiplying* n and t, not adding them.
- **Rounding r/n too early.** Rounding a decimal like $0.06/12 = 0.005$ is fine here, but with unusual rates (e.g. 7.25% monthly), rounding mid-calculation introduces error. Keep full precision until the final step.
- **Forgetting to convert the percentage to a decimal.** Entering $r = 6$ instead of $r = 0.06$ makes the growth factor enormous and the answer completely wrong.
- **Confusing A and I.** The formula gives A, the total amount. If the question asks for the interest earned, you must subtract the principal: $I = A - P$.

Worksheet

1. You invest \$1,500 at 4% per year, compounded annually, for 5 years. What is the total amount?

2. A \$4,000 deposit earns 6% per year, compounded semi-annually, for 3 years. Find the total amount.

3. \$2,500 is invested at 3% per year, compounded monthly, for 4 years. Find the total amount.

4. A \$8,000 loan charges 5% per year, compounded quarterly, for 6 years. What is the total amount owed?

5. \$1,000 is invested at 8% per year, compounded annually, for 10 years. What is the total amount?

6. How much money must be invested today at 5% per year, compounded quarterly, so that the account grows to \$10,000 in 8 years?

7. An account pays 7.2% per year, compounded monthly. If \$6,000 is deposited today, what is the total amount after 3 years?

8. (Challenge) \$5,000 is invested for 20 years at 4% per year. Calculate the total amount under both simple interest and compound interest (compounded annually). How much more does compound interest earn?

Answer Key

1. $P = \$1{,}500$, $r = 0.04$, $n = 1$, $t = 5$.

Step 1: Find the rate per period and total periods.

$$\frac{r}{n} = \frac{0.04}{1} = 0.04 \qquad \text{and} \qquad nt = 1 \times 5 = 5$$

Step 2: Substitute into the formula.

$$A = 1500\,(1.04)^5$$

Step 3: Evaluate the power, then multiply.

$$A = 1500 \times 1.216653$$

$$\boxed{A \approx \$1{,}824.98}$$

2. $P = \$4{,}000$, $r = 0.06$, $n = 2$, $t = 3$.

Step 1: Find the rate per period and total periods.

$$\frac{r}{n} = \frac{0.06}{2} = 0.03 \qquad \text{and} \qquad nt = 2 \times 3 = 6$$

Step 2: Substitute into the formula.

$$A = 4000\,(1.03)^6$$

Step 3: Evaluate.

$$A = 4000 \times 1.194052$$

$$\boxed{A \approx \$4{,}776.21}$$

3. $P = \$2{,}500$, $r = 0.03$, $n = 12$, $t = 4$.

Step 1: Find the rate per period and total periods.

$$\frac{r}{n} = \frac{0.03}{12} = 0.0025 \qquad \text{and} \qquad nt = 12 \times 4 = 48$$

Step 2: Substitute into the formula.

$$A = 2500\,(1.0025)^{48}$$

Step 3: Evaluate.

$$A = 2500 \times 1.127328$$

$$\boxed{A \approx \$2{,}818.32}$$

4. $P = \$8{,}000$, $r = 0.05$, $n = 4$, $t = 6$.

Step 1: Find the rate per period and total periods.

$$\frac{r}{n} = \frac{0.05}{4} = 0.0125 \qquad \text{and} \qquad nt = 4 \times 6 = 24$$

Step 2: Substitute into the formula.

$$A = 8000\,(1.0125)^{24}$$

Step 3: Evaluate.

$$A = 8000 \times 1.347351$$

$$\boxed{A \approx \$10{,}778.81}$$

5. $P = \$1{,}000$, $r = 0.08$, $n = 1$, $t = 10$.

Step 1: Find the rate per period and total periods.

$$\frac{r}{n} = \frac{0.08}{1} = 0.08 \qquad \text{and} \qquad nt = 1 \times 10 = 10$$

Step 2: Substitute into the formula.

$$A = 1000\,(1.08)^{10}$$

Step 3: Evaluate.

$$A = 1000 \times 2.158925$$

$$\boxed{A \approx \$2{,}158.92}$$

6. $A = \$10{,}000$, $r = 0.05$, $n = 4$, $t = 8$. Find P.

Step 1: Use the present value formula.

$$P = \frac{A}{\left(1 + \dfrac{r}{n}\right)^{nt}}$$

Step 2: Find the rate per period and total periods.

$$\frac{r}{n} = \frac{0.05}{4} = 0.0125 \qquad \text{and} \qquad nt = 4 \times 8 = 32$$

Step 3: Substitute and evaluate.

$$P = \frac{10000}{(1.0125)^{32}}$$

$$P = \frac{10000}{1.488131}$$

$$\boxed{P \approx \$6{,}719.84}$$

7. $P = \$6{,}000$, $r = 7.2\% = 0.072$, $n = 12$, $t = 3$.

Step 1: Find the rate per period and total periods.

$$\frac{r}{n} = \frac{0.072}{12} = 0.006 \qquad \text{and} \qquad nt = 12 \times 3 = 36$$

Step 2: Substitute into the formula.

$$A = 6000\,(1.006)^{36}$$

Step 3: Evaluate.

$$A = 6000 \times 1.240302$$

$$\boxed{A \approx \$7{,}441.81}$$

8. (Challenge) $P = \$5{,}000$, $r = 4\%$, $t = 20$ years.

Step 1: Calculate the simple interest total using $A = P(1 + rt)$.

$$A_{\text{simple}} = 5000\,(1 + 0.04 \times 20)$$

$$A_{\text{simple}} = 5000 \times 1.80$$

$$A_{\text{simple}} = \$9{,}000.00$$

Step 2: Calculate the compound interest total using $A = P(1 + r)^t$ with $n = 1$.

$$A_{\text{compound}} = 5000\,(1.04)^{20}$$

$$A_{\text{compound}} = 5000 \times 2.191123$$

$$A_{\text{compound}} \approx \$10{,}955.62$$

Step 3: Find the difference.

$$\$10{,}955.62 - \$9{,}000.00$$

$\boxed{\text{Compound earns \$1,955.62 more than simple interest.}}$

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