Mortgages and Loans: Applied Problems
ApexMath

Notes

Putting It All Together

This lesson applies the present value annuity formula directly to the most common real-world financial situations: mortgages and loans. You already know the formula. The goal here is to become fluent at reading a word problem, extracting the right values, and interpreting the results in a meaningful way.

The Formula — One More Time

Every problem in this lesson uses one of these two forms of the present value annuity formula:

To find the **monthly payment** given a loan amount:

$$R = PV \cdot \frac{i}{1 - (1+i)^{-n}}$$

To find the **maximum loan** you can afford given a monthly payment:

$$PV = R \cdot \frac{1 - (1+i)^{-n}}{i}$$

where $i = \dfrac{r}{12}$ for monthly payments and $n = 12 \times t$.

Three Key Quantities in Every Loan Problem

- **Principal** (PV): The amount borrowed. This is what you owe on day one.
- **Total Amount Paid**: Every dollar you pay over the life of the loan.

$$\text{Total paid} = R \times n$$

- **Total Interest Paid**: The cost of borrowing — the difference between what you pay back and what you originally borrowed.

$$\text{Total interest} = (R \times n) - PV$$

Why Amortization Period Matters

A longer repayment period (more years) means smaller monthly payments, but significantly more total interest paid. A shorter period costs more per month but saves a large amount of interest over time. Comparing two amortization periods on the same mortgage is one of the most important financial decisions a person makes.

Pro Tip

- In every problem, write out PV, r, i, t, and n before doing any calculation. Mortgage word problems contain a lot of numbers and it is easy to mix up the annual rate with the monthly rate.
- The monthly rate is always $i = r \div 12$. If the annual rate is 5%, the monthly rate is $0.05 \div 12 \approx 0.004167$. Never use 5% or 0.05 directly in the formula.
- To find total interest, you do *not* need a separate formula. Compute $R \times n$ first (total paid), then subtract PV (amount borrowed). The result is every dollar of interest.
- When comparing two amortization periods, compute both monthly payments and both total interest amounts, then state clearly which is larger and by how much. Showing the comparison is part of a complete answer.
- A rough check: for a typical mortgage, total interest over 25 years at moderate rates is often close to the original loan amount itself. If your total interest seems far too small or far too large, recheck your value of n.

Examples

Example 1 (Mortgage — Find the Monthly Payment and Total Interest): A couple takes out a \$300,000 mortgage at 5% per year, compounded monthly, amortized over 25 years. Find the monthly payment and the total interest paid over the life of the mortgage.

Step 1: Identify the values.

$$PV = 300000, \qquad r = 5\% = 0.05, \qquad t = 25 \text{ years}$$

Step 2: Find i and n.

$$i = \frac{0.05}{12} \approx 0.004167 \qquad \text{and} \qquad n = 12 \times 25 = 300$$

Step 3: Compute $(1.004167)^{-300}$.

$$(1.004167)^{-300} \approx 0.287942$$

Step 4: Find the monthly payment.

$$R = 300000 \cdot \frac{0.004167}{1 - 0.287942} = 300000 \cdot \frac{0.004167}{0.712058} = \frac{1250.10}{0.712058}$$

$$R \approx \$1{,}753.77 \text{ per month}$$

Step 5: Find the total paid and total interest.

$$\text{Total paid} = 1753.77 \times 300 = \$526{,}131.00$$

$$\text{Total interest} = 526{,}131.00 - 300{,}000 = \$226{,}131.00$$

$$\boxed{R \approx \$1{,}753.77/\text{mo}, \quad \text{Total interest} \approx \$226{,}131.04}$$

Example 2 (Car Loan — Find the Monthly Payment): Jasmine takes out an \$18,000 car loan at 7% per year, compounded monthly, to be repaid over 5 years. Find the monthly payment and total interest paid.

Step 1: Identify the values.

$$PV = 18000, \qquad r = 7\% = 0.07, \qquad t = 5 \text{ years}$$

Step 2: Find i and n.

$$i = \frac{0.07}{12} \approx 0.005833 \qquad \text{and} \qquad n = 12 \times 5 = 60$$

Step 3: Compute $(1.005833)^{-60}$.

$$(1.005833)^{-60} \approx 0.705919$$

Step 4: Find the monthly payment.

$$R = 18000 \cdot \frac{0.005833}{1 - 0.705919} = 18000 \cdot \frac{0.005833}{0.294081} = \frac{104.994}{0.294081}$$

$$R \approx \$356.42 \text{ per month}$$

Step 5: Find total interest.

$$\text{Total paid} = 356.42 \times 60 = \$21{,}385.20$$

$$\text{Total interest} = 21{,}385.20 - 18{,}000 = \$3{,}385.20$$

$$\boxed{R \approx \$356.42/\text{mo}, \quad \text{Total interest} \approx \$3{,}385.29}$$

Example 3 (Comparing Amortization Periods): A \$250,000 mortgage is available at 4.5% per year, compounded monthly. Compare the monthly payment and total interest for a 20-year amortization versus a 25-year amortization.

Step 1: Identify the shared values.

$$PV = 250000, \qquad r = 4.5\% = 0.045, \qquad i = \frac{0.045}{12} = 0.00375$$

Step 2: Compute the 20-year option. $n = 12 \times 20 = 240$.

$$(1.00375)^{-240} \approx 0.406928$$

$$R_{20} = 250000 \cdot \frac{0.00375}{1 - 0.406928} = 250000 \cdot \frac{0.00375}{0.593072} \approx \$1{,}581.62/\text{mo}$$

$$\text{Total interest}_{20} = (1581.62 \times 240) - 250000 \approx \$129{,}589.63$$

Step 3: Compute the 25-year option. $n = 12 \times 25 = 300$**.**

$$(1.00375)^{-300} \approx 0.323560$$

$$R_{25} = 250000 \cdot \frac{0.00375}{1 - 0.323560} = 250000 \cdot \frac{0.00375}{0.676440} \approx \$1{,}389.58/\text{mo}$$

$$\text{Total interest}_{25} = (1389.58 \times 300) - 250000 \approx \$166{,}874.36$$

Step 4: Compare the two options.

$$\text{Extra interest with 25-year plan} = 166{,}874.36 - 129{,}589.63 = \$37{,}284.73$$

$$\boxed{\text{20-year: \$1,581.62/mo, \$129,589.63 interest.} \quad \text{25-year: \$1,389.58/mo, \$166,874.36 interest.}}$$

Example 4 (Find Maximum Affordable Loan): Leon can afford monthly mortgage payments of $1,200. If the annual interest rate is 4%, compounded monthly, and he wants a 25-year amortization, what is the maximum loan he can take out?

Step 1: Identify the values. PV **is the unknown.**

$$R = 1200, \qquad r = 4\% = 0.04, \qquad t = 25 \text{ years}$$

Step 2: Find i **and** n**.**

$$i = \frac{0.04}{12} \approx 0.003333 \qquad \text{and} \qquad n = 12 \times 25 = 300$$

Step 3: Compute $(1.003333)^{-300}$**.**

$$(1.003333)^{-300} \approx 0.368940$$

Step 4: Substitute into the present value formula.

$$PV = 1200 \cdot \frac{1 - 0.368940}{0.003333} = 1200 \cdot \frac{0.631060}{0.003333} = 1200 \times 189.286$$

$$\boxed{PV \approx \$227{,}342.98}$$

Example 5 (Personal Loan): Tyler takes out an $8,000 personal loan at 9% per year, compounded monthly, to be repaid over 3 years. Find the monthly payment and the total interest paid.

Step 1: Identify the values.

$$PV = 8000, \qquad r = 9\% = 0.09, \qquad t = 3 \text{ years}$$

Step 2: Find i and n.

$$i = \frac{0.09}{12} = 0.0075 \qquad \text{and} \qquad n = 12 \times 3 = 36$$

Step 3: Compute $(1.0075)^{-36}$.

$$(1.0075)^{-36} \approx 0.763379$$

Step 4: Find the monthly payment.

$$R = 8000 \cdot \frac{0.0075}{1 - 0.763379} = 8000 \cdot \frac{0.0075}{0.236621} = \frac{60}{0.236621}$$

$$R \approx \$253.63 \text{ per month}$$

Step 5: Find total interest.

$$\text{Total paid} = 253.63 \times 36 = \$9{,}130.68$$

$$\text{Total interest} = 9{,}130.68 - 8{,}000 = \$1{,}130.68$$

$$\boxed{R \approx \$254.40/\text{mo}, \quad \text{Total interest} \approx \$1{,}158.32}$$

Common Mistakes

- **Using the annual rate r instead of the monthly rate i.** Always divide the annual rate by 12 before substituting. This is the most frequent error in mortgage problems.
- **Using years instead of total months for n.** For a 25-year mortgage with monthly payments, $n = 300$, not 25.
- **Reporting total paid instead of total interest.** Total paid is $R \times n$. Total interest is $(R \times n) - PV$. Read the question carefully to know which one is being asked for.
- **Assuming a longer amortization is always better.** A longer amortization does lower the monthly payment, but the total interest paid is much higher. Both facts are important and must be considered together.
- **Rounding R too early when computing total interest.** If you round the monthly payment to the nearest dollar before multiplying by n, your total interest will be off by hundreds of dollars. Keep at least two decimal places in R until the final answer.

Worksheet

1. A family takes out a \$200,000 mortgage at 5% per year, compounded monthly, amortized over 25 years. Find the monthly payment and the total interest paid over the life of the mortgage.

2. Zoe purchases a car with a \$15,000 loan at 8% per year, compounded monthly, to be repaid over 4 years. Find the monthly payment and total interest paid.

3. A \$350,000 mortgage is available at 4% per year, compounded monthly. Compare the monthly payment and total interest for a 20-year amortization versus a 25-year amortization. How much interest is saved by choosing the shorter period?

4. Sam can afford monthly payments of \$900. If the annual interest rate is 6%, compounded monthly, and he wants a 20-year amortization, what is the maximum mortgage he can afford?

5. Marcus takes out a \$12,000 personal loan at 6% per year, compounded monthly, to be repaid over 3 years. Find the monthly payment and total interest paid.

6. A \$450,000 mortgage is offered at 5% per year, compounded monthly. Compare a 20-year and a 30-year amortization. How much extra interest does the 30-year option cost?

7. Priya wants a 20-year mortgage at 4.5% per year, compounded monthly. If she can afford \$1,500 per month, what is the maximum amount she can borrow?

8. (Challenge) A couple takes out a \$320,000 mortgage at 4% per year, compounded monthly, amortized over 25 years. They decide to pay an extra \$200 per month on top of the regular payment. Approximately how many years earlier will the mortgage be paid off, and how much interest will they save?
(Hint: Find the regular payment first, then use the new total payment to solve for the new number of months using: $n = \dfrac{-\ln\left(1 - \dfrac{PV \cdot i}{R_{new}}\right)}{\ln(1+i)}$*)*

Answer Key

1. $PV = \$200{,}000$, $r = 0.05$, $t = 25$ years.

Step 1: Find i and n.

$$i = \frac{0.05}{12} \approx 0.004167 \qquad \text{and} \qquad n = 12 \times 25 = 300$$

Step 2: Compute $(1.004167)^{-300}$.

$$(1.004167)^{-300} \approx 0.287942$$

Step 3: Find the monthly payment.

$$R = 200000 \cdot \frac{0.004167}{1 - 0.287942} = 200000 \cdot \frac{0.004167}{0.712058} \approx \$1{,}169.18/\text{mo}$$

Step 4: Find total interest.

$$\text{Total paid} = 1169.18 \times 300 = \$350{,}754.00$$

$$\text{Total interest} = 350{,}754.00 - 200{,}000$$

$$\boxed{R \approx \$1{,}169.18/\text{mo}, \quad \text{Total interest} \approx \$150{,}754.02}$$

2. $PV = \$15{,}000$, $r = 0.08$, $t = 4$ years.

Step 1: Find i and n.

$$i = \frac{0.08}{12} \approx 0.006667 \qquad \text{and} \qquad n = 12 \times 4 = 48$$

Step 2: Compute $(1.006667)^{-48}$.

$$(1.006667)^{-48} \approx 0.724099$$

Step 3: Find the monthly payment.

$$R = 15000 \cdot \frac{0.006667}{1 - 0.724099} = 15000 \cdot \frac{0.006667}{0.275901} \approx \$366.19/\text{mo}$$

Step 4: Find total interest.

$$\text{Total paid} = 366.19 \times 48 = \$17{,}577.12$$

$$\text{Total interest} = 17{,}577.12 - 15{,}000$$

$$\boxed{R \approx \$366.19/\text{mo}, \quad \text{Total interest} \approx \$2{,}577.30}$$

3. $PV = \$350{,}000$, $r = 0.04$, $i = 0.04 \div 12 \approx 0.003333$.

Step 1: 20-year option. $n = 240$**.**

$$(1.003333)^{-240} \approx 0.450640$$

$$R_{20} = 350000 \cdot \frac{0.003333}{0.549360} \approx \$2{,}120.93/\text{mo}$$

$$\text{Total interest}_{20} = (2120.93 \times 240) - 350000 \approx \$159{,}023.48$$

Step 2: 25-year option. $n = 300$**.**

$$(1.003333)^{-300} \approx 0.368940$$

$$R_{25} = 350000 \cdot \frac{0.003333}{0.631060} \approx \$1{,}847.43/\text{mo}$$

$$\text{Total interest}_{25} = (1847.43 \times 300) - 350000 \approx \$204{,}228.68$$

Step 3: Interest saved by choosing 20 years.

$$204{,}228.68 - 159{,}023.48 = \$45{,}205.20$$

$$\boxed{\text{20-yr: \$2,120.93/mo, interest \$159,023.48.} \quad \text{25-yr: \$1,847.43/mo, interest \$204,228.68.} \quad \text{Savings: \$45,205}}$$

4. $R = \$900$, $r = 0.06$, $t = 20$ years. Find PV.

Step 1: Find i and n.

$$i = \frac{0.06}{12} = 0.005 \qquad \text{and} \qquad n = 12 \times 20 = 240$$

Step 2: Compute $(1.005)^{-240}$.

$$(1.005)^{-240} \approx 0.302096$$

Step 3: Substitute into the present value formula.

$$PV = 900 \cdot \frac{1 - 0.302096}{0.005} = 900 \cdot \frac{0.697904}{0.005} = 900 \times 139.581$$

$$\boxed{PV \approx \$125{,}622.69}$$

5. $PV = \$12{,}000$, $r = 0.06$, $t = 3$ years.

Step 1: Find i and n.

$$i = \frac{0.06}{12} = 0.005 \qquad \text{and} \qquad n = 12 \times 3 = 36$$

Step 2: Compute $(1.005)^{-36}$.

$$(1.005)^{-36} \approx 0.836017$$

Step 3: Find the monthly payment.

$$R = 12000 \cdot \frac{0.005}{1 - 0.836017} = 12000 \cdot \frac{0.005}{0.163983} \approx \$365.06/\text{mo}$$

Step 4: Find total interest.

$$\text{Total paid} = 365.06 \times 36 = \$13{,}142.16$$

$$\text{Total interest} = 13{,}142.16 - 12{,}000$$

$$\boxed{R \approx \$365.06/\text{mo}, \quad \text{Total interest} \approx \$1{,}142.28}$$

6. $PV = \$450{,}000$, $r = 0.05$, $i = 0.05 \div 12 \approx 0.004167$.

Step 1: 20-year option. $n = 240$.

$$(1.004167)^{-240} \approx 0.369120$$

$$R_{20} = 450000 \cdot \frac{0.004167}{0.630880} \approx \$2{,}969.80/\text{mo}$$

$$\text{Total interest}_{20} = (2969.80 \times 240) - 450000 \approx \$262{,}752.20$$

Step 2: 30-year option. $n = 360$.

$$(1.004167)^{-360} \approx 0.223828$$

$$R_{30} = 450000 \cdot \frac{0.004167}{0.776172} \approx \$2{,}415.70/\text{mo}$$

$$\text{Total interest}_{30} = (2415.70 \times 360) - 450000 \approx \$419{,}651.03$$

Step 3: Extra interest for 30-year option.

$$419{,}651.03 - 262{,}752.20 = \$156{,}898.83$$

$$\boxed{\text{20-yr: } \$2{,}969.80/\text{mo.} \quad \text{30-yr: } \$2{,}415.70/\text{mo.} \quad \text{Extra interest: } \$156{,}898.83}$$

7. $R = \$1{,}500$, $r = 0.045$, $t = 20$ years. Find PV.

Step 1: Find i and n.

$$i = \frac{0.045}{12} = 0.00375 \qquad \text{and} \qquad n = 12 \times 20 = 240$$

Step 2: Compute $(1.00375)^{-240}$.

$$(1.00375)^{-240} \approx 0.406928$$

Step 3: Substitute into the present value formula.

$$PV = 1500 \cdot \frac{1 - 0.406928}{0.00375} = 1500 \cdot \frac{0.593072}{0.00375} = 1500 \times 158.153$$

$$\boxed{PV \approx \$237{,}098.16}$$

8. (Challenge) $PV = \$320{,}000$, $r = 0.04$, $t = 25$ years, extra \$200/month.

Step 1: Find i, n, and the regular payment.

$$i = \frac{0.04}{12} \approx 0.003333, \qquad n = 300$$

$$(1.003333)^{-300} \approx 0.368940$$

$$R = 320000 \cdot \frac{0.003333}{0.631060} \approx \$1{,}689.08/\text{mo}$$

$$\text{Standard total interest} = (1689.08 \times 300) - 320000 \approx \$186{,}723.37$$

Step 2: Find the new number of months with the extra payment.

$$R_{\text{new}} = 1689.08 + 200 = \$1{,}889.08/\text{mo}$$

$$n_{\text{new}} = \frac{-\ln\left(1 - \dfrac{320000 \times 0.003333}{1889.08}\right)}{\ln(1.003333)}$$

$$= \frac{-\ln\left(1 - \dfrac{1066.56}{1889.08}\right)}{\ln(1.003333)} = \frac{-\ln(0.435291)}{0.003328} \approx \frac{0.831887}{0.003328} \approx 249.9 \text{ months}$$

Step 3: Convert to years and find interest saved.

$$\frac{249.9}{12} \approx 20.82 \text{ years} \qquad \Longrightarrow \qquad \text{Paid off about 4.18 years early}$$

$$\text{New total interest} = (1889.08 \times 249.9) - 320000 \approx \$152{,}074.22$$

$$\text{Interest saved} = 186{,}723.37 - 152{,}074.22$$

$$\boxed{\text{Paid off} \approx 4.2 \text{ years early.} \quad \text{Interest saved} \approx \$34{,}649.15}$$

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