Factoring Review
ApexMath

Notes

The Big Idea

Factoring is the process of rewriting an expression as a product of simpler expressions. It is the reverse of expanding. You will use factoring in almost every topic in this unit, so mastering it here makes everything that follows much easier.

There are four main factoring types covered in this lesson. Always check them in this order:

1. Greatest Common Factor (GCF)
2. Difference of Squares
3. Perfect Square Trinomial
4. Factoring Trinomials of the form $ax^2 + bx + c$

Always check for a GCF first before attempting any other method.

Type 1 — Greatest Common Factor (GCF)

The GCF is the largest factor that divides evenly into every term of the expression. Factor it out front.

$$6x^2 + 9x = 3x(2x + 3)$$

To verify: expand $3x(2x + 3)$ and confirm you get back the original expression.

Type 2 — Difference of Squares

A difference of squares has the form $a^2 - b^2$ and always factors as:

$$a^2 - b^2 = (a + b)(a - b)$$

The key features: two perfect square terms, separated by subtraction.
Examples:

$$x^2 - 25 = (x + 5)(x - 5)$$

$$4x^2 - 49 = (2x + 7)(2x - 7)$$

Note: A **sum** of squares $a^2 + b^2$ does **not** factor over the real numbers.

Type 3 — Perfect Square Trinomial

A perfect square trinomial has the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$ and factors as:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

To recognize one: the first and last terms are perfect squares, and the middle term is twice the product of their square roots.

Examples:

$$x^2 + 6x + 9 = (x+3)^2 \qquad \text{because } 2 \times x \times 3 = 6x$$
$$x^2 - 10x + 25 = (x-5)^2 \qquad \text{because } 2 \times x \times 5 = 10x$$

Type 4 — Factoring Trinomials ($a = 1$)

For $x^2 + bx + c$, find two numbers that:

- **Multiply** to give c
- **Add** to give b

Call these numbers p and q. Then:

$$x^2 + bx + c = (x+p)(x+q)$$

Example: Factor $x^2 + 5x + 6$.

Find two numbers that multiply to 6 and add to 5: those are 2 and 3.

$$x^2 + 5x + 6 = (x+2)(x+3)$$

Type 4 — Factoring Trinomials ($a \neq 1$)

For $ax^2 + bx + c$ where $a \neq 1$, use the **decomposition method**:

- Step 1: Multiply $a \times c$.
- Step 2: Find two numbers that multiply to ac and add to b.
- Step 3: Rewrite the middle term using those two numbers.
- Step 4: Factor by grouping.

Example: Factor $2x^2 + 7x + 3$.

Step 1: $a \times c = 2 \times 3 = 6$.

Step 2: Find two numbers that multiply to 6 and add to 7: those are 1 and 6.

Step 3: Rewrite.

$$2x^2 + x + 6x + 3$$

Step 4: Group and factor.

$$x(2x+1) + 3(2x+1) = (2x+1)(x+3)$$

Factoring Completely

An expression is **fully factored** when no factor can be broken down further. Always check each factor after factoring to see if it can be factored again.

For example:

$$2x^3 - 8x = 2x(x^2 - 4) = 2x(x-2)(x+2)$$

The first step removed the GCF. The second step applied difference of squares to the remaining factor.

Pro tip

- Always look for a GCF **first**. It simplifies every method that follows and prevents errors.
- After factoring, always verify by expanding your answer. If you get back the original expression, your factoring is correct.
- For trinomials with $a = 1$, the sign of c tells you whether the numbers add or subtract: if c is positive, the two numbers have the same sign; if c is negative, they have opposite signs.
- For difference of squares, both terms must be perfect squares and the operation between them must be subtraction. A sum of two squares does not factor.
- When using decomposition for $a \neq 1$, the order in which you split the middle term does not matter — you will always arrive at the same factored form.

Examples

Example A — Greatest Common Factor
Factor $4x^3 - 12x^2 + 8x$ completely.

Step 1: Identify the GCF of all terms. The GCF of $4x^3$, $12x^2$, and $8x$ is $4x$.
Step 2: Factor out $4x$.

$$4x^3 - 12x^2 + 8x = 4x(x^2 - 3x + 2)$$

Step 3: Factor the remaining trinomial. Find two numbers that multiply to 2 and add to -3: those are -1 and -2.

$$4x(x^2 - 3x + 2) = 4x(x - 1)(x - 2)$$

Step 4: Verify by expanding. $4x(x - 1)(x - 2) = 4x(x^2 - 3x + 2) = 4x^3 - 12x^2 + 8x$ ✓

Example B — Difference of Squares
Factor $9x^2 - 16$ completely.

Step 1: Check for GCF. There is none.
Step 2: Recognize the pattern. $9x^2 = (3x)^2$ and $16 = 4^2$. This is a difference of squares.
Step 3: Apply $a^2 - b^2 = (a + b)(a - b)$ with $a = 3x$ and $b = 4$.

$$9x^2 - 16 = (3x + 4)(3x - 4)$$

Step 4: Verify. $(3x + 4)(3x - 4) = 9x^2 - 12x + 12x - 16 = 9x^2 - 16$ ✓

Example C — Perfect Square Trinomial
Factor $x^2 - 8x + 16$ completely.

Step 1: Check for GCF. There is none.

Step 2: Check whether this is a perfect square trinomial. $\sqrt{x^2} = x$ and $\sqrt{16} = 4$. Check the middle term: $2 \times x \times 4 = 8x$. Yes, this matches.

Step 3: Apply the pattern. Since the middle term is negative:

$$x^2 - 8x + 16 = (x - 4)^2$$

Step 4: Verify. $(x - 4)^2 = x^2 - 8x + 16$ ✓

Example D — Trinomial with $a = 1$

Factor $x^2 - 3x - 18$ completely.

Step 1: Check for GCF. There is none.

Step 2: Find two numbers that multiply to -18 and add to -3.

Since c is negative, the two numbers have opposite signs. Try: -6 and 3. Check: $(-6)(3) = -18$ ✓and $-6 + 3 = -3$ ✓.

Step 3: Write the factored form.

$$x^2 - 3x - 18 = (x - 6)(x + 3)$$

Step 4: Verify. $(x - 6)(x + 3) = x^2 + 3x - 6x - 18 = x^2 - 3x - 18$ ✓

Example E — Trinomial with $a \neq 1$

Factor $3x^2 - 10x + 8$ completely.

Step 1: Check for GCF. There is none.

Step 2: Multiply $a \times c = 3 \times 8 = 24$.

Step 3: Find two numbers that multiply to 24 and add to -10. Both must be negative since c is positive and b is negative. Try -4 and -6: $(-4)(-6) = 24$ ✓and $-4 + (-6) = -10$ ✓.

Step 4: Rewrite the middle term.

$$3x^2 - 4x - 6x + 8$$

Step 5: Group and factor.

$$x(3x - 4) - 2(3x - 4) = (3x - 4)(x - 2)$$

Step 6: Verify. $(3x - 4)(x - 2) = 3x^2 - 6x - 4x + 8 = 3x^2 - 10x + 8$ ✓

Common mistakes

- **Skipping the GCF check.** Always factor out the GCF first. Missing it leads to incomplete factoring and harder arithmetic.
- **Confusing sum of squares with difference of squares.** $x^2 + 9$ does **not** factor. Only $x^2 - 9$ factors as $(x + 3)(x - 3)$.
- **Sign errors in trinomials.** When c is positive and b is negative, both numbers are negative. When c is negative, the numbers have opposite signs. Keep track carefully.

- **Stopping before fully factoring.** Always check whether any factor can be factored further. $2x(x^2-4)$ is not fully factored — it should be $2x(x-2)(x+2)$.
- **Grouping errors with decomposition.** When factoring by grouping, make sure the binomial factor that appears in both groups is identical before factoring it out.

Worksheet

1. Factor out the GCF from each expression.

   (a) $5x^2+15x$

   (b) $12x^3-8x^2+4x$

   (c) $6x^2y-9xy^2$

   (d) $-3x^2+9x-6$

2. Factor each difference of squares completely.

   (a) x^2-64

   (b) $4x^2-9$

   (c) $25x^2-1$

   (d) x^2-121

3. Factor each perfect square trinomial completely.

(a) $x^2 + 10x + 25$

(b) $x^2 - 12x + 36$

(c) $4x^2 + 12x + 9$

(d) $9x^2 - 24x + 16$

4. Factor each trinomial with $a = 1$ completely.

(a) $x^2 + 7x + 12$

(b) $x^2 - 5x + 6$

(c) $x^2 + 2x - 15$

(d) $x^2 - x - 20$

5. Factor each trinomial with $a \neq 1$ completely.

(a) $2x^2 + 5x + 3$

(b) $3x^2 - 7x + 2$

(c) $4x^2 + 4x - 3$

(d) $6x^2 - x - 2$

6. Factor each expression completely. More than one step may be required.

(a) $3x^2 - 75$

(b) $2x^3 - 8x$

(c) $5x^2 - 20x + 20$

(d) $x^3 + 3x^2 - 4x - 12$

Hint: Group the first two terms and the last two terms separately.

7. Determine whether each expression is fully factored. If not, factor it completely.

(a) $4(x^2 - 9)$

(b) $3x(x+5)(x-2)$

(c) $2(x^2+6x+9)$

8. **Challenge:** Factor $4x^4-25y^2$ completely.

9. **Challenge:** Factor $6x^2+11x-10$ completely using decomposition.

10. **Challenge:** Factor x^4-16 completely.

Answer Key

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1.

(a) $5x^2 + 15x$
Step 1: GCF of $5x^2$ and $15x$ is $5x$.

$$5x^2 + 15x = 5x(x + 3)$$

$$\boxed{5x(x + 3)}$$

(b) $12x^3 - 8x^2 + 4x$
Step 1: GCF of $12x^3$, $8x^2$, and $4x$ is $4x$.

$$12x^3 - 8x^2 + 4x = 4x(3x^2 - 2x + 1)$$

Step 2: Check whether $3x^2 - 2x + 1$ factors. Find two numbers that multiply to 3 and add to -2: there are none with integer values. It does not factor further.

$$\boxed{4x(3x^2 - 2x + 1)}$$

(c) $6x^2y - 9xy^2$
Step 1: GCF of $6x^2y$ and $9xy^2$ is $3xy$.

$$6x^2y - 9xy^2 = 3xy(2x - 3y)$$

$$\boxed{3xy(2x - 3y)}$$

(d) $-3x^2 + 9x - 6$
Step 1: GCF is -3 (factor out a negative to make the leading term positive).

$$-3x^2 + 9x - 6 = -3(x^2 - 3x + 2)$$

Step 2: Factor the trinomial. Find two numbers that multiply to 2 and add to -3: -1 and -2.

$$-3(x^2 - 3x + 2) = -3(x - 1)(x - 2)$$

$$\boxed{-3(x - 1)(x - 2)}$$

2.

(a) $x^2 - 64$
Step 1: $x^2 = (x)^2$ and $64 = 8^2$. Difference of squares.

$$x^2 - 64 = (x + 8)(x - 8)$$

$$\boxed{(x+8)(x-8)}$$

(b) $4x^2 - 9$
Step 1: $4x^2 = (2x)^2$ and $9 = 3^2$. Difference of squares.

$$4x^2 - 9 = (2x+3)(2x-3)$$

$$\boxed{(2x+3)(2x-3)}$$

(c) $25x^2 - 1$
Step 1: $25x^2 = (5x)^2$ and $1 = 1^2$. Difference of squares.

$$25x^2 - 1 = (5x+1)(5x-1)$$

$$\boxed{(5x+1)(5x-1)}$$

(d) $x^2 - 121$
Step 1: $121 = 11^2$. Difference of squares.

$$x^2 - 121 = (x+11)(x-11)$$

$$\boxed{(x+11)(x-11)}$$

3.

(a) $x^2 + 10x + 25$
Step 1: $\sqrt{x^2} = x$, $\sqrt{25} = 5$, and $2 \times x \times 5 = 10x$. Perfect square trinomial.

$$x^2 + 10x + 25 = (x+5)^2$$

$$\boxed{(x+5)^2}$$

(b) $x^2 - 12x + 36$
Step 1: $\sqrt{x^2} = x$, $\sqrt{36} = 6$, and $2 \times x \times 6 = 12x$. Middle term is negative, so:

$$x^2 - 12x + 36 = (x-6)^2$$

$$\boxed{(x-6)^2}$$

(c) $4x^2 + 12x + 9$
Step 1: $\sqrt{4x^2} = 2x$, $\sqrt{9} = 3$, and $2 \times 2x \times 3 = 12x$. Perfect square trinomial.

$$4x^2 + 12x + 9 = (2x+3)^2$$

$$\boxed{(2x+3)^2}$$

(d) $9x^2 - 24x + 16$
Step 1: $\sqrt{9x^2} = 3x$, $\sqrt{16} = 4$, and $2 \times 3x \times 4 = 24x$. Middle term is negative.

$$9x^2 - 24x + 16 = (3x - 4)^2$$

$$\boxed{(3x-4)^2}$$

4.

(a) $x^2 + 7x + 12$
Step 1: Find two numbers that multiply to 12 and add to 7: 3 and 4.

$$x^2 + 7x + 12 = (x+3)(x+4)$$

$$\boxed{(x+3)(x+4)}$$

(b) $x^2 - 5x + 6$
Step 1: Find two numbers that multiply to 6 and add to -5: -2 and -3.

$$x^2 - 5x + 6 = (x-2)(x-3)$$

$$\boxed{(x-2)(x-3)}$$

(c) $x^2 + 2x - 15$
Step 1: c is negative, so the numbers have opposite signs. Find two numbers that multiply to -15 and add to 2: 5 and -3.

$$x^2 + 2x - 15 = (x+5)(x-3)$$

$$\boxed{(x+5)(x-3)}$$

(d) $x^2 - x - 20$
Step 1: c is negative. Find two numbers that multiply to -20 and add to -1: -5 and 4.

$$x^2 - x - 20 = (x-5)(x+4)$$

$$\boxed{(x-5)(x+4)}$$

5.

(a) $2x^2 + 5x + 3$
Step 1: $a \times c = 2 \times 3 = 6$. Find two numbers that multiply to 6 and add to 5: 2 and 3.
Step 2: Rewrite and group.

$$2x^2 + 2x + 3x + 3 = 2x(x+1) + 3(x+1) = (2x+3)(x+1)$$

$$\boxed{(2x+3)(x+1)}$$

(b) $3x^2 - 7x + 2$

Step 1: $a \times c = 3 \times 2 = 6$. Find two numbers that multiply to 6 and add to -7: -1 and -6.

Step 2: Rewrite and group.

$$3x^2 - x - 6x + 2 = x(3x-1) - 2(3x-1) = (3x-1)(x-2)$$

$$\boxed{(3x-1)(x-2)}$$

(c) $4x^2 + 4x - 3$

Step 1: $a \times c = 4 \times (-3) = -12$. Find two numbers that multiply to -12 and add to 4: 6 and -2.

Step 2: Rewrite and group.

$$4x^2 + 6x - 2x - 3 = 2x(2x+3) - 1(2x+3) = (2x+3)(2x-1)$$

$$\boxed{(2x+3)(2x-1)}$$

(d) $6x^2 - x - 2$

Step 1: $a \times c = 6 \times (-2) = -12$. Find two numbers that multiply to -12 and add to -1: 3 and -4.

Step 2: Rewrite and group.

$$6x^2 + 3x - 4x - 2 = 3x(2x+1) - 2(2x+1) = (2x+1)(3x-2)$$

$$\boxed{(2x+1)(3x-2)}$$

6.

(a) $3x^2 - 75$

Step 1: Factor out the GCF first. GCF = 3.

$$3x^2 - 75 = 3(x^2 - 25)$$

Step 2: Factor the difference of squares.

$$3(x^2 - 25) = 3(x+5)(x-5)$$

$$\boxed{3(x+5)(x-5)}$$

(b) $2x^3 - 8x$

Step 1: Factor out the GCF. GCF = $2x$.

$$2x^3 - 8x = 2x(x^2 - 4)$$

Step 2: Factor the difference of squares.

$$2x(x^2-4)=2x(x+2)(x-2)$$

$$\boxed{2x(x+2)(x-2)}$$

(c) $5x^2-20x+20$
Step 1: Factor out the GCF. GCF $=5$.

$$5x^2-20x+20=5(x^2-4x+4)$$

Step 2: Recognize the perfect square trinomial. $\sqrt{x^2}=x$, $\sqrt{4}=2$, $2\times x\times 2=4x$ ✓

$$5(x^2-4x+4)=5(x-2)^2$$

$$\boxed{5(x-2)^2}$$

(d) $x^3+3x^2-4x-12$
Step 1: Group the first two terms and the last two terms.

$$(x^3+3x^2)+(-4x-12)$$

Step 2: Factor each group.

$$x^2(x+3)-4(x+3)$$

Step 3: Factor out the common binomial $(x+3)$.

$$(x+3)(x^2-4)$$

Step 4: Factor x^2-4 as a difference of squares.

$$(x+3)(x+2)(x-2)$$

$$\boxed{(x+3)(x+2)(x-2)}$$

7.

(a) $4(x^2-9)$
Step 1: Check whether x^2-9 factors further. $x^2-9=(x+3)(x-3)$.
Step 2: This is not fully factored.

$$4(x^2-9)=4(x+3)(x-3)$$

$$\boxed{4(x+3)(x-3)\text{ — not fully factored as given}}$$

(b) $3x(x+5)(x-2)$
Step 1: Check each factor. $3x$ has no further factoring. $(x+5)$ and $(x-2)$ are both linear and cannot factor further.

Step 2: This expression is already fully factored.

$$\boxed{\text{Already fully factored}}$$

(c) $2(x^2 + 6x + 9)$

Step 1: Check whether $x^2 + 6x + 9$ factors. $\sqrt{x^2} = x$, $\sqrt{9} = 3$, $2 \times x \times 3 = 6x$ ✓. Perfect square trinomial.

$$2(x^2 + 6x + 9) = 2(x + 3)^2$$

$$\boxed{2(x + 3)^2 \text{ — not fully factored as given}}$$

8. Factor $4x^4 - 25y^2$ completely.

Step 1: Check for GCF. There is none.

Step 2: Recognize the pattern. $4x^4 = (2x^2)^2$ and $25y^2 = (5y)^2$. This is a difference of squares.

Step 3: Apply $a^2 - b^2 = (a + b)(a - b)$ with $a = 2x^2$ and $b = 5y$.

$$4x^4 - 25y^2 = (2x^2 + 5y)(2x^2 - 5y)$$

Step 4: Check whether either factor can be factored further. Both are a sum and difference of terms that are not perfect squares in the usual sense, so they do not factor further over the integers.

$$\boxed{(2x^2 + 5y)(2x^2 - 5y)}$$

9. Factor $6x^2 + 11x - 10$ completely.

Step 1: $a \times c = 6 \times (-10) = -60$.

Step 2: Find two numbers that multiply to -60 and add to 11. Since the product is negative, the numbers have opposite signs. Try 15 and -4: $15 \times (-4) = -60$ ✓and $15 + (-4) = 11$ ✓.

Step 3: Rewrite the middle term.

$$6x^2 + 15x - 4x - 10$$

Step 4: Group and factor.

$$3x(2x + 5) - 2(2x + 5) = (2x + 5)(3x - 2)$$

Step 5: Verify. $(2x + 5)(3x - 2) = 6x^2 - 4x + 15x - 10 = 6x^2 + 11x - 10$ ✓

$$\boxed{(2x + 5)(3x - 2)}$$

10. Factor $x^4 - 16$ completely.

Step 1: Recognize $x^4 = (x^2)^2$ and $16 = 4^2$. This is a difference of squares.

$$x^4 - 16 = (x^2 + 4)(x^2 - 4)$$

Step 2: Check whether each factor can be factored further.
$(x^2 + 4)$: this is a sum of squares and does not factor over the real numbers.
$(x^2 - 4)$: this is a difference of squares. $x^2 - 4 = (x + 2)(x - 2)$.
Step 3: Write the fully factored form.

$$x^4 - 16 = (x^2 + 4)(x + 2)(x - 2)$$

$$\boxed{(x^2 + 4)(x + 2)(x - 2)}$$

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