Graphing Quadratics in Standard Form
ApexMath

Notes

The Standard Form of a Quadratic

A quadratic in **standard form** looks like this:

$$y = ax^2 + bx + c$$

Unlike vertex form, the vertex is not immediately visible. To graph it, we convert to vertex form by **completing the square**. Once we have vertex form, we use the same graphing strategy from the previous lesson.

Step 1 — Pull Out the Leading Coefficient (if $a \neq 1$)

If $a \neq 1$, factor a out of only the first two terms. Do not touch the constant c.

$$y = a\left(x^2 + \frac{b}{a}x\right) + c$$

If $a = 1$, skip this step and move directly to Step 2.

Step 2 — Complete the Square

Inside the parentheses, take the coefficient of x, divide it by 2, and square the result. This number is called the **completing value**.

$$\text{Completing value} = \left(\frac{\text{coefficient of } x}{2}\right)^2$$

Add this value inside the parentheses, and immediately subtract it to keep the equation balanced. When $a \neq 1$, remember that the value added inside the parentheses is being multiplied by a, so you must subtract a times the completing value outside.

Step 3 — Write the Perfect Square Trinomial as a Binomial Squared

The three terms inside the parentheses now form a perfect square trinomial. Rewrite them as a squared binomial:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Step 4 — Read Off the Vertex and Graph

The equation is now in vertex form $y = a(x-h)^2 + k$. Read the vertex (h, k), identify a, and graph using the method from Lesson 3.

Summary of the Process

Step	**What to Do**
1	Factor a out of the first two terms (skip if $a = 1$)
2	Add and subtract $\left(\frac{b}{2a}\right)^2$ to complete the square
3	Rewrite as a squared binomial
4	Read the vertex (h, k) and graph

Pro Tip

- When you add the completing value inside the parentheses, it is not free. If there is a factor of a outside the parentheses, you have actually added a times that value to the equation. You must subtract a times the completing value outside to stay balanced. This is the most common place students lose points.
- After completing the square, double-check your vertex form by expanding it back to standard form. If you get the original equation, you did it correctly.
- The vertex can also be found using the formula $x = -\frac{b}{2a}$ and then substituting back. This is a useful shortcut, but completing the square is the method this lesson focuses on.
- The sign of k in the vertex form tells you whether the vertex is above, on, or below the x-axis. Paired with the sign of a, you can immediately predict how many x-intercepts the parabola has before you even graph it.
- Always factor out a from *only* the first two terms. The constant c stays outside the parentheses untouched until the end.

Examples

Example 1: Convert $y = x^2 + 6x + 5$ to vertex form and graph.

Step 1: $a = 1$, so no factoring is needed.
Step 2: The coefficient of x is 6. Find the completing value.

$$\left(\frac{6}{2}\right)^2 = 3^2 = 9$$

Add and subtract 9 inside the expression:

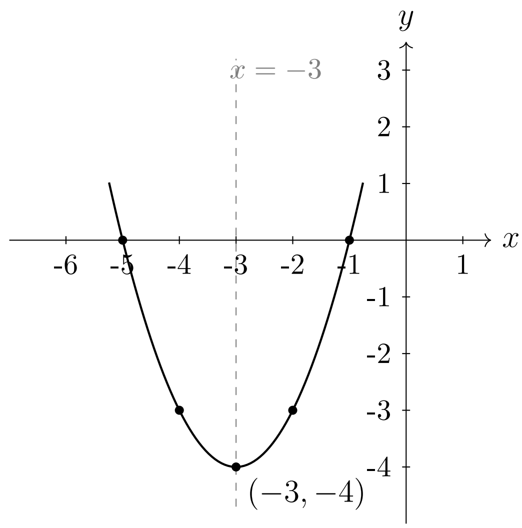
$$y = \underbrace{x^2 + 6x + 9}_{\text{perfect square}} - 9 + 5$$

Step 3: Rewrite the perfect square trinomial as a squared binomial.

$$y = (x + 3)^2 - 9 + 5$$

$$y = (x + 3)^2 - 4$$

Step 4: Identify the vertex. Rewrite as $y = (x - (-3))^2 - 4$, so $h = -3$, $k = -4$.
Vertex: $(-3, -4)$. Axis of symmetry: $x = -3$. $a = 1 > 0$, opens **upward**. Minimum.
Extra points: At $x = -2$: $y = (-2 + 3)^2 - 4 = 1 - 4 = -3$. At $x = -1$: $y = (-1 + 3)^2 - 4 = 4 - 4 = 0$.
By symmetry: $(-4, -3)$ and $(-5, 0)$.



Example 2: Convert $y = x^2 - 4x + 7$ to vertex form and graph.

Step 1: $a = 1$, so no factoring is needed.
Step 2: The coefficient of x is -4. Find the completing value.

$$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

Add and subtract 4:

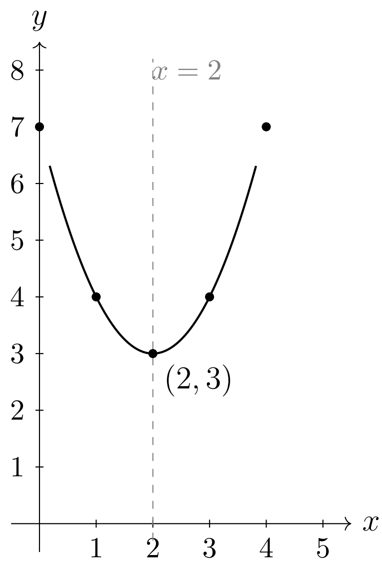
$$y = x^2 - 4x + 4 - 4 + 7$$

Step 3: Write the squared binomial.

$$y = (x-2)^2 - 4 + 7$$

$$y = (x-2)^2 + 3$$

Step 4: Vertex is $(2, 3)$. Axis of symmetry: $x = 2$. Opens **upward** ($a = 1 > 0$). Minimum.
Extra points: At $x = 3$: $y = (3-2)^2 + 3 = 4$. At $x = 4$: $y = (4-2)^2 + 3 = 7$.
By symmetry: $(1, 4)$ and $(0, 7)$.



Example 3: Convert $y = 2x^2 + 8x + 3$ to vertex form and graph.

Step 1: $a = 2 \neq 1$, so factor 2 out of the first two terms only.

$$y = 2(x^2 + 4x) + 3$$

Step 2: The coefficient of x inside the parentheses is 4. Find the completing value.

$$\left(\frac{4}{2}\right)^2 = 2^2 = 4$$

Add 4 inside the parentheses. Because of the factor of 2 outside, we have actually added $2 \times 4 = 8$ to the equation. Subtract 8 outside to compensate.

$$y = 2(x^2 + 4x + 4) - 8 + 3$$

Step 3: Write the squared binomial.

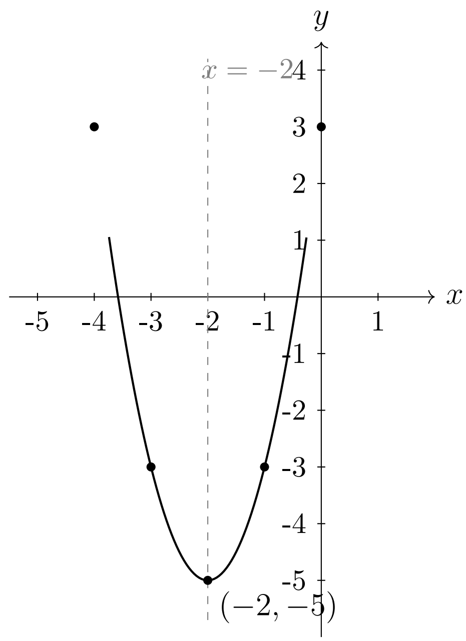
$$y = 2(x + 2)^2 - 8 + 3$$

$$y = 2(x + 2)^2 - 5$$

Step 4: $h = -2$, $k = -5$. Vertex: $(-2, -5)$. Axis of symmetry: $x = -2$. $a = 2 > 0$, opens **upward**, **narrower** than $y = x^2$. Minimum.

Extra points: At $x = -1$: $y = 2(-1 + 2)^2 - 5 = 2 - 5 = -3$. At $x = 0$: $y = 2(0 + 2)^2 - 5 = 8 - 5 = 3$.

By symmetry: $(-3, -3)$ and $(-4, 3)$.



Example 4: Convert $y = -x^2 + 4x - 1$ to vertex form and graph.

Step 1: $a = -1$. Factor -1 out of the first two terms.

$$y = -(x^2 - 4x) - 1$$

Step 2: The coefficient of x inside the parentheses is -4. Find the completing value.

$$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

Add 4 inside the parentheses. The factor outside is -1, so we have actually added $(-1) \times 4 = -4$ to the equation. Subtract -4 (i.e., add 4) outside to compensate.

$$y = -(x^2 - 4x + 4) - 1 + 4$$

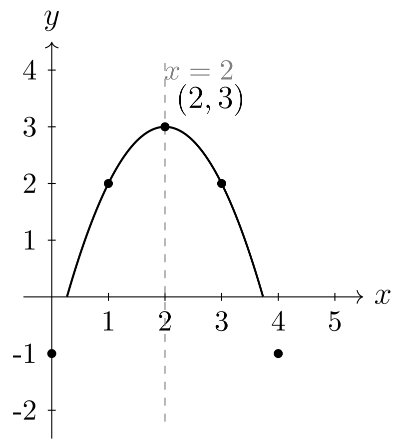
Step 3: Write the squared binomial.

$$y = -(x - 2)^2 + 3$$

Step 4: $h = 2$, $k = 3$. Vertex: $(2, 3)$. Axis of symmetry: $x = 2$. $a = -1 < 0$, opens **downward**. Maximum.

Extra points: At $x = 3$: $y = -(3-2)^2 + 3 = -1 + 3 = 2$. At $x = 4$: $y = -(4-2)^2 + 3 = -4 + 3 = -1$.

By symmetry: $(1, 2)$ and $(0, -1)$.



Example 5: Convert $y = 3x^2 - 6x + 5$ to vertex form and graph.

Step 1: $a = 3 \neq 1$. Factor 3 out of the first two terms.

$$y = 3(x^2 - 2x) + 5$$

Step 2: The coefficient of x inside the parentheses is -2. Find the completing value.

$$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

Add 1 inside the parentheses. Since the factor outside is 3, we have actually added $3 \times 1 = 3$. Subtract 3 outside.

$$y = 3(x^2 - 2x + 1) - 3 + 5$$

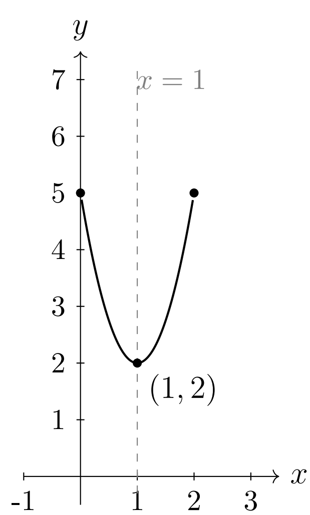
Step 3: Write the squared binomial.

$$y = 3(x - 1)^2 - 3 + 5$$

$$y = 3(x - 1)^2 + 2$$

Step 4: $h = 1$, $k = 2$. Vertex: $(1, 2)$. Axis of symmetry: $x = 1$. $a = 3 > 0$, opens **upward**, **narrower** than $y = x^2$. Minimum.

Extra points: At $x = 2$: $y = 3(2 - 1)^2 + 2 = 3 + 2 = 5$. At $x = 0$: by symmetry, $(0, 5)$.



Common Mistakes

- **Forgetting to compensate after adding inside the parentheses:** When you add the completing value inside the parentheses, you must subtract the same amount outside to keep the equation equal. If $a \neq 1$, you subtract a times the completing value, not just the completing value itself.
- **Factoring a out of all three terms including c:** When $a \neq 1$, you only factor out a from the first two terms. The constant c stays outside on its own until the very last step when you combine the constants.
- **Using the wrong coefficient when finding the completing value:** After factoring out a, the coefficient of x inside the parentheses is b/a, not b. Always look at the coefficient of x *inside* the parentheses after factoring.
- **Forgetting to halve before squaring:** The completing value is $\left(\frac{\text{coefficient of } x}{2}\right)^2$. A common error is to square the coefficient without dividing by 2 first.
- **Misreading h from vertex form:** After completing the square, re-read h carefully. The form is $(x-h)^2$, so a positive h appears as subtraction inside the parentheses. If the result is $(x+3)^2$, then $h = -3$.

Worksheet

For each problem, convert to vertex form by completing the square. Then state the vertex, axis of symmetry, direction of opening, and sketch the graph.

1. $y = x^2 + 4x + 1$

2. $y = x^2 - 8x + 10$

3. $y = x^2 + 2x - 3$

4. $y = x^2 - 6x + 9$

5. $y = -x^2 + 6x - 5$

6. $y = 2x^2 - 4x + 1$

7. $y = -2x^2 - 8x - 3$

8. $y = 3x^2 + 12x + 7$

9. (Challenge) $y = -\frac{1}{2}x^2 + 2x + 1$

10. (Challenge) Convert $y = 4x^2 - 16x + 19$ to vertex form, identify the vertex, and determine how many x-intercepts the parabola has. Explain your reasoning.

Answer Key

1. $y = x^2 + 4x + 1$
   Step 1: $a = 1$, no factoring needed.
   Step 2: Coefficient of x is 4.

$$\left(\frac{4}{2}\right)^2 = 4$$

$$y = x^2 + 4x + 4 - 4 + 1$$

Step 3: Write as squared binomial.

$$y = (x+2)^2 - 3$$

Step 4: Vertex $(-2, -3)$, axis $x = -2$, opens **upward**, minimum.

$$\boxed{y = (x+2)^2 - 3, \quad \text{Vertex } (-2, -3), \quad x = -2}$$

2. $y = x^2 - 8x + 10$
   Step 1: $a = 1$, no factoring needed.
   Step 2: Coefficient of x is -8.

$$\left(\frac{-8}{2}\right)^2 = 16$$

$$y = x^2 - 8x + 16 - 16 + 10$$

Step 3:

$$y = (x-4)^2 - 6$$

Step 4: Vertex $(4, -6)$, axis $x = 4$, opens **upward**, minimum.

$$\boxed{y = (x-4)^2 - 6, \quad \text{Vertex } (4, -6), \quad x = 4}$$

3. $y = x^2 + 2x - 3$
   Step 1: $a = 1$, no factoring needed.
   Step 2: Coefficient of x is 2.

$$\left(\frac{2}{2}\right)^2 = 1$$

$$y = x^2 + 2x + 1 - 1 - 3$$

Step 3:

$$y = (x+1)^2 - 4$$

Step 4: Vertex $(-1, -4)$, axis $x = -1$, opens **upward**, minimum.

$$\boxed{y = (x+1)^2 - 4, \quad \text{Vertex } (-1, -4), \quad x = -1}$$

4. $y = x^2 - 6x + 9$

Step 1: $a = 1$, no factoring needed.
Step 2: Coefficient of x is -6.

$$\left(\frac{-6}{2}\right)^2 = 9$$

$$y = x^2 - 6x + 9 - 9 + 9$$

Step 3:

$$y = (x-3)^2 + 0$$

$$y = (x-3)^2$$

Step 4: Vertex $(3, 0)$, axis $x = 3$, opens **upward**, minimum.

Note: $k = 0$ means the vertex sits exactly on the x-axis. This parabola touches the x-axis at exactly one point (a repeated root).

$$\boxed{y = (x-3)^2, \quad \text{Vertex } (3, 0), \quad x = 3}$$

5. $y = -x^2 + 6x - 5$
Step 1: $a = -1$. Factor -1 out of the first two terms.

$$y = -(x^2 - 6x) - 5$$

Step 2: Coefficient of x inside the parentheses is -6.

$$\left(\frac{-6}{2}\right)^2 = 9$$

Add 9 inside. The factor outside is -1, so we added $(-1)(9) = -9$ to the equation. Subtract -9 (add 9) outside.

$$y = -(x^2 - 6x + 9) - 5 + 9$$

Step 3:

$$y = -(x-3)^2 + 4$$

Step 4: Vertex $(3, 4)$, axis $x = 3$, $a = -1 < 0$ opens **downward**, maximum.

$$\boxed{y = -(x-3)^2 + 4, \quad \text{Vertex } (3, 4), \quad x = 3}$$

6. $y = 2x^2 - 4x + 1$
Step 1: $a = 2$. Factor 2 out of the first two terms.

$$y = 2(x^2 - 2x) + 1$$

Step 2: Coefficient of x inside is -2.

$$\left(\frac{-2}{2}\right)^2 = 1$$

Add 1 inside. We added $2 \times 1 = 2$ to the equation. Subtract 2 outside.

$$y = 2(x^2 - 2x + 1) - 2 + 1$$

Step 3:

$$y = 2(x - 1)^2 - 1$$

Step 4: Vertex $(1, -1)$, axis $x = 1$, opens **upward**, narrower than $y = x^2$, minimum.

$$\boxed{y = 2(x - 1)^2 - 1, \quad \text{Vertex } (1, -1), \quad x = 1}$$

7. $y = -2x^2 - 8x - 3$
Step 1: $a = -2$. Factor -2 out of the first two terms.

$$y = -2(x^2 + 4x) - 3$$

Step 2: Coefficient of x inside is 4.

$$\left(\frac{4}{2}\right)^2 = 4$$

Add 4 inside. We added $(-2)(4) = -8$ to the equation. Subtract -8 (add 8) outside.

$$y = -2(x^2 + 4x + 4) - 3 + 8$$

Step 3:

$$y = -2(x + 2)^2 + 5$$

Step 4: Vertex $(-2, 5)$, axis $x = -2$, $a = -2 < 0$ opens **downward**, narrower than $y = x^2$, maximum.

$$\boxed{y = -2(x + 2)^2 + 5, \quad \text{Vertex } (-2, 5), \quad x = -2}$$

8. $y = 3x^2 + 12x + 7$
Step 1: $a = 3$. Factor 3 out of the first two terms.

$$y = 3(x^2 + 4x) + 7$$

Step 2: Coefficient of x inside is 4.

$$\left(\frac{4}{2}\right)^2 = 4$$

Add 4 inside. We added $3 \times 4 = 12$ to the equation. Subtract 12 outside.

$$y = 3(x^2 + 4x + 4) - 12 + 7$$

Step 3:

$$y = 3(x + 2)^2 - 5$$

Step 4: Vertex $(-2,-5)$, axis $x=-2$, opens **upward**, narrower than $y=x^2$, minimum.

$$\boxed{y = 3(x+2)^2 - 5, \quad \text{Vertex } (-2,-5), \quad x=-2}$$

9. (Challenge) $y = -\dfrac{1}{2}x^2 + 2x + 1$
Step 1: $a = -\frac{1}{2}$. Factor $-\frac{1}{2}$ out of the first two terms.

$$y = -\frac{1}{2}(x^2 - 4x) + 1$$

Step 2: Coefficient of x inside is -4.

$$\left(\frac{-4}{2}\right)^2 = 4$$

Add 4 inside. We added $\left(-\frac{1}{2}\right)(4) = -2$ to the equation. Subtract -2 (add 2) outside.

$$y = -\frac{1}{2}(x^2 - 4x + 4) + 1 + 2$$

Step 3:

$$y = -\frac{1}{2}(x-2)^2 + 3$$

Step 4: Vertex $(2,3)$, axis $x=2$, $a=-\frac{1}{2}<0$ opens **downward**, wider than $y=x^2$, maximum.

$$\boxed{y = -\tfrac{1}{2}(x-2)^2 + 3, \quad \text{Vertex } (2,3), \quad x=2}$$

10. (Challenge) $y = 4x^2 - 16x + 19$
Step 1: $a=4$. Factor 4 out of the first two terms.

$$y = 4(x^2 - 4x) + 19$$

Step 2: Coefficient of x inside is -4.

$$\left(\frac{-4}{2}\right)^2 = 4$$

Add 4 inside. We added $4 \times 4 = 16$ to the equation. Subtract 16 outside.

$$y = 4(x^2 - 4x + 4) - 16 + 19$$

Step 3:

$$y = 4(x-2)^2 + 3$$

Step 4: Vertex $(2,3)$, axis $x=2$, opens **upward** ($a=4>0$), very **narrow**, minimum.

Number of x-intercepts: To find x-intercepts, set $y = 0$:

$$0 = 4(x-2)^2 + 3$$

$$4(x-2)^2 = -3$$

$$(x-2)^2 = -\frac{3}{4}$$

A squared expression can never be negative. There is **no real solution**, so the parabola has **no x-intercepts**. This makes sense: the vertex is at $(2, 3)$, which is above the x-axis, and the parabola opens upward, so it never crosses down to $y = 0$.

$$\boxed{y = 4(x-2)^2 + 3, \quad \text{Vertex } (2,3), \quad \text{no } x\text{-intercepts}}$$

Thank you for learning with ApexMath!

If you found this lesson helpful, we'd love for you to share your feedback or leave a review.

End of Lesson • ApexMath
