Graphing Quadratics in Vertex Form
ApexMath

Notes

The Vertex Form of a Quadratic

Every parabola can be written in **vertex form**:

$$\boxed{y = a(x-h)^2 + k}$$

This form is powerful because all the key features of the parabola are visible directly from the equation, without any extra calculation.

What Each Part of the Formula Means

- **The vertex** is the point (h, k). This is the highest or lowest point on the parabola.

  Important: Notice the subtraction sign in $(x-h)^2$. If the equation reads $y = a(x-3)^2 + 5$, the vertex is $(3, 5)$, not $(-3, 5)$. If the equation reads $y = a(x+3)^2 + 5$, rewrite it as $y = a(x-(-3))^2 + 5$, so the vertex is $(-3, 5)$.

- **The axis of symmetry** is the vertical line $x = h$. The parabola is a perfect mirror image on both sides of this line.

- **The value of** a controls two things:
  - *Direction of opening:* If $a > 0$, the parabola opens **upward** (U-shape). If $a < 0$, it opens **downward** (n-shape).
  - *Width:* If $|a| > 1$, the parabola is **narrower** than $y = x^2$. If $|a| < 1$, it is **wider** than $y = x^2$.

A Quick Summary Table

Feature	How to Find It
Vertex	(h, k) read directly from the equation
Axis of symmetry	$x = h$
Direction of opening	$a > 0$: upward $a < 0$: downward
Maximum or minimum	Minimum if $a > 0$; maximum if $a < 0$

How to Sketch the Parabola

Once you have the vertex and know which direction the parabola opens, you can sketch it using the following steps:

1. Plot the vertex (h, k).
2. Draw the axis of symmetry as a dashed vertical line at $x = h$.
3. Plug one or two x-values on either side of h into the equation to get extra points.
4. Use symmetry: each point you found has a mirror image on the other side of the axis.
5. Connect the points with a smooth U-shaped (or n-shaped) curve.

Pro Tip

- Read the vertex directly from the equation as (h, k). Do not substitute $x = 0$ or do any algebra. The whole point of vertex form is that the vertex is already visible.
- Watch the sign of h. The formula has $(x - h)^2$, so the h value is opposite in sign to what you see written. If you see $(x + 4)^2$, that is $(x - (-4))^2$, so $h = -4$.
- Check the sign of a before sketching. Getting the direction of opening wrong will make the entire graph incorrect.
- When finding extra points to plot, choose x-values that are exactly 1 and 2 units away from the vertex. This keeps the arithmetic simple and gives you a symmetric pair of points automatically.
- Remember: the vertex is the minimum when $a > 0$ and the maximum when $a < 0$. This helps you describe the range of the function.

Examples

Example 1: Graph $y = (x - 2)^2 + 1$.

Step 1: Identify a, h, and k.

$$a = 1, \quad h = 2, \quad k = 1$$

Step 2: State the key features.

The vertex is $(2, 1)$. The axis of symmetry is $x = 2$. Since $a = 1 > 0$, the parabola opens **upward**. The vertex is a **minimum**.

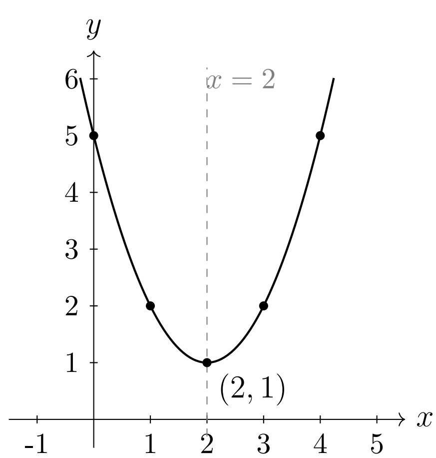
Step 3: Find two extra points by plugging in x-values near the vertex.

At $x = 3$: $\quad y = (3 - 2)^2 + 1 = 1 + 1 = 2$, giving point $(3, 2)$.

At $x = 4$: $\quad y = (4 - 2)^2 + 1 = 4 + 1 = 5$, giving point $(4, 5)$.

By symmetry, $(1, 2)$ and $(0, 5)$ are also on the graph.

Step 4: Sketch.



Example 2: Graph $y = -(x+1)^2 + 4$.

Step 1: Rewrite to identify h clearly.

$$y = -(x - (-1))^2 + 4$$

$$a = -1, \quad h = -1, \quad k = 4$$

Step 2: State the key features.

The vertex is $(-1, 4)$. The axis of symmetry is $x = -1$. Since $a = -1 < 0$, the parabola opens **downward**. The vertex is a **maximum**.

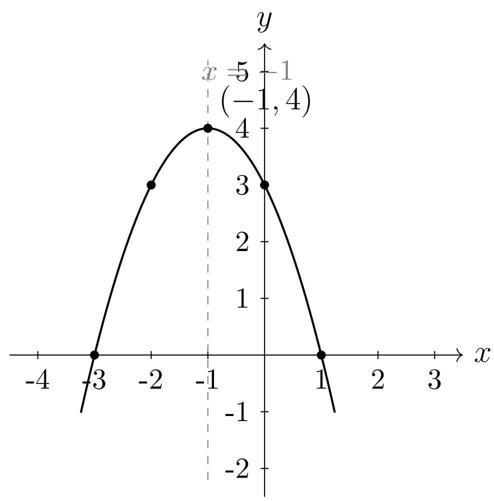
Step 3: Find extra points.

At $x = 0$: $\quad y = -(0+1)^2 + 4 = -1 + 4 = 3$, giving point $(0, 3)$.

At $x = 1$: $\quad y = -(1+1)^2 + 4 = -4 + 4 = 0$, giving point $(1, 0)$.

By symmetry, $(-2, 3)$ and $(-3, 0)$ are also on the graph.

Step 4: Sketch.



Example 3: Graph $y = 2(x-1)^2 - 3$.

Step 1: Identify a, h, and k.

$$a = 2, \quad h = 1, \quad k = -3$$

Step 2: State the key features.

The vertex is $(1, -3)$. The axis of symmetry is $x = 1$. Since $a = 2 > 0$, the parabola opens **upward**. Because $|a| = 2 > 1$, it is **narrower** than $y = x^2$. The vertex is a **minimum**.

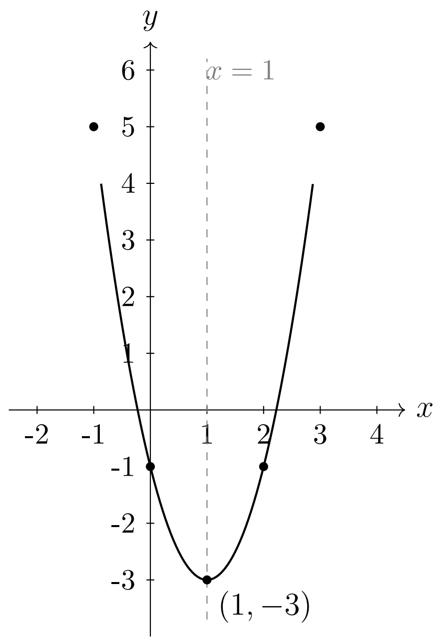
Step 3: Find extra points.

At $x = 2$: $\quad y = 2(2-1)^2 - 3 = 2(1) - 3 = -1$, giving $(2, -1)$.

At $x = 3$: $\quad y = 2(3-1)^2 - 3 = 2(4) - 3 = 5$, giving $(3, 5)$.

By symmetry, $(0, -1)$ and $(-1, 5)$ are also on the graph.

Step 4: Sketch.



Example 4: Graph $y = -\dfrac{1}{2}(x-3)^2 + 2$.

Step 1: Identify a, h, and k.

$$a = -\tfrac{1}{2}, \quad h = 3, \quad k = 2$$

Step 2: State the key features.

The vertex is $(3, 2)$. The axis of symmetry is $x = 3$. Since $a = -\frac{1}{2} < 0$, the parabola opens **downward**. Because $|a| = \frac{1}{2} < 1$, it is **wider** than $y = x^2$. The vertex is a **maximum**.

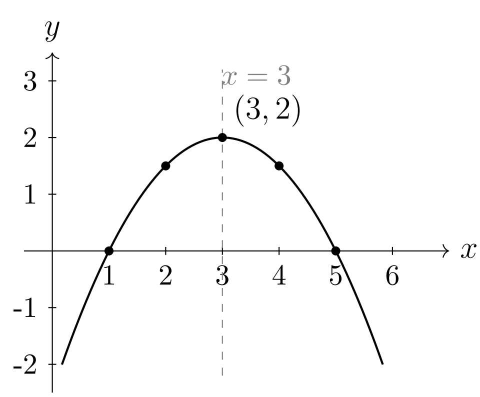
Step 3: Find extra points.

At $x = 5$: $\quad y = -\frac{1}{2}(5-3)^2 + 2 = -\frac{1}{2}(4) + 2 = -2 + 2 = 0$, giving $(5, 0)$.

At $x = 1$: $\quad y = -\frac{1}{2}(1-3)^2 + 2 = -\frac{1}{2}(4) + 2 = 0$, giving $(1, 0)$.

By symmetry these two points confirm each other. Plug in $x = 4$ for one more: $y = -\frac{1}{2}(1) + 2 = 1.5$, giving $(4, 1.5)$. By symmetry, $(2, 1.5)$.

Step 4: Sketch.



Common Mistakes

- **Getting the sign of h wrong:** The vertex form is $y = a(x - h)^2 + k$. If you see $(x + 3)^2$, that is the same as $(x - (-3))^2$, so $h = -3$, not $+3$. Always rewrite using a minus sign if needed to identify h correctly.

- **Forgetting the vertex is (h, k), not (k, h):** The x-coordinate of the vertex comes from the expression inside the parentheses (h), and the y-coordinate is the constant added outside (k). Do not swap them.

- **Ignoring the sign of a:** If a is negative, the parabola opens downward. Drawing it opening upward when $a < 0$ is one of the most common graphing errors.

- **Not using symmetry:** Every point you plot has a mirror image across the axis of symmetry. Using this symmetry saves work and makes your graph more accurate.

- **Plotting the axis of symmetry in the wrong place:** The axis is always a vertical line at $x = h$, not at $x = k$ or anywhere else. Drawing it horizontal is also a common error — the axis of symmetry is always *vertical*.

Worksheet

For each problem below, identify the vertex, axis of symmetry, direction of opening, and whether the vertex is a maximum or minimum. Then sketch the graph.

1. $y = (x - 4)^2 + 2$

2. $y = (x + 3)^2 - 1$

3. $y = -(x-1)^2 + 5$

4. $y = 3(x+2)^2 - 4$

5. $y = -2(x-3)^2 + 1$

6. $y = \frac{1}{3}(x+1)^2 - 2$

7. $y = -(x+4)^2$

8. $y = (x-2)^2$

9. (Challenge) $y = -\frac{1}{4}(x-1)^2 + 3$. State the vertex, axis of symmetry, direction of opening, and find the x-intercepts (where $y = 0$).

10. (Challenge) A parabola has vertex $(-2, 5)$, opens downward, and passes through the point $(0, 1)$. Write its equation in vertex form.

Answer Key

For problems 1–8, the answers follow the same pattern. Full steps are shown for problems 1, 3, 4, and 5. The remaining answers are summarized since the method is identical.

1. $y = (x-4)^2 + 2$
Step 1: Identify a, h, k.
$$a = 1, \quad h = 4, \quad k = 2$$
Step 2: State the features.
Vertex: $(4,2)$. Axis of symmetry: $x = 4$. $a = 1 > 0$, so the parabola opens **upward**. The vertex is a **minimum**.
Step 3: Find two extra points.
At $x = 5$: $y = (5-4)^2 + 2 = 1 + 2 = 3$. Point: $(5,3)$.
At $x = 6$: $y = (6-4)^2 + 2 = 4 + 2 = 6$. Point: $(6,6)$.
By symmetry: $(3,3)$ and $(2,6)$.
$$\boxed{\text{Vertex } (4,2),\ x = 4,\ \text{opens upward, minimum}}$$

2. $y = (x+3)^2 - 1$
Step 1: Rewrite: $y = (x-(-3))^2 - 1$. So $a = 1$, $h = -3$, $k = -1$.
Step 2: Vertex $(-3,-1)$, axis $x = -3$, opens **upward**, minimum.
$$\boxed{\text{Vertex } (-3,-1),\ x = -3,\ \text{opens upward, minimum}}$$

3. $y = -(x-1)^2 + 5$
Step 1: Identify a, h, k.
$$a = -1, \quad h = 1, \quad k = 5$$
Step 2: State the features.
Vertex: $(1,5)$. Axis of symmetry: $x = 1$. $a = -1 < 0$, so the parabola opens **downward**. The vertex is a **maximum**.
Step 3: Find extra points.
At $x = 2$: $y = -(2-1)^2 + 5 = -1 + 5 = 4$. Point: $(2,4)$.
At $x = 3$: $y = -(3-1)^2 + 5 = -4 + 5 = 1$. Point: $(3,1)$.
By symmetry: $(0,4)$ and $(-1,1)$.
$$\boxed{\text{Vertex } (1,5),\ x = 1,\ \text{opens downward, maximum}}$$

4. $y = 3(x+2)^2 - 4$
Step 1: Rewrite: $y = 3(x-(-2))^2 - 4$. So $a = 3$, $h = -2$, $k = -4$.
Step 2: Vertex $(-2,-4)$, axis $x = -2$, $a = 3 > 0$ so opens **upward**. Since $|a| = 3 > 1$, it is **narrower** than $y = x^2$. The vertex is a **minimum**.
Step 3: Find extra points.
At $x = -1$: $y = 3(-1+2)^2 - 4 = 3(1) - 4 = -1$. Point: $(-1,-1)$.

At $x = 0$: $y = 3(0+2)^2 - 4 = 3(4) - 4 = 8$. Point: $(0, 8)$.
By symmetry: $(-3, -1)$ and $(-4, 8)$.

$$\boxed{\text{Vertex } (-2, -4),\ x = -2,\ \text{opens upward, minimum}}$$

5. $y = -2(x-3)^2 + 1$
Step 1: Identify $a = -2$, $h = 3$, $k = 1$.
Step 2: Vertex $(3, 1)$, axis $x = 3$, $a = -2 < 0$ so opens **downward**. Since $|a| = 2 > 1$, it is **narrower** than $y = x^2$. The vertex is a **maximum**.
Step 3: Find extra points.
At $x = 4$: $y = -2(4-3)^2 + 1 = -2(1) + 1 = -1$. Point: $(4, -1)$.
At $x = 2$: by symmetry, $(2, -1)$.

$$\boxed{\text{Vertex } (3, 1),\ x = 3,\ \text{opens downward, maximum}}$$

6. $y = \dfrac{1}{3}(x+1)^2 - 2$
Step 1: $a = \frac{1}{3}$, $h = -1$, $k = -2$.
Step 2: Vertex $(-1, -2)$, axis $x = -1$, opens **upward**. Since $|a| = \frac{1}{3} < 1$, it is **wider** than $y = x^2$. Minimum.

$$\boxed{\text{Vertex } (-1, -2),\ x = -1,\ \text{opens upward, minimum}}$$

7. $y = -(x+4)^2$
Step 1: Rewrite: $y = -(x - (-4))^2 + 0$. So $a = -1$, $h = -4$, $k = 0$.
Step 2: Vertex $(-4, 0)$, axis $x = -4$, opens **downward**. Maximum.
Note: $k = 0$ means the vertex sits exactly on the x-axis.

$$\boxed{\text{Vertex } (-4, 0),\ x = -4,\ \text{opens downward, maximum}}$$

8. $y = (x-2)^2$
Step 1: Rewrite: $y = (x-2)^2 + 0$. So $a = 1$, $h = 2$, $k = 0$.
Step 2: Vertex $(2, 0)$, axis $x = 2$, opens **upward**. Minimum.
Note: $k = 0$ means the vertex sits exactly on the x-axis.

$$\boxed{\text{Vertex } (2, 0),\ x = 2,\ \text{opens upward, minimum}}$$

9. (Challenge) $y = -\dfrac{1}{4}(x-1)^2 + 3$
Step 1: Identify $a = -\frac{1}{4}$, $h = 1$, $k = 3$.
Step 2: Vertex $(1, 3)$, axis $x = 1$, opens **downward** ($a < 0$). Since $|a| = \frac{1}{4} < 1$, it is **wider** than $y = x^2$. Maximum.

Step 3: Find the x-intercepts. Set $y = 0$ and solve.

$$0 = -\frac{1}{4}(x-1)^2 + 3$$

Move the 3 to the left side:

$$-3 = -\frac{1}{4}(x-1)^2$$

Multiply both sides by -4:

$$12 = (x-1)^2$$

Take the square root of both sides:

$$x - 1 = \pm\sqrt{12} = \pm 2\sqrt{3}$$

Add 1 to both sides:

$$x = 1 + 2\sqrt{3} \approx 4.46 \qquad \text{or} \qquad x = 1 - 2\sqrt{3} \approx -2.46$$

$$\boxed{\text{Vertex } (1,3),\ x = 1,\ \text{opens downward; } x\text{-intercepts: } x = 1 \pm 2\sqrt{3}}$$

10. (Challenge) A parabola has vertex $(-2, 5)$, opens downward, and passes through $(0, 1)$. Find the equation in vertex form.

Step 1: Since the vertex is $(-2, 5)$, we know $h = -2$ and $k = 5$. Write the general vertex form with these values substituted:

$$y = a(x - (-2))^2 + 5 = a(x+2)^2 + 5$$

Step 2: The parabola opens downward, so we already know $a < 0$. Use the given point $(0, 1)$ to find the exact value of a. Substitute $x = 0$, $y = 1$:

$$1 = a(0+2)^2 + 5$$

$$1 = 4a + 5$$

Subtract 5 from both sides:

$$-4 = 4a$$

Divide both sides by 4:

$$a = -1$$

Step 3: Confirm $a = -1 < 0$. Yes, the parabola does open downward. Write the final equation.

$$y = -(x+2)^2 + 5$$

$$\boxed{y = -(x+2)^2 + 5}$$

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