Solving Quadratic Equations by Factoring
ApexMath

Notes

The Big Idea: Setting Up a Zero-Product Equation

A quadratic equation has the form:

$$ax^2 + bx + c = 0$$

The goal is to find the values of x that make this equation true. One of the most reliable methods is **factoring**.

The strategy rests on a single rule called the **Zero Product Property**:

$$\boxed{\text{If } A \cdot B = 0, \text{ then } A = 0 \text{ or } B = 0.}$$

In other words, if two factors multiply to zero, then at least one of them must equal zero. This is the key that lets us find solutions.

The Step-by-Step Process

1. Move all terms to one side so the equation equals zero.
2. Factor the quadratic expression completely.
3. Set each factor equal to zero.
4. Solve each resulting equation for x.

These four steps apply every time. Do not skip Step 1 — the equation *must* equal zero before you can use the Zero Product Property.

Types of Factoring You May Need

- **Common factor:** Always check for a GCF first. Example: $2x^2 + 6x = 2x(x + 3)$
- **Trinomial ($a = 1$):** Find two numbers that multiply to c and add to b. Example: $x^2 + 5x + 6 = (x + 2)(x + 3)$
- **Trinomial ($a \neq 1$):** Use AC method or trial and error. Example: $2x^2 + 5x + 3 = (2x + 3)(x + 1)$
- **Difference of squares:** $a^2 - b^2 = (a + b)(a - b)$. Example: $x^2 - 9 = (x + 3)(x - 3)$
- **Perfect square trinomial:** $a^2 \pm 2ab + b^2 = (a \pm b)^2$. Example: $x^2 + 6x + 9 = (x + 3)^2$

Pro Tip

- Always move everything to one side first. The equation must equal zero. For example, do not try to factor $x^2 + 3x = 10$ directly — subtract 10 first to get $x^2 + 3x - 10 = 0$.
- Check your GCF first. If every term shares a common factor, pull it out before anything else. Missing the GCF makes factoring much harder.
- After you set each factor equal to zero and solve, always check your answers by substituting back into the original equation.
- If a factor appears twice, like $(x - 3)^2 = 0$, then $x = 3$ is a *repeated root*. You still write it as one solution, not two different ones.

Examples

Example 1: Solve $x^2 + 5x + 6 = 0$.

Step 1: The equation is already set equal to zero. Proceed to factor.
Step 2: Find two numbers that multiply to 6 and add to 5. Those numbers are 2 and 3 because $2 \times 3 = 6$ and $2 + 3 = 5$.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Step 3: Set each factor equal to zero.

$$x + 2 = 0 \qquad \text{or} \qquad x + 3 = 0$$

Step 4: Solve each equation.

$$x = -2 \qquad \text{or} \qquad x = -3$$

Example 2: Solve $x^2 - 9 = 0$.

Step 1: The equation is already set equal to zero.
Step 2: Recognize the difference of squares pattern: $x^2 - 9 = x^2 - 3^2$.

$$x^2 - 9 = (x + 3)(x - 3)$$

Step 3: Set each factor equal to zero.

$$x + 3 = 0 \qquad \text{or} \qquad x - 3 = 0$$

Step 4: Solve.

$$x = -3 \qquad \text{or} \qquad x = 3$$

Example 3: Solve $x^2 + 3x = 10$.

Step 1: Move all terms to one side. Subtract 10 from both sides.

$$x^2 + 3x - 10 = 0$$

Step 2: Find two numbers that multiply to -10 and add to 3. Those numbers are 5 and -2 because $5 \times (-2) = -10$ and $5 + (-2) = 3$.

$$x^2 + 3x - 10 = (x + 5)(x - 2)$$

Step 3: Set each factor equal to zero.

$$x + 5 = 0 \qquad \text{or} \qquad x - 2 = 0$$

Step 4: Solve.

$$x = -5 \qquad \text{or} \qquad x = 2$$

Example 4: Solve $2x^2 + 7x + 3 = 0$.

Step 1: The equation is already set equal to zero.
Step 2: Factor the trinomial where $a = 2$, $b = 7$, $c = 3$.
Multiply $a \cdot c = 2 \cdot 3 = 6$. Find two numbers that multiply to 6 and add to 7. Those numbers are 6 and 1.
Rewrite the middle term:

$$2x^2 + 6x + x + 3$$

Group and factor:

$$2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$$

Step 3: Set each factor equal to zero.

$$2x + 1 = 0 \qquad \text{or} \qquad x + 3 = 0$$

Step 4: Solve.

$$2x = -1 \quad \Longrightarrow \quad x = -\frac{1}{2} \qquad \text{or} \qquad x = -3$$

Example 5: Solve $3x^2 - 12x = 0$.

Step 1: The equation is already set equal to zero.
Step 2: Factor out the GCF. Both terms share a factor of $3x$.

$$3x^2 - 12x = 3x(x - 4)$$

Step 3: Set each factor equal to zero.

$$3x = 0 \qquad \text{or} \qquad x - 4 = 0$$

Step 4: Solve.

$$x = 0 \qquad \text{or} \qquad x = 4$$

Common Mistakes

- **Not setting the equation equal to zero first:** The Zero Product Property only works when the product equals zero. If you factor before moving all terms to one side, your solutions will be wrong.
- **Cancelling x from both sides:** If you see $x^2 = 5x$ and divide both sides by x, you lose the solution $x = 0$. Always move terms to one side and factor instead.
- **Forgetting to check for a GCF:** If every term has a common factor and you skip it, your remaining trinomial will be harder to factor and may look like it has no solution.
- **Misidentifying the signs:** When finding two numbers that multiply to c and add to b, watch the signs carefully. A positive product with a negative sum means both numbers are negative.
- **Stopping after factoring:** Some students factor correctly but forget to set each factor equal to zero and solve. Factoring alone is not the final answer — you must use the Zero Product Property to find the values of x.

Worksheet

1. Solve: $x^2 + 7x + 12 = 0$

2. Solve: $x^2 - 5x + 6 = 0$

3. Solve: $x^2 - 16 = 0$

4. Solve: $x^2 + 6x + 9 = 0$

5. Solve: $x^2 + 2x = 8$

6. Solve: $x^2 - x = 12$

7. Solve: $4x^2 - 16x = 0$

8. Solve: $2x^2 - x - 3 = 0$

9. Solve: $3x^2 + 10x - 8 = 0$

10. (Challenge) Solve: $x^2 - 6x + 9 = 0$. How many distinct solutions does this equation have, and why?

Answer Key

1. Solve: $x^2 + 7x + 12 = 0$

Step 1: The equation equals zero. Ready to factor.

Step 2: Find two numbers that multiply to 12 and add to 7. Those numbers are 3 and 4 because $3 \times 4 = 12$ and $3 + 4 = 7$.

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

Step 3: Set each factor equal to zero.

$$x + 3 = 0 \qquad \text{or} \qquad x + 4 = 0$$

Step 4: Solve.

$$x = -3 \qquad \text{or} \qquad x = -4$$

$$\boxed{x = -3 \quad \text{or} \quad x = -4}$$

2. Solve: $x^2 - 5x + 6 = 0$

Step 1: The equation equals zero. Ready to factor.

Step 2: Find two numbers that multiply to 6 and add to -5. Both numbers must be negative. Those numbers are -2 and -3 because $(-2)(-3) = 6$ and $(-2) + (-3) = -5$.

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

Step 3: Set each factor equal to zero.

$$x - 2 = 0 \qquad \text{or} \qquad x - 3 = 0$$

Step 4: Solve.

$$x = 2 \qquad \text{or} \qquad x = 3$$

$$\boxed{x = 2 \quad \text{or} \quad x = 3}$$

3. Solve: $x^2 - 16 = 0$
Step 1: The equation equals zero.
Step 2: Recognize the difference of squares pattern: $x^2 - 16 = x^2 - 4^2$.

$$x^2 - 16 = (x + 4)(x - 4)$$

Step 3: Set each factor equal to zero.

$$x + 4 = 0 \qquad \text{or} \qquad x - 4 = 0$$

Step 4: Solve.

$$x = -4 \qquad \text{or} \qquad x = 4$$

$$\boxed{x = -4 \quad \text{or} \quad x = 4}$$

4. Solve: $x^2 + 6x + 9 = 0$
Step 1: The equation equals zero.
Step 2: Recognize the perfect square trinomial pattern. Check: $(\sqrt{9})^2 = 3$ and $2 \cdot 1 \cdot 3 = 6$. Yes, this fits.

$$x^2 + 6x + 9 = (x + 3)^2$$

Step 3: Set the factor equal to zero.

$$(x + 3)^2 = 0 \qquad \Longrightarrow \qquad x + 3 = 0$$

Step 4: Solve.

$$x = -3$$

This is a repeated root. There is only one distinct solution.

$$\boxed{x = -3}$$

5. Solve: $x^2 + 2x = 8$
Step 1: Move all terms to one side. Subtract 8 from both sides.

$$x^2 + 2x - 8 = 0$$

Step 2: Find two numbers that multiply to -8 and add to 2. Those numbers are 4 and -2 because $4 \times (-2) = -8$ and $4 + (-2) = 2$.

$$x^2 + 2x - 8 = (x+4)(x-2)$$

Step 3: Set each factor equal to zero.

$$x + 4 = 0 \qquad \text{or} \qquad x - 2 = 0$$

Step 4: Solve.

$$x = -4 \qquad \text{or} \qquad x = 2$$

$$\boxed{x = -4 \quad \text{or} \quad x = 2}$$

6. Solve: $x^2 - x = 12$
Step 1: Move all terms to one side. Subtract 12 from both sides.

$$x^2 - x - 12 = 0$$

Step 2: Find two numbers that multiply to -12 and add to -1. Those numbers are -4 and 3 because $(-4)(3) = -12$ and $(-4) + 3 = -1$.

$$x^2 - x - 12 = (x-4)(x+3)$$

Step 3: Set each factor equal to zero.

$$x - 4 = 0 \qquad \text{or} \qquad x + 3 = 0$$

Step 4: Solve.

$$x = 4 \qquad \text{or} \qquad x = -3$$

$$\boxed{x = 4 \quad \text{or} \quad x = -3}$$

7. Solve: $4x^2 - 16x = 0$
Step 1: The equation equals zero. Check for a GCF first.
Step 2: Both terms share a factor of $4x$. Factor it out.

$$4x^2 - 16x = 4x(x-4)$$

Step 3: Set each factor equal to zero.

$$4x = 0 \qquad \text{or} \qquad x - 4 = 0$$

Step 4: Solve.

$$x = 0 \qquad \text{or} \qquad x = 4$$

$$\boxed{x = 0 \quad \text{or} \quad x = 4}$$

8. Solve: $2x^2 - x - 3 = 0$

Step 1: The equation equals zero.

Step 2: Factor the trinomial where $a = 2$, $b = -1$, $c = -3$.

Multiply $a \cdot c = 2 \cdot (-3) = -6$. Find two numbers that multiply to -6 and add to -1. Those numbers are -3 and 2.

Rewrite the middle term:

$$2x^2 - 3x + 2x - 3$$

Group and factor:

$$x(2x - 3) + 1(2x - 3) = (x + 1)(2x - 3)$$

Step 3: Set each factor equal to zero.

$$x + 1 = 0 \qquad \text{or} \qquad 2x - 3 = 0$$

Step 4: Solve.

$$x = -1 \qquad \text{or} \qquad 2x = 3 \quad \Longrightarrow \quad x = \frac{3}{2}$$

$$\boxed{x = -1 \quad \text{or} \quad x = \frac{3}{2}}$$

9. Solve: $3x^2 + 10x - 8 = 0$

Step 1: The equation equals zero.

Step 2: Factor the trinomial where $a = 3$, $b = 10$, $c = -8$.

Multiply $a \cdot c = 3 \cdot (-8) = -24$. Find two numbers that multiply to -24 and add to 10. Those numbers are 12 and -2.

Rewrite the middle term:

$$3x^2 + 12x - 2x - 8$$

Group and factor:

$$3x(x + 4) - 2(x + 4) = (3x - 2)(x + 4)$$

Step 3: Set each factor equal to zero.

$$3x - 2 = 0 \qquad \text{or} \qquad x + 4 = 0$$

Step 4: Solve.

$$3x = 2 \quad \Longrightarrow \quad x = \frac{2}{3} \qquad \text{or} \qquad x = -4$$

$$\boxed{x = \frac{2}{3} \quad \text{or} \quad x = -4}$$

10. (Challenge) Solve: $x^2 - 6x + 9 = 0$.

Step 1: The equation equals zero.

Step 2: Recognize the perfect square trinomial. Check: $(\sqrt{9})^2 = 3$ and $2 \cdot 1 \cdot 3 = 6$. The pattern fits.

$$x^2 - 6x + 9 = (x - 3)^2$$

Step 3: Set the factor equal to zero.

$$(x - 3)^2 = 0 \qquad \Longrightarrow \qquad x - 3 = 0$$

Step 4: Solve.

$$x = 3$$

There is only **one distinct solution**: $x = 3$. This is called a *repeated root* or *double root*. The graph of $y = x^2 - 6x + 9$ touches the x-axis at exactly one point — the vertex — rather than crossing through it at two separate points.

$$\boxed{x = 3 \text{ (repeated root)}}$$

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