Unit Review: Quadratic Equations and Functions
ApexMath

This review covers every topic from the unit. Work through each section in order. If you get stuck, return to the lesson for that topic before checking the answer key.

Topics covered: Factoring · Solving by Factoring · Vertex Form · Standard Form · Completing the Square · The Quadratic Formula · Writing Equations from Graphs · Applications

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Part A — Factoring Review

Factor each expression completely.

1. $3x^2 + 9x$

2. $x^2 + 9x + 20$

3. $x^2 - 7x + 12$

4. $x^2 - 25$

5. $x^2 + 8x + 16$

6. $2x^2 + 7x + 3$

Part B — Solving by Factoring

Solve each equation by factoring.

7. $x^2 + x - 12 = 0$

8. $2x^2 - 5x - 3 = 0$

9. $x^2 = 4x$

10. $3x^2 + 6x = 0$

Part C — Vertex Form

For each equation in vertex form, state the vertex, axis of symmetry, direction of opening, and whether the vertex is a maximum or minimum.

11. $y = (x-3)^2 + 4$

12. $y = -2(x+1)^2 - 3$

13. $y = \frac{1}{4}(x-5)^2 - 2$

Part D — Completing the Square

Complete the square to write each expression in the form $a(x-h)^2 + k$.

14. $x^2 + 10x + 3$

15. $x^2 - 6x - 4$

16. $2x^2 + 12x + 7$

17. $-3x^2 + 12x - 1$

Part E — Graphing in Standard Form

Convert to vertex form, state the key features, and sketch the graph.

18. $y = x^2 - 4x - 5$

19. $y = -x^2 + 2x + 3$

20. $y = 2x^2 + 8x + 6$

Part F — The Quadratic Formula

Solve using the quadratic formula. Simplify all answers fully.

21. $x^2 + 3x - 10 = 0$

22. $2x^2 - 3x - 2 = 0$

23. $x^2 - 4x + 1 = 0$

24. $3x^2 + x + 1 = 0$

Part G — Writing Equations from Graphs

25. A parabola has vertex $(2, -5)$ and passes through $(4, 3)$. Write its equation in vertex form.

26. A parabola has x-intercepts at $x = -3$ and $x = 5$, and passes through $(0, -15)$. Write its equation in standard form.

27. A parabola has vertex $(-1, 6)$ and an x-intercept at $x = 1$. Write its equation in vertex form.

Part H — Applications

28. A rectangle has a length that is 5 m more than its width. The area is 84 m^2. Find the dimensions.

29. A ball is thrown upward from the ground with an initial velocity of 28 m/s. Its height is $h(t) = -4.9t^2 + 28t$. Find the maximum height and the time at which it occurs. Round to two decimal places.

30. A store sells 400 units per week at \$30 each. For every \$4 increase in price, 20 fewer units are sold. Find the price that maximises weekly revenue and state that maximum revenue.

31. (Challenge) A ball is thrown upward from a building 20 m high with an initial velocity of 15 m/s. Its height is $h(t) = -4.9t^2 + 15t + 20$. When does the ball hit the ground? Round to two decimal places.

Answer Key

Part A — Factoring

1. $3x^2 + 9x$

Factor out the GCF of $3x$:

$$3x^2 + 9x = 3x(x + 3)$$

$$\boxed{3x(x + 3)}$$

2. $x^2 + 9x + 20$

Find two numbers that multiply to 20 and add to 9: they are 4 and 5.

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

$$\boxed{(x + 4)(x + 5)}$$

3. $x^2 - 7x + 12$

Find two numbers that multiply to 12 and add to -7: they are -3 and -4.

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

$$\boxed{(x - 3)(x - 4)}$$

4. $x^2 - 25$

Difference of squares: $x^2 - 25 = x^2 - 5^2$.

$$x^2 - 25 = (x + 5)(x - 5)$$

$$\boxed{(x + 5)(x - 5)}$$

5. $x^2 + 8x + 16$

Perfect square trinomial: $\left(\dfrac{8}{2}\right)^2 = 16$. ✓

$$x^2 + 8x + 16 = (x + 4)^2$$

$$\boxed{(x + 4)^2}$$

6. $2x^2 + 7x + 3$

$a \cdot c = 2 \times 3 = 6$. Find two numbers that multiply to 6 and add to 7: they are 6 and 1. Rewrite and factor by grouping:

$$2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$$

$$\boxed{(2x + 1)(x + 3)}$$

Part B — Solving by Factoring

7. $x^2 + x - 12 = 0$

Find two numbers that multiply to -12 and add to 1: they are 4 and -3.

$$(x+4)(x-3) = 0$$

$$x = -4 \qquad \text{or} \qquad x = 3$$

$$\boxed{x = -4 \quad \text{or} \quad x = 3}$$

8. $2x^2 - 5x - 3 = 0$

$a \cdot c = 2 \times (-3) = -6$. Two numbers that multiply to -6 and add to -5: they are -6 and 1.

$$2x^2 - 6x + x - 3 = 2x(x-3) + 1(x-3) = (2x+1)(x-3) = 0$$

$$x = -\frac{1}{2} \qquad \text{or} \qquad x = 3$$

$$\boxed{x = -\frac{1}{2} \quad \text{or} \quad x = 3}$$

9. $x^2 = 4x$

Move all terms to one side first:

$$x^2 - 4x = 0$$

Factor out x:

$$x(x-4) = 0$$

$$x = 0 \qquad \text{or} \qquad x = 4$$

$$\boxed{x = 0 \quad \text{or} \quad x = 4}$$

10. $3x^2 + 6x = 0$

Factor out the GCF of $3x$:

$$3x(x+2) = 0$$

$$x = 0 \qquad \text{or} \qquad x = -2$$

$$\boxed{x = 0 \quad \text{or} \quad x = -2}$$

Part C — Vertex Form

11. $y = (x-3)^2 + 4$
$a = 1,\ h = 3,\ k = 4.$
Vertex: $(3, 4)$. Axis: $x = 3$. $a > 0$: opens **upward**. **Minimum**.

$$\boxed{\text{Vertex } (3,4),\ x = 3,\ \text{opens upward, minimum}}$$

12. $y = -2(x+1)^2 - 3$
Rewrite: $y = -2(x-(-1))^2 - 3$. So $a = -2,\ h = -1,\ k = -3.$
Vertex: $(-1, -3)$. Axis: $x = -1$. $a < 0$: opens **downward**. **Maximum**.

$$\boxed{\text{Vertex } (-1,-3),\ x = -1,\ \text{opens downward, maximum}}$$

13. $y = \dfrac{1}{4}(x-5)^2 - 2$
$a = \frac{1}{4},\ h = 5,\ k = -2.$
Vertex: $(5, -2)$. Axis: $x = 5$. $a > 0$: opens **upward**, wider than $y = x^2$. **Minimum**.

$$\boxed{\text{Vertex } (5,-2),\ x = 5,\ \text{opens upward, minimum}}$$

Part D — Completing the Square

14. $x^2 + 10x + 3$
Completing value: $\left(\dfrac{10}{2}\right)^2 = 25.$

$$x^2 + 10x + 25 - 25 + 3 = (x+5)^2 - 22$$

$$\boxed{(x+5)^2 - 22}$$

15. $x^2 - 6x - 4$
Completing value: $\left(\dfrac{-6}{2}\right)^2 = 9.$

$$x^2 - 6x + 9 - 9 - 4 = (x-3)^2 - 13$$

$$\boxed{(x-3)^2 - 13}$$

16. $2x^2 + 12x + 7$
Factor 2 from first two terms:

$$2(x^2 + 6x) + 7$$

Completing value inside: $\left(\dfrac{6}{2}\right)^2 = 9.$

Compensation: $2 \times 9 = 18$. Subtract 18 outside.

$$2(x^2 + 6x + 9) - 18 + 7 = 2(x+3)^2 - 11$$

$$\boxed{2(x+3)^2 - 11}$$

17. $-3x^2 + 12x - 1$

Factor -3 from first two terms:

$$-3(x^2 - 4x) - 1$$

Completing value inside: $\left(\frac{-4}{2}\right)^2 = 4$.

Compensation: $(-3) \times 4 = -12$. Subtract -12 (add 12) outside.

$$-3(x^2 - 4x + 4) - 1 + 12 = -3(x-2)^2 + 11$$

$$\boxed{-3(x-2)^2 + 11}$$

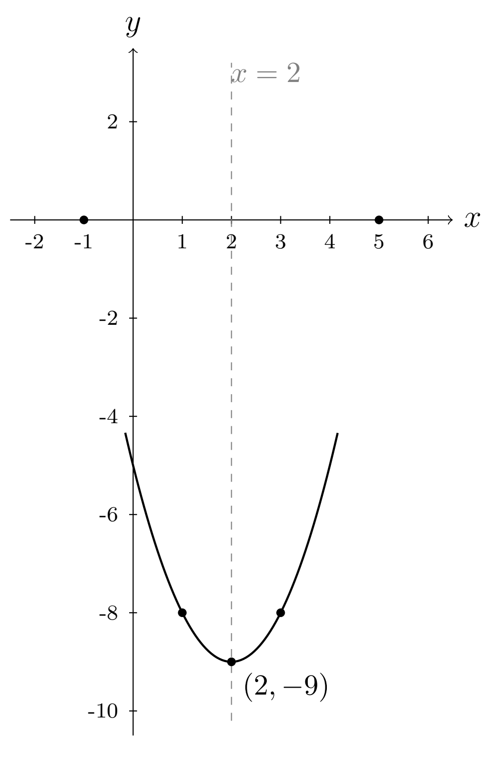
Part E — Graphing in Standard Form

18. $y = x^2 - 4x - 5$

Complete the square:

$$y = (x^2 - 4x + 4) - 4 - 5 = (x-2)^2 - 9$$

Vertex: $(2, -9)$. Axis: $x = 2$. Opens **upward**. Minimum.
Extra points: $x = 3$: $y = (3-2)^2 - 9 = -8$. $x = 5$: $y = (5-2)^2 - 9 = 0$.
By symmetry: $(1, -8)$ and $(-1, 0)$.



$$\boxed{y = (x-2)^2 - 9, \quad \text{Vertex } (2, -9), \quad x = 2, \quad \text{opens upward}}$$

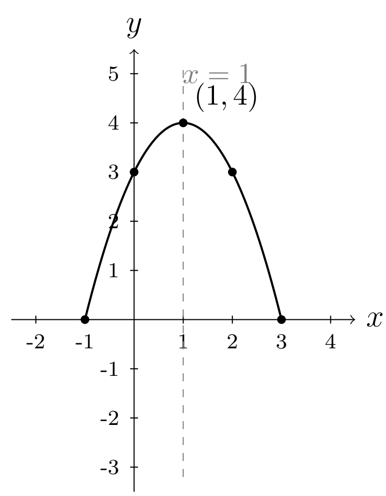
19. $y = -x^2 + 2x + 3$
Factor -1 from first two terms then complete the square:

$$y = -(x^2 - 2x) + 3$$

Completing value: $\left(\dfrac{-2}{2}\right)^2 = 1$. Compensation: $(-1)(1) = -1$. Subtract -1 (add 1) outside.

$$y = -(x^2 - 2x + 1) + 3 + 1 = -(x-1)^2 + 4$$

Vertex: $(1, 4)$. Axis: $x = 1$. Opens **downward**. Maximum.
Extra points: $x = 2$: $y = -(2-1)^2 + 4 = 3$. $\quad x = 3$: $y = -(3-1)^2 + 4 = 0$.
By symmetry: $(0, 3)$ and $(-1, 0)$.



$$\boxed{y = -(x-1)^2 + 4, \quad \text{Vertex } (1, 4), \quad x = 1, \quad \text{opens downward}}$$

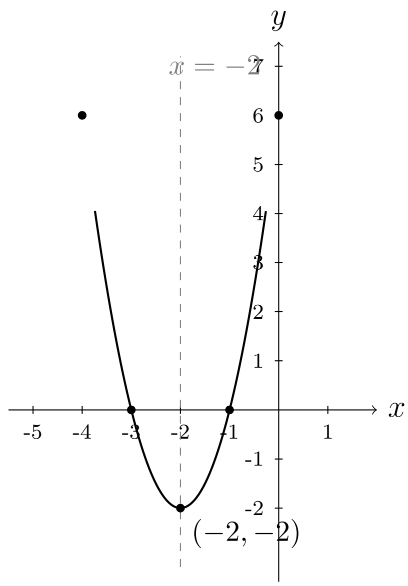
20. $y = 2x^2 + 8x + 6$
Factor 2 from first two terms:

$$y = 2(x^2 + 4x) + 6$$

Completing value inside: $\left(\dfrac{4}{2}\right)^2 = 4$. Compensation: $2 \times 4 = 8$. Subtract 8 outside.

$$y = 2(x^2 + 4x + 4) - 8 + 6 = 2(x+2)^2 - 2$$

Vertex: $(-2, -2)$. Axis: $x = -2$. Opens **upward**, narrower than $y = x^2$. Minimum.
Extra points: $x = -1$: $y = 2(-1+2)^2 - 2 = 0$. $\quad x = 0$: $y = 2(0+2)^2 - 2 = 6$.
By symmetry: $(-3, 0)$ and $(-4, 6)$.



$$\boxed{y = 2(x+2)^2 - 2, \quad \text{Vertex } (-2,-2), \quad x = -2, \quad \text{opens upward}}$$

Part F — The Quadratic Formula

21. $x^2 + 3x - 10 = 0$

$a = 1$, $b = 3$, $c = -10$. Discriminant: $3^2 - 4(1)(-10) = 9 + 40 = 49$.

$$x = \frac{-3 \pm \sqrt{49}}{2} = \frac{-3 \pm 7}{2}$$

$$x = \frac{-3+7}{2} = 2 \qquad \text{or} \qquad x = \frac{-3-7}{2} = -5$$

$$\boxed{x = 2 \quad \text{or} \quad x = -5}$$

22. $2x^2 - 3x - 2 = 0$

$a = 2$, $b = -3$, $c = -2$. Discriminant: $(-3)^2 - 4(2)(-2) = 9 + 16 = 25$.

$$x = \frac{3 \pm \sqrt{25}}{4} = \frac{3 \pm 5}{4}$$

$$x = \frac{3+5}{4} = 2 \qquad \text{or} \qquad x = \frac{3-5}{4} = -\frac{1}{2}$$

$$\boxed{x = 2 \quad \text{or} \quad x = -\frac{1}{2}}$$

23. $x^2 - 4x + 1 = 0$

$a=1$, $b=-4$, $c=1$. Discriminant: $(-4)^2-4(1)(1)=16-4=12$.

$$x=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm 2\sqrt{3}}{2}=2\pm\sqrt{3}$$

$$\boxed{x=2+\sqrt{3} \quad \text{or} \quad x=2-\sqrt{3}}$$

24. $3x^2+x+1=0$
$a=3$, $b=1$, $c=1$. Discriminant: $1^2-4(3)(1)=1-12=-11$.
Discriminant is negative: **no real solutions**.

$$\boxed{\text{No real solutions}}$$

Part G — Writing Equations from Graphs

25. Vertex $(2,-5)$, passes through $(4,3)$.
Skeleton: $y=a(x-2)^2-5$.
Substitute $(4,3)$:

$$3=a(4-2)^2-5=4a-5$$
$$8=4a \qquad \Longrightarrow \qquad a=2$$

Verify: $2(4-2)^2-5=8-5=3$. ✓

$$\boxed{y=2(x-2)^2-5}$$

26. Roots $x=-3$ and $x=5$, passes through $(0,-15)$.
Skeleton: $y=a(x+3)(x-5)$.
Substitute $(0,-15)$:

$$-15=a(0+3)(0-5)=a(3)(-5)=-15a$$
$$a=1$$

Expand:

$$y=(x+3)(x-5)=x^2-2x-15$$

Verify: $y(0)=0-0-15=-15$. ✓

$$\boxed{y=x^2-2x-15}$$

27. Vertex $(-1,6)$, x-intercept at $x=1$, meaning the point $(1,0)$.
Skeleton: $y=a(x+1)^2+6$.
Substitute $(1,0)$:

$$0=a(1+1)^2+6=4a+6$$
$$-6=4a \qquad \Longrightarrow \qquad a=-\frac{3}{2}$$

Verify: $-\frac{3}{2}(1+1)^2+6=-\frac{3}{2}(4)+6=-6+6=0$. ✓

$$\boxed{y=-\frac{3}{2}(x+1)^2+6}$$

Part H — Applications

28. Length is 5 m more than width, area = 84 m^2.
Let x = width. Length = $x+5$.

$$x(x+5) = 84 \qquad \Longrightarrow \qquad x^2 + 5x - 84 = 0$$

Discriminant: $5^2 - 4(1)(-84) = 25 + 336 = 361 = 19^2$.

$$x = \frac{-5 \pm 19}{2}$$

$$x = \frac{-5+19}{2} = 7 \qquad \text{or} \qquad x = \frac{-5-19}{2} = -12$$

Reject $x = -12$. Width = 7 m, Length = 12 m.
Verify: $7 \times 12 = 84$ m^2. ✓

$$\boxed{\text{Width: 7 m,} \quad \text{Length: 12 m}}$$

29. $h(t) = -4.9t^2 + 28t$.
Time of maximum: $t = \dfrac{28}{9.8} \approx 2.857$ s.
Maximum height:

$$h(2.857) = -4.9(2.857)^2 + 28(2.857) \approx -4.9(8.163) + 80.000 \approx -40.000 + 80.000 = 40.00 \text{ m}$$

$$\boxed{\text{Maximum height} = 40.00 \text{ m at } t \approx 2.86 \text{ s}}$$

30. Price = $30 + 4x$, Quantity = $400 - 20x$.

$$R(x) = (30+4x)(400-20x) = 12000 - 600x + 1600x - 80x^2 = -80x^2 + 1000x + 12000$$

Vertex: $x = -\dfrac{1000}{2(-80)} = \dfrac{1000}{160} = 6.25$.
Price = $30 + 4(6.25) = \$55$.
Quantity = $400 - 20(6.25) = 275$ units.
Maximum revenue = $55 \times 275 = \$15{,}125$.

$$\boxed{\text{Optimal price: \$55,} \quad \text{Maximum revenue: \$15,125/week}}$$

31. (Challenge) $h(t) = -4.9t^2 + 15t + 20 = 0$.
Multiply by -1:

$$4.9t^2 - 15t - 20 = 0$$

$a = 4.9$, $b = -15$, $c = -20$.
Discriminant: $(-15)^2 - 4(4.9)(-20) = 225 + 392 = 617$.

$$t = \frac{15 \pm \sqrt{617}}{9.8}$$

$$\sqrt{617} \approx 24.839$$

$$t = \frac{15 + 24.839}{9.8} \approx \frac{39.839}{9.8} \approx 4.06 \text{ s} \qquad \text{or} \qquad t = \frac{15 - 24.839}{9.8} \approx \frac{-9.839}{9.8} \approx -1.00 \text{ s}$$

Reject the negative time.

Verify: $h(4.06) = -4.9(4.06)^2 + 15(4.06) + 20 \approx -4.9(16.484) + 60.9 + 20 \approx -80.77 + 80.9 \approx 0.13 \approx 0.$ ✓

$$\boxed{t \approx 4.06 \text{ s}}$$

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End of Lesson • ApexMath
