Writing a Quadratic from Its Graph
ApexMath

Notes

The Big Idea

So far, you have been given an equation and asked to graph it. This lesson reverses that process: you are given information about a parabola — its vertex, its roots, or a point it passes through — and you must write the equation.

There are two main strategies, depending on what information you are given.

Strategy 1: Use Vertex Form

If you know the **vertex** (h, k), start with vertex form:

$$y = a(x - h)^2 + k$$

You already know h and k, so you only need to find a. To do that, substitute the coordinates of any other known point (x, y) into the equation and solve for a.

Strategy 2: Use Intercept Form (Roots Given)

If you know the **roots** (also called x-intercepts) $x = r$ and $x = s$, start with intercept form:

$$y = a(x - r)(x - s)$$

Again, you only need to find a. Substitute any other known point (x, y) and solve for a.

What If No Extra Point Is Given?

If only the vertex is given and no other point, you cannot determine a uniquely. Infinitely many parabolas share the same vertex. You must be given at least one additional point to pin down a.

If only the roots are given and no other point, the same issue applies. Always look for the extra point — it is what uniquely determines the parabola.

Summary

Information Given	**Form to Use**	**What to Find**
Vertex + one point	$y = a(x - h)^2 + k$	Solve for a
Two roots + one point	$y = a(x - r)(x - s)$	Solve for a
Vertex + one root	Use vertex form, sub root	Solve for a

Pro Tip

- Always set up the appropriate form first before substituting any numbers. Writing the skeleton equation with h, k, r, s already filled in — but a left as a variable — makes the algebra much cleaner and harder to mess up.
- After finding a, write out the full equation and then expand it if the problem asks for standard form. Do not try to expand and find a at the same time.

- If a comes out negative, the parabola opens downward. Check whether that makes sense given the graph or the information described in the problem.
- A root $x = r$ means the factor is $(x - r)$. If the root is negative, like $x = -3$, the factor is $(x - (-3)) = (x + 3)$. Watch the sign carefully.
- Once you have your final equation, always verify it by plugging the known points back in and confirming you get the correct y-values.

Examples

Example 1 (Vertex and one point): A parabola has vertex $(2, -3)$ and passes through the point $(4, 5)$. Write its equation in vertex form.

Step 1: Write the vertex form skeleton with $h = 2$ and $k = -3$.

$$y = a(x - 2)^2 - 3$$

Step 2: Substitute the known point $(4, 5)$, meaning $x = 4$, $y = 5$.

$$5 = a(4 - 2)^2 - 3$$

Step 3: Simplify the right side.

$$5 = a(2)^2 - 3$$

$$5 = 4a - 3$$

Step 4: Solve for a.

$$8 = 4a \qquad \Longrightarrow \qquad a = 2$$

Step 5: Write the final equation.

$$y = 2(x - 2)^2 - 3$$

Verify: At $x = 4$: $y = 2(4 - 2)^2 - 3 = 2(4) - 3 = 8 - 3 = 5$. ✓

$$\boxed{y = 2(x - 2)^2 - 3}$$

Example 2 (Vertex and one point, a is a fraction): A parabola has vertex $(-1, 4)$ and passes through the point $(1, 2)$. Write its equation in vertex form.

Step 1: Write the skeleton with $h = -1$, $k = 4$.

$$y = a(x - (-1))^2 + 4 = a(x + 1)^2 + 4$$

Step 2: Substitute $(1, 2)$, meaning $x = 1$, $y = 2$.

$$2 = a(1 + 1)^2 + 4$$

Step 3: Simplify.

$$2 = a(2)^2 + 4$$

$$2 = 4a + 4$$

Step 4: Solve for a.

$$-2 = 4a \qquad \Longrightarrow \qquad a = -\frac{1}{2}$$

Since $a < 0$, the parabola opens downward. This is consistent with the vertex at $(-1, 4)$ being a maximum.

Step 5: Write the final equation.

$$y = -\frac{1}{2}(x+1)^2 + 4$$

Verify: At $x = 1$: $y = -\frac{1}{2}(1+1)^2 + 4 = -\frac{1}{2}(4) + 4 = -2 + 4 = 2$. ✓

$$\boxed{y = -\frac{1}{2}(x+1)^2 + 4}$$

Example 3 (Two roots and one point): A parabola has x-intercepts at $x = 1$ and $x = 5$, and passes through $(3, -4)$. Write its equation.

Step 1: Write the intercept form skeleton with $r = 1$ and $s = 5$.

$$y = a(x-1)(x-5)$$

Step 2: Substitute $(3, -4)$, meaning $x = 3$, $y = -4$.

$$-4 = a(3-1)(3-5)$$

Step 3: Simplify.

$$-4 = a(2)(-2)$$

$$-4 = -4a$$

Step 4: Solve for a.

$$a = 1$$

Step 5: Write the final equation.

$$y = (x-1)(x-5)$$

If standard form is needed, expand:

$$y = x^2 - 6x + 5$$

Verify: At $x = 3$: $y = (3-1)(3-5) = (2)(-2) = -4$. ✓

$$\boxed{y = (x-1)(x-5) \quad \text{or} \quad y = x^2 - 6x + 5}$$

Example 4 (Two roots and one point, $a \neq 1$): A parabola has x-intercepts at $x = -2$ and $x = 3$, and passes through $(1, 6)$. Write its equation in standard form.

Step 1: Write the intercept form with $r = -2$, $s = 3$.

$$y = a(x - (-2))(x - 3) = a(x + 2)(x - 3)$$

Step 2: Substitute $(1, 6)$, meaning $x = 1$, $y = 6$.

$$6 = a(1 + 2)(1 - 3)$$

Step 3: Simplify.

$$6 = a(3)(-2)$$

$$6 = -6a$$

Step 4: Solve for a.

$$a = -1$$

Step 5: Write the equation.

$$y = -(x + 2)(x - 3)$$

Expand:

$$y = -(x^2 - 3x + 2x - 6) = -(x^2 - x - 6) = -x^2 + x + 6$$

Verify: At $x = 1$: $y = -(1 + 2)(1 - 3) = -(3)(-2) = 6$. ✓

$$\boxed{y = -x^2 + x + 6}$$

Example 5 (Vertex and a root given): A parabola has vertex $(3, -1)$ and one x-intercept at $x = 2$. Write its equation in vertex form.

Step 1: Write the vertex form skeleton with $h = 3$, $k = -1$.

$$y = a(x - 3)^2 - 1$$

Step 2: An x-intercept means $y = 0$ at that x-value. Substitute $(2, 0)$, meaning $x = 2$, $y = 0$.

$$0 = a(2 - 3)^2 - 1$$

Step 3: Simplify.

$$0 = a(-1)^2 - 1$$

$$0 = a - 1$$

Step 4: Solve for a.

$$a = 1$$

Step 5: Write the final equation.

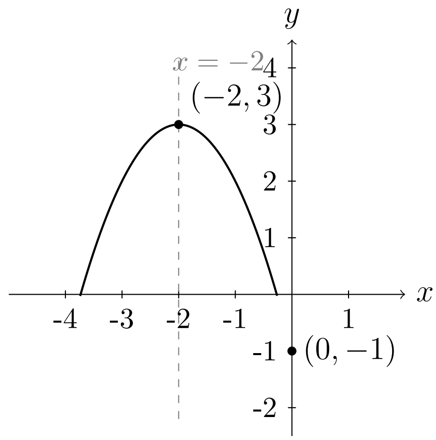
$$y = (x - 3)^2 - 1$$

Verify: At $x = 2$: $y = (2 - 3)^2 - 1 = 1 - 1 = 0$. ✓

Note: By symmetry, the other x-intercept is at $x = 4$ (since the axis of symmetry is $x = 3$ and $x = 2$ is one unit to the left).

$$\boxed{y = (x-3)^2 - 1}$$

Example 6 (Reading from a graph): The graph of a parabola is shown below. Its vertex is $(-2, 3)$ and it passes through the point $(0, -1)$. Write its equation.



Step 1: Write the vertex form skeleton with $h = -2$, $k = 3$.

$$y = a(x+2)^2 + 3$$

Step 2: Substitute $(0, -1)$, meaning $x = 0$, $y = -1$.

$$-1 = a(0+2)^2 + 3$$

Step 3: Simplify.

$$-1 = 4a + 3$$

Step 4: Solve for a.

$$-4 = 4a \qquad \Longrightarrow \qquad a = -1$$

Step 5: Write the final equation.

$$y = -(x+2)^2 + 3$$

Verify: At $x = 0$: $y = -(0+2)^2 + 3 = -4 + 3 = -1$. ✓

$$\boxed{y = -(x+2)^2 + 3}$$

Common Mistakes

- **Substituting the vertex as the extra point:** The vertex is already built into h and k. You must use a *different* point to find a. Plugging the vertex back in gives $0 = 0$, which tells you nothing about a.
- **Confusing the sign of h:** In vertex form, the expression is $(x-h)^2$. If the vertex is $(-1, 4)$, then $h = -1$ and you write $(x-(-1))^2 = (x+1)^2$, not $(x-1)^2$.
- **Confusing the sign of a root in intercept form:** A root at $x = -3$ gives the factor $(x-(-3)) = (x+3)$, not $(x-3)$. Always think: "what value of x makes this factor zero?"
- **Forgetting to solve for a:** Leaving the answer as $y = a(x-2)^2 + 1$ is incomplete. You must use the extra point to find the numerical value of a.
- **Not verifying:** After writing the final equation, always substitute the known points back in. A quick check takes ten seconds and will catch sign errors or arithmetic mistakes before they cost you marks.

Worksheet

1. A parabola has vertex $(1, -2)$ and passes through $(3, 6)$. Write its equation in vertex form.

2. A parabola has vertex $(-3, 5)$ and passes through $(-1, 1)$. Write its equation in vertex form.

3. A parabola has vertex $(0, -4)$ and passes through $(2, 0)$. Write its equation in vertex form.

4. A parabola has x-intercepts at $x = 2$ and $x = 6$, and passes through $(4, -4)$. Write its equation in standard form.

5. A parabola has x-intercepts at $x = -1$ and $x = 4$, and passes through $(1, 6)$. Write its equation in standard form.

6. A parabola has x-intercepts at $x = -3$ and $x = -1$, and passes through $(-2, 3)$. Write its equation in standard form.

7. A parabola has vertex $(2, 5)$ and an x-intercept at $x = 4$. Write its equation in vertex form.

8. A parabola has vertex $(-1, -3)$ and passes through $(2, 15)$. Write its equation in vertex form.

9. (Challenge) A parabola has x-intercepts at $x = -4$ and $x = 2$, and a y-intercept at $y = -16$. Write its equation in standard form.

10. (Challenge) A parabola opens upward, has its vertex at $(1, -9)$, and passes through the point $(4, 0)$. Write the equation in vertex form and then convert it to standard form.

Answer Key

1. Vertex $(1, -2)$, passes through $(3, 6)$.
Step 1: Write the skeleton.
$$y = a(x-1)^2 - 2$$
Step 2: Substitute $(3, 6)$.
$$6 = a(3-1)^2 - 2$$
Step 3: Simplify.
$$6 = 4a - 2$$
Step 4: Solve for a.
$$8 = 4a \qquad \Longrightarrow \qquad a = 2$$
Step 5: Final equation.
$$y = 2(x-1)^2 - 2$$
Verify at $(3, 6)$: $2(3-1)^2 - 2 = 2(4) - 2 = 6$. ✓
$$\boxed{y = 2(x-1)^2 - 2}$$

2. Vertex $(-3, 5)$, passes through $(-1, 1)$.
Step 1:
$$y = a(x+3)^2 + 5$$
Step 2: Substitute $(-1, 1)$.
$$1 = a(-1+3)^2 + 5$$
Step 3:
$$1 = 4a + 5$$
Step 4:
$$-4 = 4a \qquad \Longrightarrow \qquad a = -1$$
Step 5:
$$y = -(x+3)^2 + 5$$
Verify at $(-1, 1)$: $-(-1+3)^2 + 5 = -(4) + 5 = 1$. ✓
$$\boxed{y = -(x+3)^2 + 5}$$

3. Vertex $(0, -4)$, passes through $(2, 0)$.
Step 1: $h = 0$, $k = -4$.
$$y = a(x-0)^2 - 4 = ax^2 - 4$$
Step 2: Substitute $(2, 0)$.
$$0 = a(2)^2 - 4$$
Step 3:
$$0 = 4a - 4$$

Step 4:
$$4 = 4a \qquad \Longrightarrow \qquad a = 1$$
Step 5:
$$y = x^2 - 4$$
Verify at $(2, 0)$: $(2)^2 - 4 = 0$. ✓
$$\boxed{y = x^2 - 4}$$

4. Roots $x = 2$ and $x = 6$, passes through $(4, -4)$.
Step 1:
$$y = a(x - 2)(x - 6)$$
Step 2: Substitute $(4, -4)$.
$$-4 = a(4 - 2)(4 - 6)$$
Step 3:
$$-4 = a(2)(-2) = -4a$$
Step 4:
$$a = 1$$
Step 5:
$$y = (x - 2)(x - 6)$$
Expand:
$$y = x^2 - 8x + 12$$
Verify at $(4, -4)$: $(4 - 2)(4 - 6) = (2)(-2) = -4$. ✓
$$\boxed{y = x^2 - 8x + 12}$$

5. Roots $x = -1$ and $x = 4$, passes through $(1, 6)$.
Step 1:
$$y = a(x + 1)(x - 4)$$
Step 2: Substitute $(1, 6)$.
$$6 = a(1 + 1)(1 - 4)$$
Step 3:
$$6 = a(2)(-3) = -6a$$
Step 4:
$$a = -1$$
Step 5:
$$y = -(x + 1)(x - 4)$$
Expand:
$$y = -(x^2 - 4x + x - 4) = -(x^2 - 3x - 4) = -x^2 + 3x + 4$$

Verify at $(1,6)$: $-(1+1)(1-4) = -(2)(-3) = 6$. ✓

$$\boxed{y = -x^2 + 3x + 4}$$

6. Roots $x = -3$ and $x = -1$, passes through $(-2, 3)$.
Step 1:

$$y = a(x+3)(x+1)$$

Step 2: Substitute $(-2, 3)$.

$$3 = a(-2+3)(-2+1)$$

Step 3:

$$3 = a(1)(-1) = -a$$

Step 4:

$$a = -3$$

Step 5:

$$y = -3(x+3)(x+1)$$

Expand:

$$y = -3(x^2 + 4x + 3) = -3x^2 - 12x - 9$$

Verify at $(-2, 3)$: $-3(-2+3)(-2+1) = -3(1)(-1) = 3$. ✓

$$\boxed{y = -3x^2 - 12x - 9}$$

7. Vertex $(2, 5)$, x-intercept at $x = 4$ (so the point is $(4, 0)$).
Step 1:

$$y = a(x-2)^2 + 5$$

Step 2: Substitute $(4, 0)$.

$$0 = a(4-2)^2 + 5$$

Step 3:

$$0 = 4a + 5$$

Step 4:

$$-5 = 4a \qquad \Longrightarrow \qquad a = -\frac{5}{4}$$

Step 5:

$$y = -\frac{5}{4}(x-2)^2 + 5$$

Verify at $(4, 0)$: $-\frac{5}{4}(4-2)^2 + 5 = -\frac{5}{4}(4) + 5 = -5 + 5 = 0$. ✓

$$\boxed{y = -\frac{5}{4}(x-2)^2 + 5}$$

8. Vertex $(-1, -3)$, passes through $(2, 15)$.
Step 1:

$$y = a(x+1)^2 - 3$$

Step 2: Substitute $(2, 15)$.

$$15 = a(2+1)^2 - 3$$

Step 3:

$$15 = 9a - 3$$

Step 4:

$$18 = 9a \qquad \Longrightarrow \qquad a = 2$$

Step 5:

$$y = 2(x+1)^2 - 3$$

Verify at $(2, 15)$: $2(2+1)^2 - 3 = 2(9) - 3 = 18 - 3 = 15$. ✓

$$\boxed{y = 2(x+1)^2 - 3}$$

9. (Challenge) Roots $x = -4$ and $x = 2$, y-intercept at $y = -16$.
A y-intercept at $y = -16$ means the parabola passes through $(0, -16)$.
Step 1:

$$y = a(x+4)(x-2)$$

Step 2: Substitute $(0, -16)$.

$$-16 = a(0+4)(0-2)$$

Step 3:

$$-16 = a(4)(-2) = -8a$$

Step 4:

$$a = 2$$

Step 5:

$$y = 2(x+4)(x-2)$$

Expand:

$$y = 2(x^2 + 2x - 8) = 2x^2 + 4x - 16$$

Verify at $(0, -16)$: $2(0)^2 + 4(0) - 16 = -16$. ✓
Verify roots: at $x = -4$: $2(-4+4)(-4-2) = 0$. ✓ At $x = 2$: $2(2+4)(2-2) = 0$. ✓

$$\boxed{y = 2x^2 + 4x - 16}$$

10. (Challenge) Vertex $(1, -9)$, passes through $(4, 0)$.
Step 1:

$$y = a(x-1)^2 - 9$$

Step 2: Substitute $(4, 0)$.

$$0 = a(4-1)^2 - 9$$

Step 3:

$$0 = 9a - 9$$

Step 4:

$$9 = 9a \qquad \Longrightarrow \qquad a = 1$$

Since $a = 1 > 0$, the parabola opens upward. ✓
Step 5: Vertex form.

$$y = (x-1)^2 - 9$$

Convert to standard form by expanding:

$$y = x^2 - 2x + 1 - 9$$

$$y = x^2 - 2x - 8$$

Verify at $(4, 0)$: $(4)^2 - 2(4) - 8 = 16 - 8 - 8 = 0$. ✓
Verify at $(1, -9)$: $(1)^2 - 2(1) - 8 = 1 - 2 - 8 = -9$. ✓

$$\boxed{y = (x-1)^2 - 9 \quad \text{or} \quad y = x^2 - 2x - 8}$$

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End of Lesson • ApexMath
