Adding and Subtracting Radicals
ApexMath

Notes

The Big Idea

Adding and subtracting radicals works almost exactly like adding and subtracting variables in algebra. The key rule is simple:

You can only add or subtract radicals that have the same number under the radical symbol.

These are called **like radicals**, just like like terms in algebra.

For example:

$$3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$$

Think of $\sqrt{5}$ like a variable. This is exactly the same as $3x + 2x = 5x$. The radical part does not change — only the number in front (called the **coefficient**) changes.

Like Radicals vs. Unlike Radicals

- **Like radicals** have the same radicand. They **can** be combined.

$$4\sqrt{3} + 7\sqrt{3} = 11\sqrt{3}$$

- **Unlike radicals** have different radicands. They **cannot** be combined and must be left as is.

$$4\sqrt{3} + 7\sqrt{2} \quad \text{cannot be simplified further}$$

What if the Radicals Do Not Look Like?

Sometimes two radicals look different but are actually like radicals in disguise. This happens when one or both radicals have not been simplified yet.

The rule is: **always simplify each radical first, then check whether they are like radicals.**

For example:

$$\sqrt{12} + \sqrt{75}$$

At first glance, these look unlike. But after simplifying:

$$\sqrt{12} = 2\sqrt{3} \qquad \sqrt{75} = 5\sqrt{3}$$

Now they are like radicals and can be combined:

$$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$$

How to Add or Subtract Radicals — The Method

- Step 1: Simplify every radical in the expression completely.

- Step 2: Identify which radicals are like radicals (same radicand).
- Step 3: Add or subtract the coefficients of like radicals.
- Step 4: Keep the radical part unchanged.
- Step 5: Leave any unlike radicals as separate terms in your answer.

Pro tip

- Always simplify **before** you try to combine. Skipping this step is the most common source of errors in this topic.
- Think of the radical like a label or a unit. Just like 3 cm + 5 cm = 8 cm, we have $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$. The label stays the same; only the number in front changes.
- If after simplifying you still have two different radicands, the expression is already in its simplest form. Do not try to force them together.
- Watch your signs carefully when subtracting. A negative coefficient is still a valid coefficient.

Examples

Example A

Simplify $5\sqrt{7} + 3\sqrt{7}$.

Step 1: Check whether the radicals are like radicals. Both have $\sqrt{7}$, so yes, they are like radicals.

Step 2: Add the coefficients.

$$5 + 3 = 8$$

Step 3: Keep the radical part.

$$5\sqrt{7} + 3\sqrt{7} = 8\sqrt{7}$$

Example B

Simplify $9\sqrt{11} - 4\sqrt{11}$.

Step 1: Both radicals have $\sqrt{11}$, so they are like radicals.

Step 2: Subtract the coefficients.

$$9 - 4 = 5$$

Step 3: Keep the radical part.

$$9\sqrt{11} - 4\sqrt{11} = 5\sqrt{11}$$

Example C

Simplify $\sqrt{50} + \sqrt{8}$.

Step 1: Simplify $\sqrt{50}$.

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

Step 2: Simplify $\sqrt{8}$.

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

Step 3: Now both radicals have $\sqrt{2}$. They are like radicals. Add the coefficients.

$$5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$$

Example D

Simplify $3\sqrt{12} - \sqrt{75} + 2\sqrt{3}$.

Step 1: Simplify $3\sqrt{12}$.

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3} \qquad \text{so} \qquad 3\sqrt{12} = 3 \times 2\sqrt{3} = 6\sqrt{3}$$

Step 2: Simplify $\sqrt{75}$.

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

Step 3: The last term $2\sqrt{3}$ is already simplified.

Step 4: Rewrite the full expression using the simplified radicals.

$$6\sqrt{3} - 5\sqrt{3} + 2\sqrt{3}$$

Step 5: All three terms are like radicals. Combine the coefficients from left to right.

$$6 - 5 + 2 = 3$$

Step 6: Write the final answer.

$$3\sqrt{3}$$

Example E

Simplify $\sqrt{18} + \sqrt{5} + \sqrt{50}$.

Step 1: Simplify $\sqrt{18}$.

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

Step 2: Simplify $\sqrt{5}$. It has no perfect square factors, so it stays as $\sqrt{5}$.

Step 3: Simplify $\sqrt{50}$.

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

Step 4: Rewrite the full expression.

$$3\sqrt{2} + \sqrt{5} + 5\sqrt{2}$$

Step 5: Combine like radicals. $3\sqrt{2}$ and $5\sqrt{2}$ are like radicals. $\sqrt{5}$ is unlike and stays separate.

$$3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$$

Step 6: Write the final answer.

$$8\sqrt{2} + \sqrt{5}$$

Common mistakes

- **Trying to add unlike radicals.** $\sqrt{2}+\sqrt{3}$ does not equal $\sqrt{5}$. You cannot add the numbers inside the radical. The radicands must match.
- **Forgetting to simplify first.** Always simplify every term before deciding whether the radicals are like or unlike.
- **Changing the radical when adding.** When you add $4\sqrt{6}+3\sqrt{6}$, the answer is $7\sqrt{6}$, not $7\sqrt{12}$. The radical part never changes during addition or subtraction.
- **Sign errors when subtracting.** Be careful with negative coefficients. $2\sqrt{5}-6\sqrt{5}=-4\sqrt{5}$, not $4\sqrt{5}$.

Worksheet

1. Add or subtract the following like radicals.

   (a) $6\sqrt{3}+2\sqrt{3}$

   (b) $10\sqrt{5}-4\sqrt{5}$

   (c) $\sqrt{11}+8\sqrt{11}$

   (d) $7\sqrt{2}-7\sqrt{2}$

2. Simplify each expression. Simplify any radicals first.

   (a) $\sqrt{12}+\sqrt{27}$

   (b) $\sqrt{45}-\sqrt{20}$

(c) $\sqrt{50} + \sqrt{32}$

(d) $\sqrt{75} - \sqrt{48}$

3. Simplify each expression completely.

(a) $2\sqrt{18} + 3\sqrt{2}$

(b) $5\sqrt{8} - 2\sqrt{50}$

(c) $4\sqrt{12} + \sqrt{75}$

(d) $3\sqrt{28} - 2\sqrt{63}$

4. Simplify each expression with three terms.

(a) $\sqrt{8} + \sqrt{18} + \sqrt{50}$

(b) $3\sqrt{12} - \sqrt{75} + 2\sqrt{3}$

(c) $\sqrt{20} + \sqrt{45} - \sqrt{5}$

5. Simplify $\sqrt{98} + \sqrt{2} - \sqrt{50}$.

6. Simplify $2\sqrt{48} - 3\sqrt{27} + \sqrt{75}$.

7. State whether the following expression can be simplified further. Explain your reasoning.

$$5\sqrt{3} + 4\sqrt{7}$$

8. A student claims that $\sqrt{2} + \sqrt{8} = \sqrt{10}$. Show whether this is correct or incorrect, and find the right answer.

9. **Challenge:** Simplify $\sqrt{200} - \sqrt{72} + \sqrt{128}$.

10. **Challenge:** Simplify $3\sqrt{24} + 2\sqrt{54} - \sqrt{96}$.

Answer Key

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1.

(a) $6\sqrt{3} + 2\sqrt{3}$
Step 1: Both terms have $\sqrt{3}$. They are like radicals.
Step 2: Add the coefficients.

$$6 + 2 = 8$$

$$\boxed{8\sqrt{3}}$$

(b) $10\sqrt{5} - 4\sqrt{5}$
Step 1: Both terms have $\sqrt{5}$. They are like radicals.
Step 2: Subtract the coefficients.

$$10 - 4 = 6$$

$$\boxed{6\sqrt{5}}$$

(c) $\sqrt{11} + 8\sqrt{11}$
Step 1: Both terms have $\sqrt{11}$. Note that $\sqrt{11}$ has a coefficient of 1.
Step 2: Add the coefficients.

$$1 + 8 = 9$$

$$\boxed{9\sqrt{11}}$$

(d) $7\sqrt{2} - 7\sqrt{2}$
Step 1: Both terms have $\sqrt{2}$. They are like radicals.
Step 2: Subtract the coefficients.

$$7 - 7 = 0$$

$$\boxed{0}$$

2.

(a) $\sqrt{12} + \sqrt{27}$
Step 1: Simplify $\sqrt{12}$.

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$$

Step 2: Simplify $\sqrt{27}$.

$$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

Step 3: Both are now like radicals with $\sqrt{3}$. Add the coefficients.

$$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$$

$$\boxed{5\sqrt{3}}$$

(b) $\sqrt{45} - \sqrt{20}$
Step 1: Simplify $\sqrt{45}$.

$$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

Step 2: Simplify $\sqrt{20}$.

$$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

Step 3: Both are like radicals with $\sqrt{5}$. Subtract the coefficients.

$$3\sqrt{5} - 2\sqrt{5} = 1\sqrt{5} = \sqrt{5}$$

$$\boxed{\sqrt{5}}$$

(c) $\sqrt{50} + \sqrt{32}$
Step 1: Simplify $\sqrt{50}$.

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

Step 2: Simplify $\sqrt{32}$.

$$\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$$

Step 3: Both are like radicals with $\sqrt{2}$. Add the coefficients.

$$5\sqrt{2} + 4\sqrt{2} = 9\sqrt{2}$$

$$\boxed{9\sqrt{2}}$$

(d) $\sqrt{75} - \sqrt{48}$
Step 1: Simplify $\sqrt{75}$.

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

Step 2: Simplify $\sqrt{48}$.

$$\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$$

Step 3: Both are like radicals with $\sqrt{3}$. Subtract the coefficients.

$$5\sqrt{3} - 4\sqrt{3} = 1\sqrt{3} = \sqrt{3}$$

$$\boxed{\sqrt{3}}$$

3.

(a) $2\sqrt{18} + 3\sqrt{2}$
Step 1: Simplify $2\sqrt{18}$.

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2} \qquad \text{so} \qquad 2\sqrt{18} = 2 \times 3\sqrt{2} = 6\sqrt{2}$$

Step 2: $3\sqrt{2}$ is already simplified.

Step 3: Both are like radicals with $\sqrt{2}$. Add the coefficients.

$$6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$$

$$\boxed{9\sqrt{2}}$$

(b) $5\sqrt{8} - 2\sqrt{50}$
Step 1: Simplify $5\sqrt{8}$.

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2} \qquad \text{so} \qquad 5\sqrt{8} = 5 \times 2\sqrt{2} = 10\sqrt{2}$$

Step 2: Simplify $2\sqrt{50}$.

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2} \qquad \text{so} \qquad 2\sqrt{50} = 2 \times 5\sqrt{2} = 10\sqrt{2}$$

Step 3: Both are like radicals with $\sqrt{2}$. Subtract the coefficients.

$$10\sqrt{2} - 10\sqrt{2} = 0$$

$$\boxed{0}$$

(c) $4\sqrt{12} + \sqrt{75}$
Step 1: Simplify $4\sqrt{12}$.

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3} \qquad \text{so} \qquad 4\sqrt{12} = 4 \times 2\sqrt{3} = 8\sqrt{3}$$

Step 2: Simplify $\sqrt{75}$.

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

Step 3: Both are like radicals with $\sqrt{3}$. Add the coefficients.

$$8\sqrt{3} + 5\sqrt{3} = 13\sqrt{3}$$

$$\boxed{13\sqrt{3}}$$

(d) $3\sqrt{28} - 2\sqrt{63}$
Step 1: Simplify $3\sqrt{28}$.

$$\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7} \qquad \text{so} \qquad 3\sqrt{28} = 3 \times 2\sqrt{7} = 6\sqrt{7}$$

Step 2: Simplify $2\sqrt{63}$.

$$\sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7} \qquad \text{so} \qquad 2\sqrt{63} = 2 \times 3\sqrt{7} = 6\sqrt{7}$$

Step 3: Both are like radicals with $\sqrt{7}$. Subtract the coefficients.

$$6\sqrt{7} - 6\sqrt{7} = 0$$

$$\boxed{0}$$

4.

(a) $\sqrt{8} + \sqrt{18} + \sqrt{50}$

Step 1: Simplify $\sqrt{8}$.

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

Step 2: Simplify $\sqrt{18}$.

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

Step 3: Simplify $\sqrt{50}$.

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

Step 4: All three are like radicals with $\sqrt{2}$. Add the coefficients.

$$2 + 3 + 5 = 10$$

$$\boxed{10\sqrt{2}}$$

(b) $3\sqrt{12} - \sqrt{75} + 2\sqrt{3}$

Step 1: Simplify $3\sqrt{12}$.

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3} \qquad \text{so} \qquad 3\sqrt{12} = 3 \times 2\sqrt{3} = 6\sqrt{3}$$

Step 2: Simplify $\sqrt{75}$.

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

Step 3: $2\sqrt{3}$ is already simplified.

Step 4: Rewrite the expression.

$$6\sqrt{3} - 5\sqrt{3} + 2\sqrt{3}$$

Step 5: Combine coefficients from left to right.

$$6 - 5 + 2 = 3$$

$$\boxed{3\sqrt{3}}$$

(c) $\sqrt{20} + \sqrt{45} - \sqrt{5}$

Step 1: Simplify $\sqrt{20}$.

$$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

Step 2: Simplify $\sqrt{45}$.

$$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

Step 3: $\sqrt{5}$ is already simplified. Its coefficient is 1.

Step 4: Rewrite the expression.

$$2\sqrt{5} + 3\sqrt{5} - 1\sqrt{5}$$

Step 5: Combine coefficients.

$$2 + 3 - 1 = 4$$

$$\boxed{4\sqrt{5}}$$

5. Simplify $\sqrt{98} + \sqrt{2} - \sqrt{50}$.
Step 1: Simplify $\sqrt{98}$.

$$\sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2}$$

Step 2: $\sqrt{2}$ is already simplified. Its coefficient is 1.
Step 3: Simplify $\sqrt{50}$.

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

Step 4: Rewrite the expression.

$$7\sqrt{2} + 1\sqrt{2} - 5\sqrt{2}$$

Step 5: Combine coefficients.

$$7 + 1 - 5 = 3$$

$$\boxed{3\sqrt{2}}$$

6. Simplify $2\sqrt{48} - 3\sqrt{27} + \sqrt{75}$.
Step 1: Simplify $2\sqrt{48}$.

$$\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3} \qquad \text{so} \qquad 2\sqrt{48} = 2 \times 4\sqrt{3} = 8\sqrt{3}$$

Step 2: Simplify $3\sqrt{27}$.

$$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3} \qquad \text{so} \qquad 3\sqrt{27} = 3 \times 3\sqrt{3} = 9\sqrt{3}$$

Step 3: Simplify $\sqrt{75}$.

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

Step 4: Rewrite the expression.

$$8\sqrt{3} - 9\sqrt{3} + 5\sqrt{3}$$

Step 5: Combine coefficients from left to right.

$$8 - 9 + 5 = 4$$

$$\boxed{4\sqrt{3}}$$

7. Can $5\sqrt{3} + 4\sqrt{7}$ be simplified further?
Step 1: Check whether the radicals are like radicals.
$\sqrt{3}$ and $\sqrt{7}$ have different radicands, so they are **unlike radicals**.
Step 2: Check whether either radical can be simplified.
$\sqrt{3}$: 3 has no perfect square factors. Already simplified.
$\sqrt{7}$: 7 has no perfect square factors. Already simplified.

Step 3: Since the radicands are different and neither can be simplified to match the other, the expression cannot be combined.

$$\boxed{5\sqrt{3} + 4\sqrt{7} \text{ cannot be simplified further.}}$$

8. A student claims $\sqrt{2} + \sqrt{8} = \sqrt{10}$. Is this correct?
Step 1: Test the student's claim. Simplify $\sqrt{8}$ first.

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

Step 2: Rewrite the expression.

$$\sqrt{2} + 2\sqrt{2} = 1\sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$$

Step 3: Check whether $3\sqrt{2} = \sqrt{10}$.

$$3\sqrt{2} = \sqrt{9 \times 2} = \sqrt{18} \neq \sqrt{10}$$

Step 4: The student is incorrect. They added the numbers inside the radicals, which is not allowed. You must simplify first and then add the coefficients.

$$\boxed{3\sqrt{2} \quad \text{(the student's answer is incorrect)}}$$

9. Simplify $\sqrt{200} - \sqrt{72} + \sqrt{128}$.
Step 1: Simplify $\sqrt{200}$.

$$\sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$$

Step 2: Simplify $\sqrt{72}$.

$$\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$$

Step 3: Simplify $\sqrt{128}$.

$$\sqrt{128} = \sqrt{64 \times 2} = 8\sqrt{2}$$

Step 4: Rewrite the expression.

$$10\sqrt{2} - 6\sqrt{2} + 8\sqrt{2}$$

Step 5: Combine coefficients from left to right.

$$10 - 6 + 8 = 12$$

$$\boxed{12\sqrt{2}}$$

10. Simplify $3\sqrt{24} + 2\sqrt{54} - \sqrt{96}$.
Step 1: Simplify $3\sqrt{24}$.

$$\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6} \qquad \text{so} \qquad 3\sqrt{24} = 3 \times 2\sqrt{6} = 6\sqrt{6}$$

Step 2: Simplify $2\sqrt{54}$.

$$\sqrt{54} = \sqrt{9 \times 6} = 3\sqrt{6} \qquad \text{so} \qquad 2\sqrt{54} = 2 \times 3\sqrt{6} = 6\sqrt{6}$$

Step 3: Simplify $\sqrt{96}$.

$$\sqrt{96} = \sqrt{16 \times 6} = 4\sqrt{6}$$

Step 4: Rewrite the expression.

$$6\sqrt{6} + 6\sqrt{6} - 4\sqrt{6}$$

Step 5: Combine coefficients from left to right.

$$6 + 6 - 4 = 8$$

$$\boxed{8\sqrt{6}}$$

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