

Multiplying Radicals

ApexMath

Notes

The Big Idea

Multiplying radicals is based on one simple rule that you already saw in the simplifying lesson. Now we use it in the forward direction:

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

When you multiply two radicals, you multiply the numbers inside together and place the result under one radical symbol. Then you simplify the result if possible.

Multiplying Simple Radicals

When both terms are just radicals with no coefficients:

- Multiply the radicands together.
- Place the product under one radical.
- Simplify the result completely.

For example:

$$\sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12} = \sqrt{36} = 6$$

Multiplying Radicals with Coefficients

When each term has a coefficient out front, you handle the two parts separately:

- Multiply the coefficients together.
- Multiply the radicands together.
- Combine and simplify.

For example:

$$3\sqrt{2} \times 4\sqrt{5} = (3 \times 4)\sqrt{2 \times 5} = 12\sqrt{10}$$

A Special Case: Multiplying a Radical by Itself

When you multiply a radical by itself, the result is always the radicand. The radical disappears entirely.

$$\sqrt{a} \times \sqrt{a} = \sqrt{a \times a} = \sqrt{a^2} = a$$

For example:

$$\sqrt{7} \times \sqrt{7} = 7$$

$$3\sqrt{5} \times 3\sqrt{5} = 9 \times 5 = 45$$

Multiplying a Radical by a Whole Number

A whole number can be thought of as a radical with 1 under the radical symbol. More practically, just multiply the whole number by the coefficient of the radical.

$$4 \times 3\sqrt{6} = 12\sqrt{6}$$

Multiplying Binomials Involving Radicals

Sometimes you will need to multiply two expressions that each have two terms, such as:

$$(2 + \sqrt{3})(1 + \sqrt{3})$$

Use the distributive property, multiplying each term in the first bracket by each term in the second bracket. This is the same process as expanding two brackets in algebra (FOIL).

$$\begin{aligned}(2 + \sqrt{3})(1 + \sqrt{3}) &= 2 \times 1 + 2 \times \sqrt{3} + \sqrt{3} \times 1 + \sqrt{3} \times \sqrt{3} \\ &= 2 + 2\sqrt{3} + \sqrt{3} + 3 \\ &= 5 + 3\sqrt{3}\end{aligned}$$

Always collect like terms at the end.

Always Simplify the Final Answer

After multiplying, always check whether the radical in your answer can be simplified. For example:

$$\begin{aligned}\sqrt{6} \times \sqrt{6} &= \sqrt{36} = 6 \\ 2\sqrt{3} \times \sqrt{6} &= 2\sqrt{18} = 2 \times 3\sqrt{2} = 6\sqrt{2}\end{aligned}$$

Pro tip

- Always multiply the coefficients separately from the radicands. Keep them organized to avoid errors.
- After multiplying, always check whether the resulting radical can be simplified. Forgetting this step is the most common mistake.
- When a radical multiplies itself, the answer is simply the radicand. This is a shortcut worth remembering: $\sqrt{n} \times \sqrt{n} = n$.
- For binomial multiplication, write out every product term by term before collecting like terms. Skipping steps leads to missed terms.

Examples

Example A

Simplify $\sqrt{5} \times \sqrt{20}$.

Step 1: Multiply the radicands.

$$\sqrt{5} \times \sqrt{20} = \sqrt{5 \times 20} = \sqrt{100}$$

Step 2: Evaluate $\sqrt{100}$.

$$\sqrt{100} = 10$$

Example B

Simplify $3\sqrt{2} \times 5\sqrt{8}$.

Step 1: Multiply the coefficients.

$$3 \times 5 = 15$$

Step 2: Multiply the radicands.

$$\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16}$$

Step 3: Evaluate $\sqrt{16} = 4$.

Step 4: Multiply the results together.

$$15 \times 4 = 60$$

Example C

Simplify $2\sqrt{3} \times 4\sqrt{15}$.

Step 1: Multiply the coefficients.

$$2 \times 4 = 8$$

Step 2: Multiply the radicands.

$$\sqrt{3} \times \sqrt{15} = \sqrt{3 \times 15} = \sqrt{45}$$

Step 3: Simplify $\sqrt{45}$.

$$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

Step 4: Multiply the coefficient from Step 1 by the result from Step 3.

$$8 \times 3\sqrt{5} = 24\sqrt{5}$$

Example D

Expand and simplify $(1 + \sqrt{5})(3 + \sqrt{5})$.

Step 1: Multiply each term in the first bracket by each term in the second bracket.

$$1 \times 3 + 1 \times \sqrt{5} + \sqrt{5} \times 3 + \sqrt{5} \times \sqrt{5}$$

Step 2: Simplify each product.

$$3 + \sqrt{5} + 3\sqrt{5} + 5$$

Step 3: Collect the whole number terms.

$$3 + 5 = 8$$

Step 4: Collect the like radical terms.

$$\sqrt{5} + 3\sqrt{5} = 4\sqrt{5}$$

Step 5: Write the final answer.

$$8 + 4\sqrt{5}$$

Example E

Expand and simplify $(3 - \sqrt{2})(3 + \sqrt{2})$.

Step 1: Multiply each term in the first bracket by each term in the second bracket.

$$3 \times 3 + 3 \times \sqrt{2} + (-\sqrt{2}) \times 3 + (-\sqrt{2}) \times \sqrt{2}$$

Step 2: Simplify each product.

$$9 + 3\sqrt{2} - 3\sqrt{2} - 2$$

Step 3: Collect the whole number terms.

$$9 - 2 = 7$$

Step 4: Collect the radical terms.

$$3\sqrt{2} - 3\sqrt{2} = 0$$

Step 5: Write the final answer.

$$7$$

Note: This is called a **difference of squares** pattern: $(a - b)(a + b) = a^2 - b^2$. When applied to radicals, the radical always disappears.

Common mistakes

- **Forgetting to simplify after multiplying.** After you multiply the radicands, always check if the result can be simplified. $\sqrt{18}$ is not a final answer — it simplifies to $3\sqrt{2}$.
- **Multiplying the coefficient into the radical.** $3\sqrt{2}$ means $3 \times \sqrt{2}$. The 3 is outside the radical. Do not write $\sqrt{6}$ when you mean $3\sqrt{2}$.
- **Missing terms when expanding binomials.** Each term in the first bracket must multiply every term in the second bracket. Write out all four products before simplifying.
- **Sign errors in binomial expansion.** Watch negative signs carefully, especially when a term like $-\sqrt{2}$ is multiplied by another negative.

Worksheet

1. Multiply and simplify each expression.

(a) $\sqrt{3} \times \sqrt{3}$

(b) $\sqrt{4} \times \sqrt{9}$

(c) $\sqrt{5} \times \sqrt{45}$

(d) $\sqrt{6} \times \sqrt{6}$

2. Multiply and simplify each expression.

(a) $2\sqrt{3} \times 3\sqrt{3}$

(b) $4\sqrt{5} \times 2\sqrt{5}$

(c) $3\sqrt{2} \times 5\sqrt{8}$

(d) $2\sqrt{6} \times 3\sqrt{6}$

3. Multiply and simplify each expression.

(a) $\sqrt{2} \times \sqrt{18}$

(b) $\sqrt{3} \times \sqrt{27}$

(c) $2\sqrt{5} \times 3\sqrt{20}$

(d) $4\sqrt{3} \times 2\sqrt{12}$

4. Expand and simplify each expression.

(a) $\sqrt{2}(3 + \sqrt{2})$

(b) $\sqrt{3}(2\sqrt{3} - 5)$

(c) $2\sqrt{5}(\sqrt{5} + \sqrt{20})$

5. Expand and simplify each binomial product.

(a) $(1 + \sqrt{3})(2 + \sqrt{3})$

(b) $(2 + \sqrt{5})(2 - \sqrt{5})$

(c) $(\sqrt{6} + 1)(\sqrt{6} - 4)$

6. Expand and simplify $(3 + \sqrt{2})^2$.

Hint: $(a + b)^2 = (a + b)(a + b)$. Expand as a binomial product.

7. Expand and simplify $(5 - \sqrt{3})(5 + \sqrt{3})$.

8. Expand and simplify $(\sqrt{2} + \sqrt{5})(\sqrt{2} + \sqrt{5})$.

9. **Challenge:** Expand and simplify $(2\sqrt{3} + \sqrt{6})(3\sqrt{3} - 2\sqrt{6})$.

10. **Challenge:** Expand and simplify $(\sqrt{5} + \sqrt{2})^2 - (\sqrt{5} - \sqrt{2})^2$.

Answer Key

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1.

(a) $\sqrt{3} \times \sqrt{3}$

Step 1: A radical multiplied by itself equals the radicand.

$$\sqrt{3} \times \sqrt{3} = \sqrt{3 \times 3} = \sqrt{9} = 3$$

$$\boxed{3}$$

(b) $\sqrt{4} \times \sqrt{9}$

Step 1: Multiply the radicands.

$$\sqrt{4} \times \sqrt{9} = \sqrt{4 \times 9} = \sqrt{36}$$

Step 2: Evaluate.

$$\sqrt{36} = 6$$

$$\boxed{6}$$

(c) $\sqrt{5} \times \sqrt{45}$

Step 1: Multiply the radicands.

$$\sqrt{5} \times \sqrt{45} = \sqrt{5 \times 45} = \sqrt{225}$$

Step 2: Evaluate.

$$\sqrt{225} = 15$$

$$\boxed{15}$$

(d) $\sqrt{6} \times \sqrt{6}$

Step 1: A radical multiplied by itself equals the radicand.

$$\sqrt{6} \times \sqrt{6} = \sqrt{6 \times 6} = \sqrt{36} = 6$$

$$\boxed{6}$$

2.

(a) $2\sqrt{3} \times 3\sqrt{3}$

Step 1: Multiply the coefficients.

$$2 \times 3 = 6$$

Step 2: Multiply the radicands.

$$\sqrt{3} \times \sqrt{3} = 3$$

Step 3: Multiply the results.

$$6 \times 3 = 18$$

$$\boxed{18}$$

(b) $4\sqrt{5} \times 2\sqrt{5}$

Step 1: Multiply the coefficients.

$$4 \times 2 = 8$$

Step 2: Multiply the radicands.

$$\sqrt{5} \times \sqrt{5} = 5$$

Step 3: Multiply the results.

$$8 \times 5 = 40$$

$$\boxed{40}$$

(c) $3\sqrt{2} \times 5\sqrt{8}$

Step 1: Multiply the coefficients.

$$3 \times 5 = 15$$

Step 2: Multiply the radicands.

$$\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$$

Step 3: Multiply the results.

$$15 \times 4 = 60$$

$$\boxed{60}$$

(d) $2\sqrt{6} \times 3\sqrt{6}$

Step 1: Multiply the coefficients.

$$2 \times 3 = 6$$

Step 2: Multiply the radicands.

$$\sqrt{6} \times \sqrt{6} = 6$$

Step 3: Multiply the results.

$$6 \times 6 = 36$$

$$\boxed{36}$$

3.

(a) $\sqrt{2} \times \sqrt{18}$

Step 1: Multiply the radicands.

$$\sqrt{2} \times \sqrt{18} = \sqrt{2 \times 18} = \sqrt{36}$$

Step 2: Evaluate.

$$\sqrt{36} = 6$$
$$\boxed{6}$$

(b) $\sqrt{3} \times \sqrt{27}$

Step 1: Multiply the radicands.

$$\sqrt{3} \times \sqrt{27} = \sqrt{3 \times 27} = \sqrt{81}$$

Step 2: Evaluate.

$$\sqrt{81} = 9$$
$$\boxed{9}$$

(c) $2\sqrt{5} \times 3\sqrt{20}$

Step 1: Multiply the coefficients.

$$2 \times 3 = 6$$

Step 2: Multiply the radicands.

$$\sqrt{5} \times \sqrt{20} = \sqrt{100} = 10$$

Step 3: Multiply the results.

$$6 \times 10 = 60$$
$$\boxed{60}$$

(d) $4\sqrt{3} \times 2\sqrt{12}$

Step 1: Multiply the coefficients.

$$4 \times 2 = 8$$

Step 2: Multiply the radicands.

$$\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$$

Step 3: Multiply the results.

$$8 \times 6 = 48$$
$$\boxed{48}$$

4.

(a) $\sqrt{2}(3 + \sqrt{2})$

Step 1: Distribute $\sqrt{2}$ to each term inside the brackets.

$$\sqrt{2} \times 3 + \sqrt{2} \times \sqrt{2}$$

Step 2: Simplify each product.

$$3\sqrt{2} + 2$$

Step 3: Write in standard form (whole number first).

$$\boxed{2 + 3\sqrt{2}}$$

(b) $\sqrt{3}(2\sqrt{3} - 5)$

Step 1: Distribute $\sqrt{3}$ to each term inside the brackets.

$$\sqrt{3} \times 2\sqrt{3} + \sqrt{3} \times (-5)$$

Step 2: Simplify each product.

$$2 \times 3 - 5\sqrt{3} = 6 - 5\sqrt{3}$$

$$\boxed{6 - 5\sqrt{3}}$$

(c) $2\sqrt{5}(\sqrt{5} + \sqrt{20})$

Step 1: Simplify $\sqrt{20}$ first.

$$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

Step 2: Rewrite the expression.

$$2\sqrt{5}(\sqrt{5} + 2\sqrt{5}) = 2\sqrt{5}(3\sqrt{5})$$

Step 3: Multiply.

$$2\sqrt{5} \times 3\sqrt{5} = 6 \times 5 = 30$$

$$\boxed{30}$$

5.

(a) $(1 + \sqrt{3})(2 + \sqrt{3})$

Step 1: Expand using the distributive property.

$$1 \times 2 + 1 \times \sqrt{3} + \sqrt{3} \times 2 + \sqrt{3} \times \sqrt{3}$$

Step 2: Simplify each term.

$$2 + \sqrt{3} + 2\sqrt{3} + 3$$

Step 3: Collect whole number terms.

$$2 + 3 = 5$$

Step 4: Collect like radical terms.

$$\sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$$

Step 5: Write the final answer.

$$\boxed{5 + 3\sqrt{3}}$$

(b) $(2 + \sqrt{5})(2 - \sqrt{5})$

Step 1: Expand using the distributive property.

$$2 \times 2 + 2 \times (-\sqrt{5}) + \sqrt{5} \times 2 + \sqrt{5} \times (-\sqrt{5})$$

Step 2: Simplify each term.

$$4 - 2\sqrt{5} + 2\sqrt{5} - 5$$

Step 3: Collect whole number terms.

$$4 - 5 = -1$$

Step 4: Collect like radical terms.

$$-2\sqrt{5} + 2\sqrt{5} = 0$$

Step 5: Write the final answer.

$$\boxed{-1}$$

(c) $(\sqrt{6} + 1)(\sqrt{6} - 4)$

Step 1: Expand using the distributive property.

$$\sqrt{6} \times \sqrt{6} + \sqrt{6} \times (-4) + 1 \times \sqrt{6} + 1 \times (-4)$$

Step 2: Simplify each term.

$$6 - 4\sqrt{6} + \sqrt{6} - 4$$

Step 3: Collect whole number terms.

$$6 - 4 = 2$$

Step 4: Collect like radical terms.

$$-4\sqrt{6} + \sqrt{6} = -3\sqrt{6}$$

Step 5: Write the final answer.

$$\boxed{2 - 3\sqrt{6}}$$

6. Expand and simplify $(3 + \sqrt{2})^2$.

Step 1: Rewrite as a binomial product.

$$(3 + \sqrt{2})(3 + \sqrt{2})$$

Step 2: Expand using the distributive property.

$$3 \times 3 + 3 \times \sqrt{2} + \sqrt{2} \times 3 + \sqrt{2} \times \sqrt{2}$$

Step 3: Simplify each term.

$$9 + 3\sqrt{2} + 3\sqrt{2} + 2$$

Step 4: Collect whole number terms.

$$9 + 2 = 11$$

Step 5: Collect like radical terms.

$$3\sqrt{2} + 3\sqrt{2} = 6\sqrt{2}$$

Step 6: Write the final answer.

$$\boxed{11 + 6\sqrt{2}}$$

7. Expand and simplify $(5 - \sqrt{3})(5 + \sqrt{3})$.

Step 1: Expand using the distributive property.

$$5 \times 5 + 5 \times \sqrt{3} + (-\sqrt{3}) \times 5 + (-\sqrt{3}) \times \sqrt{3}$$

Step 2: Simplify each term.

$$25 + 5\sqrt{3} - 5\sqrt{3} - 3$$

Step 3: Collect whole number terms.

$$25 - 3 = 22$$

Step 4: Collect like radical terms.

$$5\sqrt{3} - 5\sqrt{3} = 0$$

Step 5: Write the final answer.

$$\boxed{22}$$

8. Expand and simplify $(\sqrt{2} + \sqrt{5})(\sqrt{2} + \sqrt{5})$.

Step 1: Rewrite as a binomial product.

$$(\sqrt{2} + \sqrt{5})(\sqrt{2} + \sqrt{5})$$

Step 2: Expand using the distributive property.

$$\sqrt{2} \times \sqrt{2} + \sqrt{2} \times \sqrt{5} + \sqrt{5} \times \sqrt{2} + \sqrt{5} \times \sqrt{5}$$

Step 3: Simplify each term.

$$2 + \sqrt{10} + \sqrt{10} + 5$$

Step 4: Collect whole number terms.

$$2 + 5 = 7$$

Step 5: Collect like radical terms.

$$\sqrt{10} + \sqrt{10} = 2\sqrt{10}$$

Step 6: Write the final answer.

$$\boxed{7 + 2\sqrt{10}}$$

9. Expand and simplify $(2\sqrt{3} + \sqrt{6})(3\sqrt{3} - 2\sqrt{6})$.

Step 1: Expand using the distributive property.

$$2\sqrt{3} \times 3\sqrt{3} + 2\sqrt{3} \times (-2\sqrt{6}) + \sqrt{6} \times 3\sqrt{3} + \sqrt{6} \times (-2\sqrt{6})$$

Step 2: Simplify each term one at a time.

First term:

$$2\sqrt{3} \times 3\sqrt{3} = 6 \times 3 = 18$$

Second term:

$$2\sqrt{3} \times (-2\sqrt{6}) = -4\sqrt{18} = -4 \times 3\sqrt{2} = -12\sqrt{2}$$

Third term:

$$\sqrt{6} \times 3\sqrt{3} = 3\sqrt{18} = 3 \times 3\sqrt{2} = 9\sqrt{2}$$

Fourth term:

$$\sqrt{6} \times (-2\sqrt{6}) = -2 \times 6 = -12$$

Step 3: Collect all terms.

$$18 - 12\sqrt{2} + 9\sqrt{2} - 12$$

Step 4: Collect whole number terms.

$$18 - 12 = 6$$

Step 5: Collect like radical terms.

$$-12\sqrt{2} + 9\sqrt{2} = -3\sqrt{2}$$

Step 6: Write the final answer.

$$\boxed{6 - 3\sqrt{2}}$$

10. Expand and simplify $(\sqrt{5} + \sqrt{2})^2 - (\sqrt{5} - \sqrt{2})^2$.

Step 1: Expand $(\sqrt{5} + \sqrt{2})^2$.

$$(\sqrt{5} + \sqrt{2})(\sqrt{5} + \sqrt{2}) = 5 + \sqrt{10} + \sqrt{10} + 2 = 7 + 2\sqrt{10}$$

Step 2: Expand $(\sqrt{5} - \sqrt{2})^2$.

$$(\sqrt{5} - \sqrt{2})(\sqrt{5} - \sqrt{2}) = 5 - \sqrt{10} - \sqrt{10} + 2 = 7 - 2\sqrt{10}$$

Step 3: Subtract the second result from the first.

$$(7 + 2\sqrt{10}) - (7 - 2\sqrt{10})$$

Step 4: Distribute the negative sign carefully.

$$7 + 2\sqrt{10} - 7 + 2\sqrt{10}$$

Step 5: Collect whole number terms.

$$7 - 7 = 0$$

Step 6: Collect like radical terms.

$$2\sqrt{10} + 2\sqrt{10} = 4\sqrt{10}$$

Step 7: Write the final answer.

$$\boxed{4\sqrt{10}}$$

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