Simplifying Radicals
ApexMath

Notes

The Big Idea

Not every radical gives a clean whole number answer. When that happens, we do not just leave it as is — we **simplify** it. Simplifying a radical means pulling out any perfect square factors that are hiding inside the radicand.

The goal is to write the radical in its **simplest form**: a whole number on the outside and the smallest possible number left on the inside.

The Product Rule for Radicals

The key rule that makes simplifying possible is:

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

This means you can split a radical into two separate radicals, as long as you are multiplying inside. We use this in reverse — we look for a perfect square factor inside the radicand, split it off, and evaluate it.

The Method: Find the Largest Perfect Square Factor

To simplify $\sqrt{n}$, follow these steps every time:

- Step 1: Look at the radicand. Find the **largest perfect square** that divides evenly into it.
- Step 2: Rewrite the radicand as a product of that perfect square and the remaining factor.
- Step 3: Split the radical using the product rule.
- Step 4: Evaluate the perfect square root. Write the result on the outside.
- Step 5: Leave the remaining factor under the radical.

Example of the Method

Simplify $\sqrt{72}$.

Step 1: What is the largest perfect square that divides into 72?

$$36 \times 2 = 72 \qquad \text{and} \qquad 36 \text{ is a perfect square}$$

Step 2: Rewrite.

$$\sqrt{72} = \sqrt{36 \times 2}$$

Step 3: Split using the product rule.

$$\sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2}$$

Step 4: Evaluate $\sqrt{36}$.

$$\sqrt{36} = 6$$

Step 5: Write the simplified form.

$$\sqrt{72} = 6\sqrt{2}$$

What if You Miss the Largest Factor?

If you do not find the largest perfect square right away, that is okay. You can simplify in multiple steps. For example, with $\sqrt{72}$, you might notice 4 first:

$$\sqrt{72} = \sqrt{4 \times 18} = 2\sqrt{18}$$

But $\sqrt{18}$ can still be simplified because $18 = 9 \times 2$:

$$2\sqrt{18} = 2 \times \sqrt{9 \times 2} = 2 \times 3\sqrt{2} = 6\sqrt{2}$$

You still arrive at the same answer. However, finding the **largest** perfect square factor first saves you steps.

When is a Radical Fully Simplified?

A radical $\sqrt{n}$ is fully simplified when the radicand has **no perfect square factors** other than 1. For example:

- $\sqrt{5}$ is already simplified. 5 has no perfect square factors.
- $\sqrt{12}$ is not simplified. $12 = 4 \times 3$, and 4 is a perfect square.
- $3\sqrt{7}$ is fully simplified. 7 has no perfect square factors.

Pro tip

- Always look for the **largest** perfect square factor first. It gives you the answer in one step instead of two or three.
- After simplifying, always check your answer. Multiply the outside number by itself and then by the inside number. You should get the original radicand. For example: $6\sqrt{2}$ means $6^2 \times 2 = 36 \times 2 = 72$. ✓
- If the radicand is odd, it cannot have 4, 16, 36, or any even perfect square as a factor. This narrows your search quickly.
- Perfect squares to keep in mind: $4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225$.

Examples

Example A

Simplify $\sqrt{50}$.

Step 1: Find the largest perfect square factor of 50.

$$25 \times 2 = 50 \qquad \text{and} \qquad 25 \text{ is a perfect square}$$

Step 2: Rewrite the radicand.

$$\sqrt{50} = \sqrt{25 \times 2}$$

Step 3: Split using the product rule.

$$\sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2}$$

Step 4: Evaluate $\sqrt{25}$.

$$\sqrt{25} = 5$$

Step 5: Write the simplified form.

$$\sqrt{50} = 5\sqrt{2}$$

Check: $5^2 \times 2 = 25 \times 2 = 50$ ✓

Example B

Simplify $\sqrt{48}$.

Step 1: Find the largest perfect square factor of 48.

$$16 \times 3 = 48 \qquad \text{and} \qquad 16 \text{ is a perfect square}$$

Step 2: Rewrite the radicand.

$$\sqrt{48} = \sqrt{16 \times 3}$$

Step 3: Split using the product rule.

$$\sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3}$$

Step 4: Evaluate $\sqrt{16}$.

$$\sqrt{16} = 4$$

Step 5: Write the simplified form.

$$\sqrt{48} = 4\sqrt{3}$$

Check: $4^2 \times 3 = 16 \times 3 = 48$ ✓

Example C

Simplify $\sqrt{200}$.

Step 1: Find the largest perfect square factor of 200.

$$100 \times 2 = 200 \qquad \text{and} \qquad 100 \text{ is a perfect square}$$

Step 2: Rewrite the radicand.

$$\sqrt{200} = \sqrt{100 \times 2}$$

Step 3: Split using the product rule.

$$\sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2}$$

Step 4: Evaluate $\sqrt{100}$.

$$\sqrt{100} = 10$$

Step 5: Write the simplified form.

$$\sqrt{200} = 10\sqrt{2}$$

Check: $10^2 \times 2 = 100 \times 2 = 200$ ✓

Example D

Simplify $3\sqrt{98}$.

Step 1: Focus on simplifying the radical part first. Find the largest perfect square factor of 98.

$$49 \times 2 = 98 \qquad \text{and} \qquad 49 \text{ is a perfect square}$$

Step 2: Rewrite.

$$3\sqrt{98} = 3\sqrt{49 \times 2}$$

Step 3: Split using the product rule.

$$3 \times \sqrt{49} \times \sqrt{2}$$

Step 4: Evaluate $\sqrt{49}$.

$$\sqrt{49} = 7$$

Step 5: Multiply the whole numbers together.

$$3 \times 7\sqrt{2} = 21\sqrt{2}$$

Check: $21^2 \times 2 = 441 \times 2 = 882$. Also $3^2 \times 98 = 9 \times 98 = 882$ ✓

Common mistakes

- **Using a perfect square factor that is not the largest.** This gives a correct but not fully simplified answer. Always check that the remaining radicand has no more perfect square factors.
- **Forgetting to multiply the outside number.** In Example D, students sometimes write $3\sqrt{49 \times 2} = 3 + 7\sqrt{2}$ instead of $3 \times 7\sqrt{2}$. The outside numbers multiply — they do not add.
- **Thinking $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.** This is **wrong**. The product rule only works for multiplication inside the radical, not addition. For example, $\sqrt{9+16} \neq \sqrt{9} + \sqrt{16}$ because $\sqrt{25} = 5$ but $3 + 4 = 7$.
- **Stopping too early.** After one round of simplifying, always check that the new radicand is fully simplified before writing your final answer.

Worksheet

1. Simplify each radical completely.

   (a) $\sqrt{12}$

   (b) $\sqrt{45}$

   (c) $\sqrt{75}$

   (d) $\sqrt{98}$

2. Simplify each radical completely.

   (a) $\sqrt{128}$

(b) $\sqrt{180}$

(c) $\sqrt{243}$

(d) $\sqrt{363}$

3. Simplify each expression completely.

(a) $2\sqrt{18}$

(b) $5\sqrt{20}$

(c) $3\sqrt{50}$

(d) $4\sqrt{75}$

4. Determine whether each radical is already in simplest form. If it is not, simplify it.

(a) $\sqrt{11}$

(b) $\sqrt{60}$

(c) $\sqrt{17}$

(d) $\sqrt{90}$

5. A student simplified $\sqrt{72}$ and got $3\sqrt{8}$. Is this fully simplified? If not, find the correct answer and explain the mistake.

6. Simplify $\sqrt{300}$.

7. Simplify $6\sqrt{48}$.

8. The diagonal of a square with side length 6 cm is $\sqrt{72}$ cm. Write this length in simplest radical form.

9. **Challenge:** Simplify $\sqrt{720}$.

10. **Challenge:** Simplify $2\sqrt{392}$.

Answer Key

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1.

(a) Simplify $\sqrt{12}$.
Step 1: Find the largest perfect square factor of 12.

$$4 \times 3 = 12 \qquad \text{and} \qquad 4 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3}$$

Step 3: Evaluate $\sqrt{4} = 2$ and write the simplified form.

$$\boxed{2\sqrt{3}}$$

(b) Simplify $\sqrt{45}$.
Step 1: Find the largest perfect square factor of 45.

$$9 \times 5 = 45 \qquad \text{and} \qquad 9 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5}$$

Step 3: Evaluate $\sqrt{9} = 3$ and write the simplified form.

$$\boxed{3\sqrt{5}}$$

(c) Simplify $\sqrt{75}$.
Step 1: Find the largest perfect square factor of 75.

$$25 \times 3 = 75 \qquad \text{and} \qquad 25 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3}$$

Step 3: Evaluate $\sqrt{25} = 5$ and write the simplified form.

$$\boxed{5\sqrt{3}}$$

(d) Simplify $\sqrt{98}$.

Step 1: Find the largest perfect square factor of 98.

$$49 \times 2 = 98 \qquad \text{and} \qquad 49 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{98} = \sqrt{49 \times 2} = \sqrt{49} \times \sqrt{2}$$

Step 3: Evaluate $\sqrt{49} = 7$ and write the simplified form.

$$\boxed{7\sqrt{2}}$$

2.

(a) Simplify $\sqrt{128}$.
Step 1: Find the largest perfect square factor of 128.

$$64 \times 2 = 128 \qquad \text{and} \qquad 64 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{128} = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2}$$

Step 3: Evaluate $\sqrt{64} = 8$ and write the simplified form.

$$\boxed{8\sqrt{2}}$$

(b) Simplify $\sqrt{180}$.
Step 1: Find the largest perfect square factor of 180.

$$36 \times 5 = 180 \qquad \text{and} \qquad 36 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{180} = \sqrt{36 \times 5} = \sqrt{36} \times \sqrt{5}$$

Step 3: Evaluate $\sqrt{36} = 6$ and write the simplified form.

$$\boxed{6\sqrt{5}}$$

(c) Simplify $\sqrt{243}$.
Step 1: Find the largest perfect square factor of 243.

$$81 \times 3 = 243 \qquad \text{and} \qquad 81 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{243} = \sqrt{81 \times 3} = \sqrt{81} \times \sqrt{3}$$

Step 3: Evaluate $\sqrt{81} = 9$ and write the simplified form.

$$\boxed{9\sqrt{3}}$$

(d) Simplify $\sqrt{363}$.
Step 1: Find the largest perfect square factor of 363.

$$121 \times 3 = 363 \qquad \text{and} \qquad 121 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{363} = \sqrt{121 \times 3} = \sqrt{121} \times \sqrt{3}$$

Step 3: Evaluate $\sqrt{121} = 11$ and write the simplified form.

$$\boxed{11\sqrt{3}}$$

3.

(a) Simplify $2\sqrt{18}$.
Step 1: Find the largest perfect square factor of 18.

$$9 \times 2 = 18 \qquad \text{and} \qquad 9 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$2\sqrt{18} = 2 \times \sqrt{9 \times 2} = 2 \times \sqrt{9} \times \sqrt{2}$$

Step 3: Evaluate $\sqrt{9} = 3$.

$$2 \times 3 \times \sqrt{2}$$

Step 4: Multiply the whole numbers.

$$2 \times 3 = 6$$

$$\boxed{6\sqrt{2}}$$

(b) Simplify $5\sqrt{20}$.
Step 1: Find the largest perfect square factor of 20.

$$4 \times 5 = 20 \qquad \text{and} \qquad 4 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$5\sqrt{20} = 5 \times \sqrt{4 \times 5} = 5 \times \sqrt{4} \times \sqrt{5}$$

Step 3: Evaluate $\sqrt{4} = 2$.

$$5 \times 2 \times \sqrt{5}$$

Step 4: Multiply the whole numbers.

$$5 \times 2 = 10$$

$$\boxed{10\sqrt{5}}$$

(c) Simplify $3\sqrt{50}$.
Step 1: Find the largest perfect square factor of 50.

$$25 \times 2 = 50 \qquad \text{and} \qquad 25 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$3\sqrt{50} = 3 \times \sqrt{25 \times 2} = 3 \times \sqrt{25} \times \sqrt{2}$$

Step 3: Evaluate $\sqrt{25} = 5$.

$$3 \times 5 \times \sqrt{2}$$

Step 4: Multiply the whole numbers.

$$3 \times 5 = 15$$

$$\boxed{15\sqrt{2}}$$

(d) Simplify $4\sqrt{75}$.
Step 1: Find the largest perfect square factor of 75.

$$25 \times 3 = 75 \qquad \text{and} \qquad 25 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$4\sqrt{75} = 4 \times \sqrt{25 \times 3} = 4 \times \sqrt{25} \times \sqrt{3}$$

Step 3: Evaluate $\sqrt{25} = 5$.

$$4 \times 5 \times \sqrt{3}$$

Step 4: Multiply the whole numbers.

$$4 \times 5 = 20$$

$$\boxed{20\sqrt{3}}$$

4.

(a) Is $\sqrt{11}$ in simplest form?
Step 1: Check whether 11 has any perfect square factors. The perfect squares are $4, 9, 16, \ldots$ None of these divide evenly into 11.

Step 2: 11 has no perfect square factors other than 1.

$$\boxed{\sqrt{11} \text{ is already in simplest form.}}$$

(b) Simplify $\sqrt{60}$.
Step 1: Find the largest perfect square factor of 60.

$$4 \times 15 = 60 \qquad \text{and} \qquad 4 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{60} = \sqrt{4 \times 15} = \sqrt{4} \times \sqrt{15}$$

Step 3: Evaluate $\sqrt{4} = 2$. Check that 15 has no perfect square factors ($15 = 3 \times 5$, neither is a perfect square).

$$\boxed{2\sqrt{15}}$$

(c) Is $\sqrt{17}$ in simplest form?
Step 1: Check whether 17 has any perfect square factors. 17 is a prime number, so its only factors are 1 and 17. Neither 4, 9, nor any other perfect square divides into 17.

$$\boxed{\sqrt{17} \text{ is already in simplest form.}}$$

(d) Simplify $\sqrt{90}$.
Step 1: Find the largest perfect square factor of 90.

$$9 \times 10 = 90 \qquad \text{and} \qquad 9 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{90} = \sqrt{9 \times 10} = \sqrt{9} \times \sqrt{10}$$

Step 3: Evaluate $\sqrt{9} = 3$. Check that 10 has no perfect square factors ($10 = 2 \times 5$).

$$\boxed{3\sqrt{10}}$$

5. A student simplified $\sqrt{72}$ and got $3\sqrt{8}$. Is this fully simplified?
Step 1: Check whether the student's answer is correct so far.

$$9 \times 8 = 72 \qquad \sqrt{9} = 3 \qquad \text{so } 3\sqrt{8} \text{ is a valid first step}$$

Step 2: Check whether $\sqrt{8}$ can be simplified further.

$$4 \times 2 = 8 \qquad \text{and} \qquad 4 \text{ is a perfect square}$$

Step 3: Simplify $\sqrt{8}$.

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

Step 4: Substitute back.

$$3\sqrt{8} = 3 \times 2\sqrt{2} = 6\sqrt{2}$$

Step 5: State the mistake. The student stopped too early. They used 9 as their perfect square factor instead of 36, which is the largest perfect square factor of 72.

$$\boxed{6\sqrt{2} \quad \text{(the student's answer was not fully simplified)}}$$

6. Simplify $\sqrt{300}$.
Step 1: Find the largest perfect square factor of 300.

$$100 \times 3 = 300 \qquad \text{and} \qquad 100 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{300} = \sqrt{100 \times 3} = \sqrt{100} \times \sqrt{3}$$

Step 3: Evaluate $\sqrt{100} = 10$.

$$\boxed{10\sqrt{3}}$$

7. Simplify $6\sqrt{48}$.
Step 1: Find the largest perfect square factor of 48.

$$16 \times 3 = 48 \qquad \text{and} \qquad 16 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$6\sqrt{48} = 6 \times \sqrt{16 \times 3} = 6 \times \sqrt{16} \times \sqrt{3}$$

Step 3: Evaluate $\sqrt{16} = 4$.

$$6 \times 4 \times \sqrt{3}$$

Step 4: Multiply the whole numbers.

$$6 \times 4 = 24$$

$$\boxed{24\sqrt{3}}$$

8. The diagonal of a square is $\sqrt{72}$ cm. Write in simplest form.
Step 1: Find the largest perfect square factor of 72.

$$36 \times 2 = 72 \qquad \text{and} \qquad 36 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2}$$

Step 3: Evaluate $\sqrt{36} = 6$.

$$\boxed{6\sqrt{2} \text{ cm}}$$

9. Simplify $\sqrt{720}$.
Step 1: Find the largest perfect square factor of 720.

$$144 \times 5 = 720 \qquad \text{and} \qquad 144 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$\sqrt{720} = \sqrt{144 \times 5} = \sqrt{144} \times \sqrt{5}$$

Step 3: Evaluate $\sqrt{144} = 12$.

$$\boxed{12\sqrt{5}}$$

10. Simplify $2\sqrt{392}$.
Step 1: Find the largest perfect square factor of 392.

$$196 \times 2 = 392 \qquad \text{and} \qquad 196 \text{ is a perfect square}$$

Step 2: Rewrite and split.

$$2\sqrt{392} = 2 \times \sqrt{196 \times 2} = 2 \times \sqrt{196} \times \sqrt{2}$$

Step 3: Evaluate $\sqrt{196} = 14$.

$$2 \times 14 \times \sqrt{2}$$

Step 4: Multiply the whole numbers.

$$2 \times 14 = 28$$

$$\boxed{28\sqrt{2}}$$

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